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Fundamentals of Surveying

220 ILLUSTRATIONS

By

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CHAIN SURVEYING
LEVELING
COMPASS SURVEYING
TRANSIT SURVEYING
OFFICE WORK IN ANGULAR SURVEYING
CIRCULAR CURVES
STADIA AND PLANE-TABLE SURVEYING
TOPOGRAPHIC SURVEYING

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CHAIN SURVEYING

CHAINING

INTRODUCTION

1. Definition.—Surveying is the art of determining the relative positions of points and lines on the earth's surface. By the term *the earth's surface* is meant all that part of the earth that can be explored; therefore, the term includes the bottoms of rivers and seas and the interiors of mines, as well as the more accessible points on the actual surface. From the information obtained by the methods of surveying, the locations, sizes, and areas of objects can be found either graphically or by calculation.

2. Classification.—Surveying may be classified, according to its purpose, as land surveying, topographic surveying, railroad surveying, hydrographic surveying, mine surveying, etc. The general principles employed in all these classes of surveying are the same, but according to the methods and instruments used, surveying is divided into three branches, as follows:

1. *Chain*, or *linear*, *surveying*, in which no other measuring instrument is employed than a chain or tape.
2. *Angular surveying*, in which angle-measuring instruments are employed in connection with distance-measuring instruments.
3. *Leveling*, in which the elevations of points or the vertical distances between two or more points are determined.

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Linear surveying without the aid of angle-measuring instruments is seldom practiced, but the process of measuring distances is an important part of angular surveying. Therefore, the methods of measuring with a chain or tape described in this Section will include those used in angular surveying.

3. Instruments.—The measuring instruments used in determining distances are the chain and the tape. In addition, plumb-bobs, range poles, and marking pins are usually employed to facilitate the work.

DESCRIPTION OF INSTRUMENTS

CHAINS

4. Description.—A chain, Fig. 1, is composed of links of steel or iron wire, each two adjacent links being connected by small rings. The best chains are made of No. 12 steel wire and have all joints in the links and rings brazed in order to prevent their opening when a pull is applied to the chain. Some chains have two, and some three, rings between adjoining links. At the ends of the chain are handles, usually made of brass, which are attached to the chain by means of swivels having nuts and threads. Each handle forms part of the end link, the length of the chain extending to the outer edges of the handles; by means of the swivels, the length of the chain can be adjusted. Chains are classified as engineers' chains and surveyors' chains.

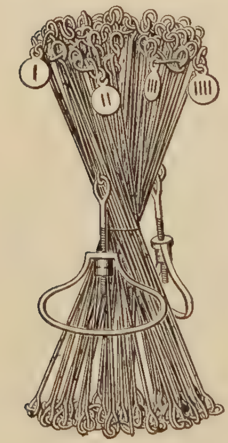


FIG. 1

5. The engineers' chain is 50 or 100 feet long. In each foot there is one link; that is, one link and the rings on one end between that link and the next make a foot. Every tenth link is marked with a brass tag to indicate its distance from the nearer end of the chain. Thus, in the 100-foot chain, shown in Fig. 1, the tags that are 10 feet from either end of the chain are marked |, those at 20 feet | |, those at 30 feet | | |, and at 40 feet | | | |. At 50 feet, which is the middle of the tape, a plain tag, usually of different shape from the others, is used. Therefore, when a measurement is greater than half the length of the chain, the tag marked | | | | next beyond the center will represent 60 feet; and then, continuing in the same direction, tag | | | will be 70 feet; tag | |, 80 feet; and tag |, 90 feet. Hence it is important to observe on which side of the 50-foot point the tag is, in order to get the correct reading.

Engineers' chains were formerly used on all kinds of surveys where the foot was the unit of measurement, but they have been generally superseded by the steel tape.

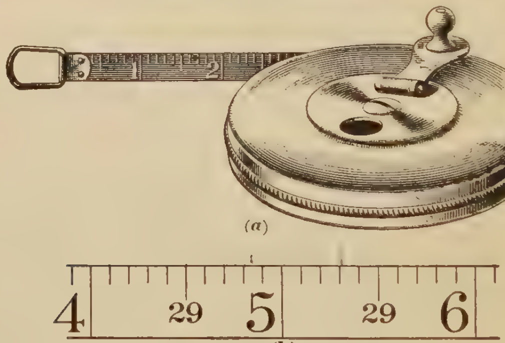
6. The surveyors' chain, often called Gunter's chain, from the name of its inventor, is 66 feet, or 4 rods, long. It is divided into 100 links and, consequently, the length of a link and the connecting rings is .66 foot or 7.92 inches. This type of chain was used in all old United States land surveys, but now specially graduated steel tapes are employed instead. Whenever the word *chain* occurs in a deed, lease, or other legal document, it is understood to mean a surveyors' chain of 66 feet. The advantages of the 66-foot chain as a unit in land surveys are that there are 80 chains in 1 statute mile, and that areas expressed in square chains can be changed to acres by simply moving the decimal point one place to the left, since there are 10 square chains in 1 acre.

7. Folding a Chain.—A chain may be folded either from one end or from the center. To fold it from the center, take the middle pair of links together in the left hand, grasp the third pair of links from the middle with the right hand and fold the second and third pairs of links across the middle links and nearly parallel to them; then grasp the fifth pair and fold the fourth and fifth pairs across and nearly parallel to those already in place; proceed in the same way until the end is reached. The chain should then be secured by a cord or strap around the centers of the links, as shown in Fig. 1. A chain may be folded from one end in a similar manner.

TAPES

8. Classification.—Tapes for measuring are usually made of steel, although cloth tapes and so-called metallic tapes are sometimes used. Since the cloth tape stretches easily and shrinks when wet, it is of little value in surveying. *Metallic tapes* are composed of linen with fine brass threads woven in lengthwise to reduce the stretching. They are made in lengths of 25, 50, 75, and 100 feet and are usually graduated to feet, tenths, and half-tenths of a foot. Their use is limited to short measurements where great accuracy is not required.

Steel tapes are ribbons of steel, varying in width from $\frac{1}{8}$ inch to $\frac{1}{2}$ inch, and are obtainable in various lengths from 1 yard to 1,000 feet. They are graduated in various ways, depending on the purposes for which they are used.



(a)

(b)

Fig. 2

9. Pocket Tapes.—Where many short measurements are taken, a thin tape, 25 feet, 50 feet, or 100 feet long and graduated to feet, tenths of a foot, and hundredths of a foot throughout its length, is most convenient. Such tapes are usually a little less than $\frac{3}{8}$ inch wide and are enclosed in a hard-leather case with a folding crank for winding up the tape. A tape in its case is shown in Fig. 2 (*a*); it can be conveniently carried in a pocket. The graduations are etched on the tape, so that it winds easily. Often the number of the preceding foot is marked on the tape at intervals of a tenth of a foot, as illustrated in Fig. 2 (*b*), which shows part of a tape between 29 and 30 feet from the end. The zero point is generally at the end of the tape, but it is sometimes at the end of the small ring. Before the tape is used, the location of the zero mark must be determined.

The small ring on any tape serves for attaching a handle. A metal handle is best, but a rawhide strip tied through the ring is suitable and gives a good grip. The inner end of the tape is held on the reel by a small pin which fits in a hole in the ribbon. An additional length of ribbon is provided beyond the last graduation so that a small part remains coiled after the graduated part has been unwound.

10. Band Chains.—For general work it is common to use 100-foot tapes of greater thickness than the pocket tapes described in the preceding article. These tapes are graduated every 5 or 10 feet, with the first and last intervals subdivided into feet, and the first and last feet into tenths of a foot. Sometimes the tapes are marked every foot for the entire length. These heavier tapes are usually between $\frac{1}{8}$ and $\frac{1}{4}$ inch in width and are commonly called *band chains*. The graduations are generally marked on small sleeves of brass or copper soldered on the tape or on Babbitt metal brazed on both sides of the tape. In Fig. 3 is shown the graduation 56 feet from the end of a tape. Sometimes small brass or copper rivets are used to mark the feet and every 5-foot or 10-foot mark is numbered. Rivets are not so good as sleeves, since the holes for the rivets weaken the tape. On band chains the last graduations are usually some distance from the ends of the tape, and metal or rawhide handles are attached through the end rings of the tape. When long lines are to be measured and the slope of the ground permits, 300-foot or 500-foot band chains can be used to advantage. The band chains used in land surveying are graduated to links and are 1, 2, 5, or 8 chains in length. Usually, each 5-link mark is numbered.

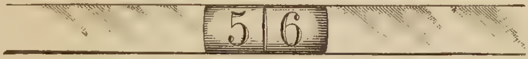


Fig. 3

Band chains are wound on reels of metal or wood. A typical metal reel is shown in Fig. 4, and a wooden reel is shown in Fig. 5. The sides of the guides *a* are just far enough apart to permit one width of the tape to pass between them. In order to hold the tape on the wooden reel shown in Fig. 5 after it has been wound up, it is tied near the ends with cords or leather strips *b*.

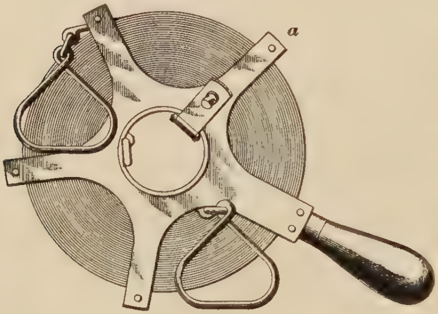


FIG. 4

11. Handling Tape. When a tape is being used, it is unrolled to its full length and detached from the reel. For convenience in carrying the wooden reel when it is empty, the pin *c*, Fig. 5, can be removed and the reel can then be folded. There are many other types of reels, some of which have arm straps to assist in holding the reel when long tapes are being wound. On some reels the sides of the guides are farther apart and permit the tape to spread on the reel in winding.

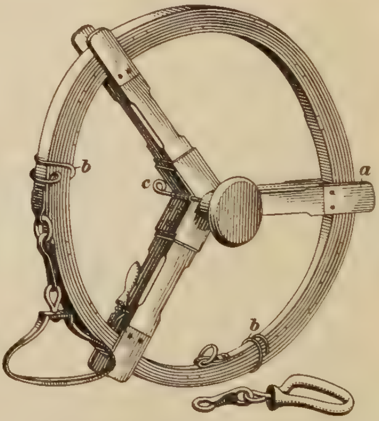


FIG. 5

When only part of the tape is being used, or when the tape is being carried from place to place for measurements, it is usually inconvenient to keep the tape on the reel and to wind and unwind it continually as the occasion requires. If it is not advisable to leave the tape spread out, an easy way to hold the tape when off the reel is to coil it somewhat like a rope. To do this, take the zero end of the tape in the left hand with the graduated face upward and, without twisting the tape, run it through the fingers of the right hand until the 5-foot mark is reached. Then, always keeping the graduations upward, place the 5-foot mark directly over the 0 mark in the left hand, run more of the tape between the fingers of the right hand, and place the 10-foot mark over the 5-foot division. Continue this operation for the entire length of the tape, placing each 5-foot division over the preceding one. Then when the tape is held in the hand at the 5-foot graduations, it will fall in the approximate form of a figure 8. The tape can also be held conveniently at the center of the 8 as shown in Fig. 6. The tape can easily be uncoiled by releasing the loops one at a time as required. Several coils should not be dropped at once, since then the tape is likely to become tangled.

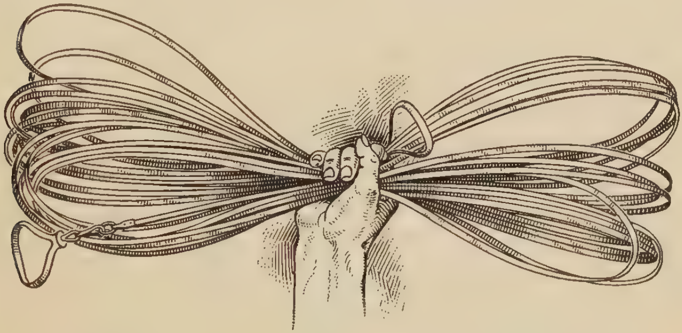


Fig. 6

Cloth tapes should be used only in places where they can be kept clean and dry. They are easily broken and need careful handling. A break can be repaired by stitching on an extra piece. Pocket tapes may be employed in wet or dirty work. But, when exposed to the least bit of moisture, they should be wiped perfectly dry and then rubbed with some light oil, which will not stain the hands. They are fragile and break very easily; if run over by a vehicle or struck by an axe or stake, they are quite likely to be broken.

Band tapes of suitable material can be injured only by gross carelessness. If they are pulled while looped, they are likely to kink and break. They may be used all day in the rain and mud, and they only need to be hung where they will dry during the night. Still many surveyors prefer to clean them.

Pocket and band tapes are mended by attaching an extra piece at the break with two rivets on each side of the joint or by soldering a sleeve over the broken portion. Care should be taken that the proper distance between the two adjoining marks is maintained and that the tape is kept straight. For splicing a tape in the field, for a few hours or even a day or two, a device consisting of a sleeve with a setscrew at each end is sometimes used. Since repaired instruments are seldom as good as before they were damaged, the utmost care should be used to avoid accidents.

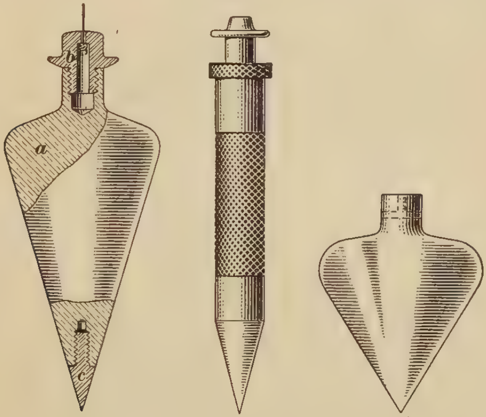
12. Although tapes have practically supplanted chains, the word *chaining* is still commonly used to describe the operation of measuring distances. The methods employed in measuring are substantially the same, no matter what style of tape is used, and, therefore, the term *tape* will here be used in its general meaning unless otherwise stated. Measurements are usually recorded in feet and decimals, but those taken with a tape graduated to links are given in chains and links.

ACCESSORIES

13. Plumb-Bobs.—A plumb-bob can be made by attaching any kind of a weight to the end of a string; when the weight is allowed to hang freely, the line of the string points toward the center of the earth and is vertical, or *plumb*. Typical plumb-bobs for surveying work are shown in Fig. 7. The type commonly used is shown in (*a*); it consists of a brass body *a*, into which are screwed the brass cap *b* and the steel point *c*. Part of the bob is shown in cross-section to illustrate the method of inserting and fastening the string. In view (*b*) is shown a steel bob which is suitable for work near walls or other surfaces and which has also the advantage that there is not so much surface exposed to the wind. The bob shown in (*c*) is made of iron, and is solid except for a hole in the top to allow the string to be fastened. This style of bob is used only for very rough work. In all types of bobs, it is important that the bob should be exactly centered on a line through the point and the hole in the top where the string is inserted. The weight of bobs varies from about 8 ounces to 3 pounds. Light plumb-bobs sway in the wind while heavy ones are inconvenient to carry. A weight of 1 or 1½ pounds is preferred by most surveyors.

The cord for plumb-bobs should be fine and light but should be strong enough to resist a pull in excess of the weight of the bob. A special plumb-bob string is manufactured. For very accurate work piano wire is often employed.

14. Range Poles.—Wooden or steel poles or rods called range poles are placed at points to which measurements or sights are taken when the point cannot be seen otherwise. When made of wood, they are usually 8 or 10 feet long, and are either circular or octagonal in section with a diameter of about 1 inch. A pointed iron shoe is attached to the lower end so that the rod can be stuck in the ground and will then stand erect without being held. Steel rods are 6 or 8 feet long and are usually hexagonal in section with a thickness of about ½ inch. They are pointed at the lower end so that they can be



(a) *(b)* *(c)*

FIG. 7

stuck in the ground in any desired position. Both wooden and metal poles are divided into foot-long spaces painted alternately red and white to make them visible at a long distance. Often a piece of cloth is tied at the top of the pole to make the pole more easily seen and, therefore, range poles are sometimes called *flagpoles*, or simply *flags*, and the men who handle them are called *flagmen*. If it is desired to sight past the pole, the flagman should stand to one side; otherwise, he should stand behind the pole. Poles should be held as nearly plumb as possible. If the pole is not stuck in the ground it can be kept vertical by resting the point on the ground and balancing the pole between the fingers; if the pole is secured in the ground, it can be made to stand vertical by shifting it until it is parallel to a plumb-bob string held near it

15. Marking Pins.—For marking temporarily the ends of measurements, where only the distance between certain points is needed, marking pins are used. These are slender rods of iron or steel about 12 or 14 inches long, pointed at one end and bent into a ring at the other. They can be stuck in the ground where needed. When used in grassy or weedy ground, pieces of cloth are tied to the rings so that the pins can be readily found.

16. Stakes and Monuments.—When the points set in chaining are to be used later, their location should be fixed. For permanent points, concrete or stone monuments are commonly placed in the ground with a cross mark chiseled in the top to indicate the exact point. If the location of the point is needed for a short time only, wooden stakes are driven and left in the ground. A definite point can be marked by a small tack in the top of the stake or by a small cross in pencil. Stakes must be suitably marked for identification, and for this purpose a specially prepared crayon, called *keel*, is commonly used. Often, to aid in identifying a point, an additional stake, called a *witness stake*, is placed near the point.

MEASURING DISTANCES

METHODS OF CHAINING

17. Distances.—Unless otherwise stated, when the distance between two points on the earth's surface is given, as on a map, the horizontal distance is meant. For example, if in Fig. 8 the surface of the ground is represented by the line AB, the distance from A to B is represented on a map as the length AB' along the horizontal line $A\,X$, the line $B\,B'$ being vertical. Therefore, in surveying, the horizontal distance between A and B must be determined.

18. Distances in surveying must be measured in a straight line, as well as horizontally, because a straight line between two points is shorter than any other line and always has the same length. Therefore, when a distance is measured, it is important to see that there are no bends or twists in the tape; if the measurement is made in several parts, the intermediate points should lie on the straight line between the ends.

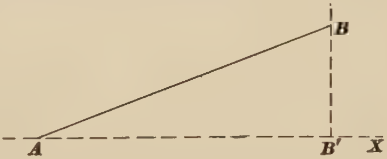


FIG. 8

19. Measuring on Level Ground.—Two men are necessary for all chaining; they are called the *head chainman*, or *front chainman*, and the *rear chainman*, according to their positions in reference to the direction in which the measurement is being taken. If the tape is still on the reel, the head chainman takes the exposed end and walks ahead, while the rear chainman remains at the beginning of the line and holds the reel so that the tape unwinds. If the tape is coiled in 5-foot loops, the head chainman goes ahead with one end, while the rear chainman releases one loop at a time. A chain is undone by holding both handles in one hand and throwing it forcibly with the other hand so that the links will be free from each other. The chain should be thrown in the direction opposite to that in which the measurement is to be made, so that when the head chainman takes one handle and drags the chain ahead, the rear chainman can stand at the point of beginning and straighten out any kinks in the chain as it is drawn past him.

When almost all of the tape or chain has been drawn past, the rear chainman calls, "Chain," and the head chainman stops and straightens out the tape. The rear chainman holds his end of the tape firmly at the stake or pin that marks the beginning of the measurement, and the head chainman holds a stake or pin at his end of the tape, keeping his body to one side of the line. The tape is then pulled taut, and the rear chainman, sighting along the line from the point he occupies to a range pole or some other object that determines the direction, signals to the head chainman to move to the right or left as required, until his stake or pin is on line. When the tape is taut and straight and the stake or pin held by the head chainman is on line, the rear chainman calls, "All right." The head chainman then places the stake or pin in the ground and replies, "All right." When the line to be measured is longer than one tape length, the operation is repeated as often as required.

The length of any line can thus be determined by a series of measurements similar to that just described. For each measurement the rear chainman holds his end of the tape at the point marking the end of the last preceding tape length, and lines in the head chainman, who sets the next point ahead. When the end of the line is reached, the head chainman walks on past that point with the tape until the rear end is at the last stake or pin. He then returns, and the men measure the distance from the last pin to the end of the line. Sometimes the head chainman holds his end of the tape at the end of the line and the rear chainman takes the reading opposite the last pin, it thus being unnecessary for the head chainman to walk past the end of the line. The total length of the line will be equal to a number of full tape lengths, plus a single partial tape length at the end. In order to avoid mistakes in the number of full tape lengths, it is important to have a system of counting them by which the result can be readily checked.

20. When a stake is placed at each tape length, it is numbered so that the distance from the beginning of the line is shown. Thus the starting point is called 0, and when the head chainman has placed the stake at the end of the first tape length, he marks it with the figure 1. When the rear chainman arrives at this stake to start the second measurement he calls, "One;" the head chainman sets the next stake and, after announcing, "Two," so that the rear chainman can correct him if it is not the proper number, he marks the stake 2. This operation is repeated at each measurement and thus the distance to the last stake is always known. Then the length of the partial measurement at the end of the line can be added to the distance represented by the number of full tape lengths. For example, if the stake at the end of the last full 100-foot tape length is marked 8 and the distance from this stake to the end of the line is found to be 36 feet, the length of the line equals $8\times100+36=836$ feet.

21. If chaining pins are used to mark the ends of measurements, it is convenient to use a set of eleven pins. When the head chainman starts, he carries ten pins, leaving the eleventh to mark the beginning of the line; or, if the starting point is otherwise located, the eleventh pin is left with the rear chainman. The rear chainman pulls out each pin after the measurement from it has been taken. When ten pins have been set, the head chainman calls, "Tally." He then receives from the rear chainman ten pins with which to start again, the eleventh pin being left in the ground to mark the end of the tenth tape length. Each tally should be recorded, either by writing or in some other convenient manner. By counting the number of tallies and the number of pins in the possession of the rear chainman, exclusive of the one in the ground, the distance to the last pin is calculated; then the distance from the last pin to the end of the line is added.

Example 1.—A 100-foot tape is used and there have been 3 tallies, the rear chainman has 6 pins, and the distance from the last pin to the end of the line is 37.8 feet. Find the length of the line.

Solution.—Each tally represents 10 full tape lengths and each pin held by the rear chainman represents 1 full tape length. Hence, the total number of full tape lengths equals $3\times10+6=36$ and the distance to the last pin equals $36\times100=3{,}600$ ft. Since the distance from the last pin to the end of the line is 37.8 ft., the length of the line equals $3{,}600+37.8=3{,}637.8$ ft. Ans.

Example 2.—What distance has been measured with a tape, 1 chain long, if 4 tallies have been recorded, the rear chainman has 7 pins, and the last partial measurement is 47 links?

Solution.—The 4 tallies represent 4×10 or 40 ch. and the 7 pins represent 7 ch.; each link is .01 ch. and 47 links are .47 ch. Hence, the total distance equals $40+7+.47=47.47$ ch. Ans.

22. Reading Tape for Partial Measurements.—In measuring a line by chaining, it is usually necessary at the end to take a measurement that is not a full tape length. In many cases, intermediate partial measurements are also required because of the impracticability of setting a point at the full tape length; for instance, the point may come in a stream or other inaccessible place. It is also common to locate various objects near a line by short measurements from points along the line. These points are seldom at the end of a full tape length. In general work, almost all measurements are partial tape lengths.

The method of measuring a distance less than a full tape length is the same as for a tape length, except that the rear chainman holds some graduation at his point instead of the end of the tape. When a tape divided in links is used, the measurement is usually taken to the nearest link, and the number of links can be readily determined by counting from one of the 5-link graduations. For example, if the head chainman holds the zero end of the tape, and the point occupied by the rear chainman is near the third link past the 40-link mark the measurement is 40+3, or 43, links. If tenths of a link are desired, they can be estimated by eye; the value will be close enough, since these tapes are never used in accurate work.

If the tape is graduated to feet, tenths, and hundredths throughout its length, the measurement to hundredths of a foot is read directly on the tape; if desired, thousandths of a foot can be estimated with a little practice. If a band chain, graduated to feet, and the end foot to tenths, is employed, the rear chainman holds some foot mark at his point so that the end of the measurement falls within the end foot held by the head chainman, as shown in Fig. 9, where *A* is the end of the line to be measured. Then the head chainman reads the number of tenths, and, if desired, estimates the hundredths, from the end of the tape to the end of the measurement; and this value is subtracted from the number of feet indicated by the graduation held by the rear chainman. For example, suppose the rear chainman holds the 54-foot graduation at his point and the other end of the measurement falls between 7 tenths and 8 tenths at the zero end of the tape, as in Fig. 9. If the head chainman estimates that the reading is 73 hundredths, the length of the measurement is recorded as 54−.73, or 53.27 feet.

23. Stations.—Important points on a survey line are called *stations*. These may be at the ends of tape lengths, as mentioned in Art. **20**, or at such definite points as may be

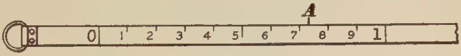


FIG. 9

required for future reference. In the latter case they are marked permanently by stakes or monuments. Stations may be identified by giving them either letters or numbers, but the method generally used for referencing is to number stations according to the hundreds of feet they are distant from the starting point of the survey; these distances are measured along the line of the survey. Thus, the starting point is Station 0, a point 400 feet from the start is Station 4, and a point 1,200 feet from the start is Station 12. A point whose distance from Station 0 is not a multiple of 100 feet is marked by the number of the 100-foot station immediately preceding, plus the distance from that station to the point in question. For example, a point between Stations 5 and 6, and 47 feet from 5, is marked 5+47 and is called Station 5 plus 47. Its distance from Station 0 is 547 feet.

EXAMPLE.—Find the distance in feet between Stations 3+76.4 and 6+13.1.

Solution.—Sta. 6+13.1 is 613.1 ft. from the beginning of the line and Sta. 3+76.4 is 376.4 ft. from the start. Hence, the distance between the given points equals 613.1−376.4=236.7 ft. Ans.

24. Measurement on Sloping Ground.—Very often the two points on the ground between which a measurement is taken, as *A* and *B* in Fig. 10, are at different elevations. One method of determining the horizontal distance from *A* to *B* is to measure between *A* and *B* with the tape held horizontally.

In taking a horizontal measurement on sloping ground, three things must be considered: (1) The tape must be horizontal, as represented by line *A″B* in Fig. 10; (2) the point *A″*, indicating the end of the measurement on the tape, must be vertically above point *A* on the ground; (3) the pull on the tape should be such that the stretch will be equal to the shortening of the length due to the fact that a tape or chain suspended only at the ends hangs in a curve, or *sags;* the straight-line distance between the ends is, therefore, less than the length of the tape.

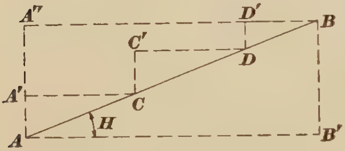


Fig. 10

It is a common error to hold the down-hill end of the tape too low. To aid in judging when the tape is horizontal, lines on nearby buildings may sometimes be used.

The end of the measurement is best transferred to the ground by means of a plumb-bob. Sometimes, a range pole held vertically is used instead of a plumb-bob, or a chaining pin is allowed to drop head downwards, but these methods are not accurate enough for careful work. The plumb-bob is held so that its point is close to the ground, but not touching it; when the tape is taut and horizontal and the end properly lined, the plumb-bob string is released by the chainman and the pin or stake is placed where the point of the bob strikes the ground. If the wind is blowing, it is necessary to stand so as to protect the bob from its force. The bob can be stopped from swinging by a slight counter movement of the hand holding the string.

Measurements are more easily made down-hill because then the rear chainman can hold his end of the tape firmly and the head chainman can pull steadily; when measuring up-hill, the rear chainman must hold a plumb-bob over his point, and it is difficult to keep it steady while the head chainman is pulling on the tape. When measuring across a gully, much time will be saved if a measurement can be taken between points at the same elevation on opposite slopes.

25. If the difference in elevation between two points a full tape-length apart is too great to permit the tape to be held horizontal, the measurement can be made by *breaking chain.* Suppose that, in Fig. 10, *A* and *B* are a tape length apart, but *B* is much higher than *A*. In such a case, the distance *A B* is measured in sections by establishing intermediate points, such as *C* and *D*, as far apart as possible but preferably at distances that are multiples of 10 feet, as 40 feet or 70 feet. Then distance *A B′* is the sum of distances *A′C*, *C′D*, and *D′B*. However, it is sometimes more convenient, especially when stakes are driven, to select the points on the line, as *C* and *D*, and then measure the distances *A′C*, *C′D*, and *D′B*. The method of measuring by breaking chain is illustrated in Fig. 11. The points on the ground, between which the measurement is taken, are *A* and *B*. The rear chainman holds a point of the tape at *B* and the head chainman holds a plumb-

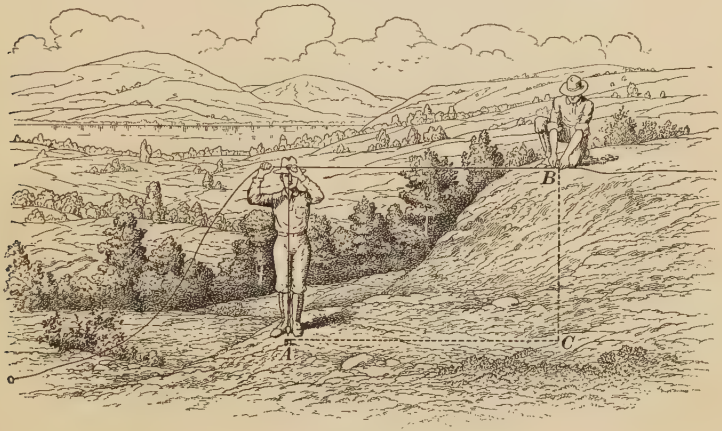


FIG. 11

bob over A. The distance recorded on the tape is the horizontal distance $A\,C$.

When a line is being measured by breaking chain, mistakes in reading the tape and adding the distances will be avoided if the rear chainman holds the zero point of the tape at the beginning of the measurement, and then at each intermediate point, he holds the same graduation as the head chainman had when the preceding distance was measured. This operation is repeated until the required distance is found or the end of the tape is reached. It is then unnecessary to record the partial measurements prior to the end of the line. When pins are used to mark the intermediate points in measuring by breaking chain, the rear chainman should keep only those pins that mark the full tape lengths; the others should be given back to the head chainman as soon as they are removed from the ground.

26. If the ground is very steep, the horizontal measurements must be very short. Therefore, when the slope is uniform, it is better to measure the distance along the surface as if the ground were level and to determine the angle of inclination of the ground. (The method of measuring this angle will be described in another Section.) Then the horizontal distance can be calculated by multiplying the inclined distance by the cosine of the angle of inclination; for example, in Fig. 10,

$$A\,B' = A\,B \cos H \qquad (1)$$

Sometimes the difference in elevation, or the vertical distance, between the two points is known. Then it is unnecessary to measure the angle of inclination, the horizontal distance being found from the inclined distance and the vertical distance as follows: In the right triangle $A\,B'B$, Fig. 10, $\overline{AB}^2 = \overline{AB'}^2 + \overline{B'B}^2$, or $\overline{AB'}^2 = \overline{AB}^2 - \overline{B'B}^2$. Hence,

$$A\,B' = \sqrt{\overline{AB}^2 - \overline{B'B}^2} \qquad (2)$$

Example 1.—A distance, measured along the ground, is 118 feet and the angle of inclination is 18° 12′. If the line is to be plotted on a map, what length would be used?

Solution.—Distances on maps are horizontal and the horizontal projection of an inclined line is found by formula **1**. Then with Fig. 10 as a diagram, $AB = 118$ and $H = 18^\circ\ 12'$. Therefore, $AB' = 118 \times \cos 18^\circ\ 12' = 118 \times .950 = 112.1$ ft. Ans.

Example 2.—If the difference in elevation between two points A and B is 60 feet and the distance AB measured along the ground is 170.9 feet, what is the horizontal distance from A to B?

Solution.—The hypotenuse and one leg of a right triangle are known and, therefore, the length of the other leg can be found by formula **2**. With reference to Fig. 10, $AB = 170.9$ and $B'B = 60$; then $\overline{AB}^2 = 29{,}207$, $\overline{B'B}^2 = 3{,}600$, and $AB' = \sqrt{29{,}207 - 3{,}600} = \sqrt{25{,}607} = 160.0$ ft. Ans.

EXAMPLES FOR PRACTICE

1. The distance between two points, measured along the ground, is 216.3 feet. If the ground slopes at an angle of 7° 39′ to the horizontal, what length should be used on a map in locating one point from the other?
Ans. 214.4 ft.

2. Find the horizontal distance between two points if the inclined distance between them is 411.8 feet and the difference in elevation is 81.5 feet.
Ans. 403.7 ft.

3. A line was measured with a 200-foot tape; there were 2 tallies, the rear chainman had 3 pins, and the distance from the last pin to the end of the line was 19.4 feet. Find the length of the line.
Ans. 4,619.4 ft.

4. A 2-chain tape was used to measure a line; 3 tallies were recorded, the rear chainman had 5 pins, and the distance from the last pin to the end of the line was 81.1 links. What was the length of the line?
Ans. 70.811 ch.

ERRORS AND CORRECTIONS

27. Sources of Errors.—Errors in chained distances are of two classes: (1) errors that are due to faulty chaining and to natural conditions; and (2) mistakes in reading or recording measurements. The chief sources of errors of the first class are: (*a*) having the forward end of the tape off line or not having the tape horizontal; (*b*) having a pull different from that required to compensate for the effect of sag and of the wind; (*c*) careless plumbing; (*d*) incorrect length of tape, and (*e*) variation in temperature. Errors in reading or recording measurements may be caused by: (*a*) using the wrong zero point on the tape: (*b*) reading the wrong foot-mark, as 46.8

instead of 45.8; (*c*) reading in the wrong direction from a graduation, as 38 instead of 42 or 38.3 for 37.7; (*d*) reading the tape upside down, thus mistaking some figure, perhaps a 6 for a 9 or vice versa; (*e*) transposing figures in recording, as 71.23 instead of 72.13; (*f*) the note keeper misunderstanding the reading called by the chainman; and (*g*) mistakes in counting the full tape lengths.

28. **Preventing Mistakes.**—The zero point of the tape should be carefully located before starting any measurement. Mistakes in reading the wrong foot-mark may best be avoided by looking at the foot-mark on each side of the reading; then it is certain between which two feet the reading must be. Similarly the possibility of reading in the wrong direction from a graduation is removed by noting the graduations on both sides of the reading. The transposition of numbers in recording is purely a mental slip and can be prevented only by exercising great care.

Except when the head chainman is keeping the notes, he should call all readings distinctly and in such terms that there can be no doubt of the value. For example, the chainman calls "thirty nine three," meaning 39.3, but the note keeper might write 30.93 if readings are taken to hundredths; also, "thirty nine three" at a distance might sound like "thirty-nine feet." Hence, 39.3 should be called "thirty-nine, thirty." The note keeper should always call back the reading for the chainman's approval before he records it, and if possible should repeat it differently. Thus, if the chainman gives a reading as "twenty five," the note keeper could answer "twenty and a half" so that if the chainman meant 25.0 or 20.05 the difference would be readily noticed. In order to avoid mistakes of this kind, the correct way to call the following values is given: 20.05, "twenty, naught five;" 20.5, "twenty fifty" or "twenty and a half;" 25.0, "twenty-five, naught" or "twenty-five feet, even." Mistakes in counting the tape lengths should not occur if the pins are handled correctly or if the stakes are marked and the numbers called as previously explained.

29. Alinement.—The error in length due to the tape being not quite horizontal or not quite in the proper direction is very small. If one end of a 100-foot tape is held 1 foot out of line or 1 foot lower than the other end, the error in a tape length is about $\frac{1}{16}$ inch. Therefore, too much time should not be wasted in lining in the intermediate pins or stakes when measuring the line. However, if it is desired to take a measurement from an intermediate point on a line, that point should be lined in accurately. For example, the point *C*, Fig. 12, set on the line *A B* for the purpose of measuring distance *C D*, should be located carefully.

30. Sag and Pull.—The pull required to compensate for the effect of sag depends on the length and the cross-sectional area of the tape. In general, a pull of about 10 pounds is best. The pull can be measured accurately by means of a spring balance attached to the handle at one end of the tape. The proper pull for a given tape may be determined by measuring between two points that are a known distance apart. This refinement is necessary only in very accurate work. When a chain is used, it should be taut, but should not be pulled with so much force that the rings stretch. An experienced chainman can apply the proper pull by judgment.

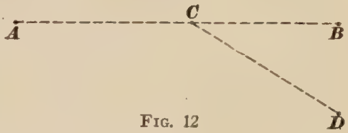


Fig. 12

31. Correction for Erroneous Length of Tape.—The length of a chain or a steel tape is altered by wear and distortion, and by changes in temperature. The length is increased when the temperature rises and decreased when the temperature falls. The length of a steel tape is specified at a given temperature, the change in the length of a 100-foot tape for a difference in temperature of 15 degrees Fahrenheit (15° F.) being about $\frac{1}{8}$ inch, or .01 foot. In accurate surveying the variation due to temperature must be considered.

32. The permanent change in length in a 100-foot tape due to wear and distortion is sometimes as much as $\frac{1}{2}$ inch. This shows the necessity of handling the tape carefully; if it is caught, as in a stump, in a rail joint, or in any other way, pulling from the ends may stretch it. The length of a tape should therefore be tested frequently, either by comparing it with a tape of standard length or by measuring between two points a known distance apart. It is advisable to have two such points marked permanently on a smooth pavement, curb, or some other convenient place. The length of a chain can be adjusted to the standard length by means of the nuts or swivels on the handles.

After a line has been measured, it is often found that the length of the tape is in error; the true length of the line must then be determined. For example, suppose it is required to calculate the true length of a line that was measured as 634.2 feet with a 100-foot tape that was afterwards found to be .05 foot too long. Since a 100-foot tape was used, there were $\frac{634.2}{100}$, or 6.342, tape lengths. But each tape length was .05 foot more than 100 feet. Hence, the true measurement was $6.342 \times 100 + 6.342 \times .05$. In general, when the tape is too long, the correct distance equals the measured distance plus the number of tape lengths times the amount by which the tape exceeds its proper length. This statement may be expressed in a formula.

Let T = true, or correct, distance, in feet;
M = measured distance, in feet;
L = tape length, in feet;
and e = error in one tape length, in feet.

Then,
$$T = M + \frac{Me}{L} \qquad (1)$$

If the tape is too short, similar reasoning will show that the correct distance is given by the equation

$$T = M - \frac{Me}{L} \qquad (2)$$

If the tape is too long, the true distance will be longer than the field measurement, while if the tape is too short, the true distance will be less than the recorded value. In using these formulas, other units, such as inches, chains, or meters, may be employed instead of feet, but the units must be the same for all quantities in any one formula.

Example 1.—The length of a line measured with a 50-foot tape was recorded as 1,048.4 feet. It was afterwards found that the length of the tape was 50.03 feet. What is the true length of the line?

Solution.—The error e in one tape length equals $50.03-50.00=.03$ ft.; $M=1{,}048.4$ ft.; and $L=50$ ft. Then, by formula **1**,

$$T=M+\frac{Me}{L}=1{,}048.4+\frac{1{,}048.4\times.03}{50}=1{,}048.4+0.6=1{,}049.0 \text{ ft.} \quad \text{Ans.}$$

Example 2.—The length of a line measured with a 66-foot tape was recorded as 19.89 chains. If the tape was $1\frac{3}{4}$ inches too short, what is the true length of the line?

Solution.—Since the measurement is in chains, e must likewise be expressed in chains. Then, the error e in each tape length equals $1\frac{3}{4}$ in. $=1.75$ in. $=\frac{1.75}{12\times66}$ ch. $=.0022$ ch.; $M=19.89$ ch.; and $L=1$ ch. Hence, by formula **2**,

$$T=M-\frac{Me}{L}=19.89-\frac{19.89\times.0022}{1}=19.89-0.04=19.85 \text{ ch.} \quad \text{Ans.}$$

33. When a given distance is to be laid off with a tape, the length of which is in error, the corrected measurement may be calculated as follows: Suppose that a distance of 456.22 feet is to be measured with a 100-foot tape which is known to be .05 foot too long. If a tape exactly 100 feet long were used, the number of tape lengths would be $\frac{456.22}{100}$, or 4.5622. Since each tape length is .05 foot too long, the actual length of the line if measured with the erroneous tape would be $4.5622\times100+4.5622\times.05$. When the tape is too long, the corrected measurement will therefore be obtained if, from the given distance, is subtracted the number of tape lengths times the error in each. This statement may be expressed in a formula.

Let T = given distance, in feet;
M = distance, in feet, to be measured with the erroneous tape, to equal the given distance;
L = tape length, in feet;
and e = error in one tape length, in feet.

Then, $$M=T-\frac{Te}{L} \qquad (1)$$

If the tape is too short, similar reasoning will show that the corrected measurement is given by the equation

$$M=T+\frac{Te}{L} \qquad (2)$$

If the tape is too long, the corrected measurement will be shorter than the true distance, and formula **1** applies. If the tape is too short, the corrected measurement will be greater than the given value and formula **2** must be used. As in the preceding article, other units may be employed instead of feet, provided that the units are the same for all quantities in a formula.

EXAMPLE.—A city block 400 feet in length is to be measured with a 100-foot tape that is 100.021 feet long. What distance should be laid off?

SOLUTION.—Here, the error e in one tape length equals 100.021 − 100.000 = .021 ft.; $T=400$ ft.; and $L=100$ ft. Then, by formula **1**,

$$M=T-\frac{Te}{L}=400-\frac{400\times.021}{100}=399.916 \text{ ft.} \quad \text{Ans.}$$

34. Precision Required.—No survey can be made absolutely free from errors, but the allowable error varies with the kind of work. Since the accuracy with which measurements are taken affects the cost of the survey, the work should be done to give the desired degree of precision with the least expense. For land surveys where the value of the land is about $40 or $50 an acre, the permissible error in chaining may be as much as 1 foot in 1,000 feet; but if the land is worth about $500 an acre, the error should be less than 1 foot in 2,000 feet. The error in measurements for engineering structures is often limited to 1 foot in 10,000 feet.

If the allowable error is 1 in 1,000, no refinements in chaining are necessary. The error can easily be made less than 1 in 5,000 if ordinary care is taken in lining, plumbing, and correcting for large errors in the length of the tape. To obtain results in which the error is less than 1 in 10,000, allowance must be made for variations in length caused by differences in pull and in temperature.

EXAMPLES FOR PRACTICE

1. The length of a line measured with a 100-foot tape was recorded as 1,946.2 feet. If the tape was .03 foot too short, what is the true length of the line?
Ans. 1,945.6 ft.

2. A line is measured with a 66-foot tape and its length is found to be 8.94 chains. If the tape is .4 link too long, find the true length of the line.
Ans. 8.98 ch.

3. A certain line is to be laid off with a 100-foot tape that is 99.987 feet long. If the required distance is 336 feet, what length should be laid off?
Ans. 336.044 ft.

FIELD PROBLEMS

35. To Prolong a Line.—In Fig. 13, *A B* is a line that it is desired to prolong beyond its extremity *B*. Having marked *A* and *B* by vertical range poles at these points, the surveyor walks some distance back of *A* and places himself at a point *P* in line with *A* and *B*, by sighting to the pole at *A*, and stepping to the right or to the left until the pole at *B* is covered by that at *A*. He then directs a pole to be held beyond *B* and signals to the flagman until the latter has the pole in such position that it is covered by the poles at *A* and *B*. Let *Q* be the point thus determined. A pole being stuck at *Q*, the surveyor moves to *A*, and, sighting along *B Q*, lines in the flagman at a point *R* beyond *Q*. The process may be repeated and the line prolonged as far as necessary. The distances *A B*, *B Q*, *Q R*, etc., should be as long as they can be conveniently made. Steel poles are thinner than wooden poles and are therefore preferable.

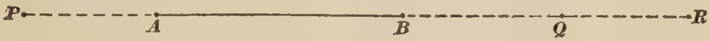


Fig. 13

36. To Run a Line Over a Hill Between Two Points, Neither of Which Is Visible from the Other.—The points *A* and *B*, Fig. 14, are supposed to be invisible from each other because they are on opposite sides of a hill. A line can be run between *A* and *B*, and intermediate points on the line can be set, as follows: One pole is placed at *A* and another at *B*; then two men with poles station themselves at points, as *C* and *D*, which they judge to be in line and in such positions that the poles at *B* and *D* are visible from *C* and the poles at *A* and *C* are visible from *D*. The man at *C* first lines in the pole at *D* between *C* and *B*, and then the man at *D* lines in the pole at *C* between *D* and *A*. From his new position, the man at *C* again lines in the pole at *D*, and so on, until the pole at *C* is in line between *D* and *A* at the same time that the pole at *D* is in line between *C* and *B*. The points *C* and *D* are then in line with *A* and *B*, and any other points can be lined in between *A* and *C*, *C* and *D*, or *B* and *D*.

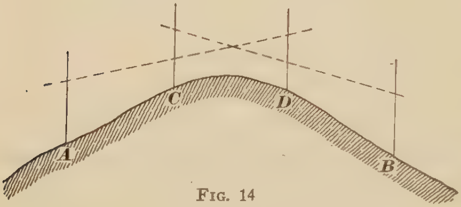
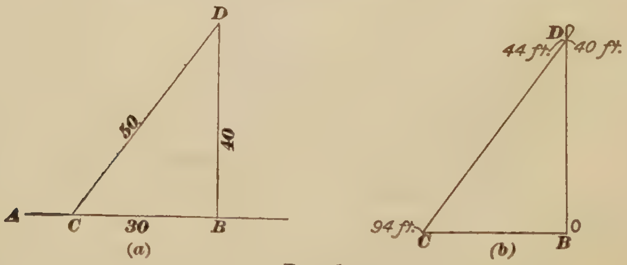


FIG. 14

37. To Erect a Perpendicular to a Line at a Given Point. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs. Therefore, since $5^2 = 3^2 + 4^2$, a triangle having sides proportional to the numbers 5, 3, and 4 is a right triangle with the right angle opposite the longest side. For example, if, in Fig. 15 (*a*), $BC = 30$ feet, $BD = 40$ feet, and $CD = 50$ feet, then BCD is a right triangle and BD is at right angles, or perpendicular, to AB. Hence, to run a line from point *B* perpendicular to the line AB, locate point *C* on line between *A* and *B* and 30 feet from *B*.



(*a*) (*b*)

FIG. 15

If one man holds the zero mark of a 100-foot tape at point *B* and another holds the 90-foot mark at point *C*, a third, holding the 40-foot mark, can locate point *D* at that mark by moving about until both parts of the tape are taut; for, since $BD = 40$ feet and $CD = 90 - 40 = 50$ feet, *BD* is then perpendicular to *AB*. However, a steel tape cannot be bent to a sharp angle, as would be necessary at *D*. For this reason, the work must be done in the following manner: One man holds the zero mark at point *B*, but the second man, instead of holding the 90-foot mark at point *C*, holds the 93-, 94-, or 95-foot mark at that point. The extra 3, 4, or 5 feet is allowed to form a loop at *D*, as shown in (*b*), in order to avoid a sharp bend in the tape. If, as indicated in (*b*), the man at point *C* holds the 94-foot mark on the point, the man at *D* holds the 40-foot and the 44-foot marks together, with the part of the tape between 40 and 44 feet in the form of a single loop; if the 93-foot mark is held at *C*, the 40- foot and 43-foot marks are brought together at *D* so that the length of the line *CD* is 50 feet.

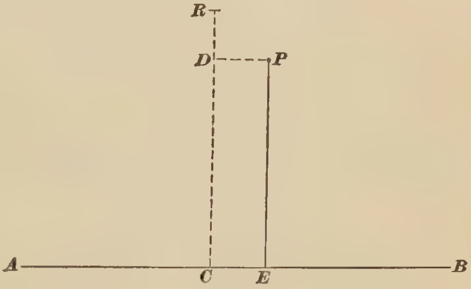


FIG. 16

The distances *BC*, *BD*, and *CD* may be any multiples of 3, 4, and 5, but the distance *BD* should be as long as possible to give the best indication of direction; with a 100-foot tape, the lengths 30, 40, and 50 feet are most convenient for the sides of the triangle.

38. To Drop a Perpendicular to a Given Line From a Given Point.—In Fig. 16, let it be required to drop a perpendicular to the line *AB* from the point *P*. First, select by judgment a point *C* on *AB* where it seems that the perpendicular will meet *AB*. Then at *C* erect a perpendicular to *AB* by the method described in the preceding article; make this perpendicular long enough to extend beyond P, as to R. Next, measure the distance PD from P to CR; in measuring this distance, D should be on the line CR, but since PD will be short, its direction can be determined by eye with sufficient accuracy. Finally lay off CE equal to PD. Then PE is the desired perpendicular.

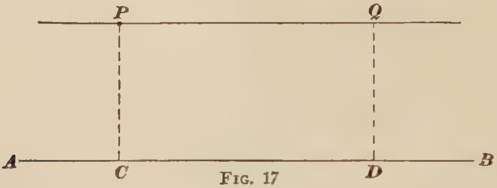


FIG. 17

39. Through a Given Point, to Run a Line Parallel to a Given Line.—Let AB, Fig. 17, be a given line to which a parallel through the point P is to be run. First, drop a perpendicular from P to AB by the method described in the preceding article; let C be the point where this perpendicular meets AB. Measure CP. At any other point D in the line AB, erect a perpendicular to the latter line, and measure on this perpendicular a distance DQ, equal to CP. A line through P and Q will then be the required parallel.

40. To Determine the Angle Between Two Lines.—In Fig. 18, AD and AE are two lines on the ground, forming an angle at A, the value of which is required. To find this angle, lay off along AD and AE the equal distances AB and AC, and measure the distance BC. Then the angle DAE is calculated from the relation

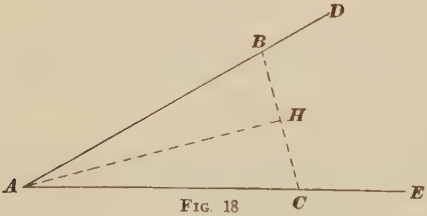


FIG. 18

$$\sin\frac{1}{2}DAE=\frac{\frac{1}{2}BC}{AB}$$

The derivation of this equation is as follows: The triangle BAC being isosceles, the perpendicular AH on BC bisects both the angle DAE and the base BC; that is, $BAH=\frac{1}{2}DAE$, and $BH=\frac{1}{2}BC$.

Then in the right triangle BAH, $\sin BAH = \frac{BH}{AB}$; that is,

$$\sin \frac{1}{2} DAE = \frac{\frac{1}{2} BC}{AB}$$

The value of $\frac{1}{2} DAE$ may be found from a table of natural sines; the result multiplied by 2 is the angle DAE. For convenience in the calculation, the distances AB and AC should be made 100 feet.

EXAMPLE.—If AB and AC, Fig. 18, are each 100 feet long and BC measures 57.6 feet, what is the value of the angle DAE?

SOLUTION.—When the values of AB and BC are substituted in the equation, $\sin \frac{1}{2} DAE = \frac{\frac{1}{2} BC}{AB}$, the result is

$$\sin \frac{1}{2} DAE = \frac{\frac{1}{2} \times 57.6}{100} = .28800$$

Then, $\frac{1}{2} DAE = 16° 44'$ (to the nearest minute) and $DAE = 2 \times 16° 44' = 33° 28'$. Ans.

41. To Lay Out an Angle.—From the line AB, Fig. 19, let it be required to lay out an angle BAC less than 90°.

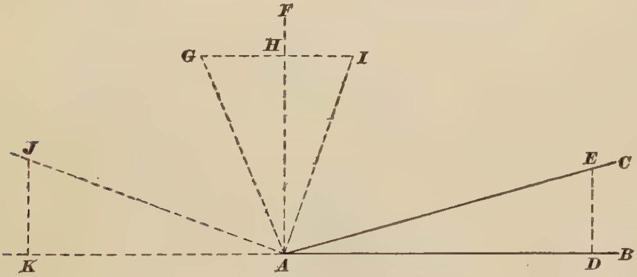


FIG. 19

Lay off any convenient distance AD on AB, and at D erect a perpendicular to AB. On this perpendicular, lay off DE equal to $AD \tan BAC$; and join A with E. Then DAE is the required angle.

If the angle is between 90° and 180°, the line AB is prolonged to K; then from AK, an angle is laid off equal to the difference between 180° and the given angle. For instance, the angle BAJ is laid off by erecting the perpendicular KJ and taking the distance KJ equal to AK tan $(180° - BAJ)$.

If there are several angles at A not far from 90°, it will be found more economical in the field to erect a perpendicular AF first. If an angle is less than 90°, as BAI, lay off HI equal to AH tan $(90° - BAI)$; then AI is a side of the required angle. If an angle is more than 90°, as BAG, lay off HG equal to AH tan $(BAG - 90°)$; then AG is a side of the required angle.

For convenience in computing, the distance AD, AK, or AH should be chosen 100 feet, except when it makes the offset DE or JK too large. In such a case, AD, AK, or AH may be reduced to 50 feet, or perhaps 20 feet.

EXAMPLE 1.—If the angle BAG, Fig. 19, is 114° 45′, and the distance AH is 100 feet, what is the length of the perpendicular HG to be laid off from AF in constructing the angle BAG?

SOLUTION.—The angle FAG equals $114° 45' - 90° = 24° 45'$, and $HG = 100 \tan 24° 45' = 46.10$ ft. Ans.

EXAMPLE 2.—What is the length of the perpendicular KJ, Fig. 19, to be laid off from AK in order to make the angle BAJ equal to 152° 30′, the distance AK being 100 feet?

SOLUTION.—The angle KAJ equals $180° - 152° 30' = 27° 30'$. Therefore, $KJ = 100 \tan 27° 30' = 52.1$ ft. Ans.

42. To Prolong a Line Through an Obstacle.—Suppose that the line AB, Fig. 20, is to be prolonged and that an obstacle K, such as a building, makes it impossible to produce the line by a direct sight on poles held at A and B. Under such circumstances, the line AB is run to a point C as near the obstacle as possible. At points C and M on the line AB, perpendiculars are then erected; the distance CM should be made equal to 30 or 40 feet for convenience in erecting per-

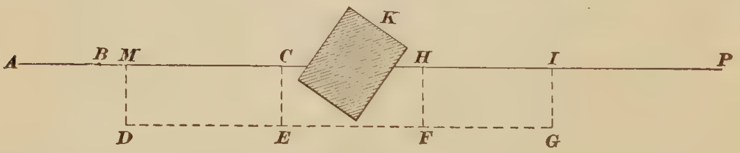


FIG. 20

pendiculars by the method explained in Art. **37.** Along these perpendiculars equal distances MD and CE are laid off, of such length that a line through D and E will clear the obstacle; the line through D and E is then parallel to AC. Next, the line DE is prolonged and on it are located point F, just beyond the obstacle, and point G, 30 or 40 feet from F. Finally, perpendiculars to DE are erected at F and G, and on these perpendiculars, the distances FH and GI are laid off equal to MD and CE. The points H and I are then in the prolongation of AB; and any point such as P can be located on line with A and B by lining in with poles at H and I, as explained in Art. **35.** The distance CH is equal to EF, which may be measured if required.

43. To Determine the Distance Between Two Points, Each of Which is Invisible From the Other.—In Fig. 21,

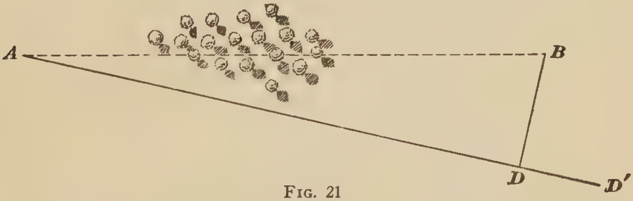


FIG. 21

the points A and B are separated by an obstruction which makes each invisible from the other. A convenient method of determining the distance between these points under such circumstances is as follows: From one of the points, as A, a line AD', called a *random line*, is run so that it passes as close as possible to the obstacle. Next, from B a line BD perpendicular to AD' is laid off, and the distances AD and BD are measured. Then in the right triangle ABD.

$$\overline{AB}^2 = \overline{AD}^2 + \overline{BD}^2, \text{ or } AB = \sqrt{\overline{AD}^2 + \overline{BD}^2}.$$

As an example, let AD be 206.1 feet and BD, 35.1 feet. Then

$$AB = \sqrt{206.1^2 + 35.1^2} = 209.1 \text{ feet.}$$

44. To Determine the Distance Between Two Points When One is Inaccessible.—Let it be required to determine the distance between points B and P, Fig. 22, which are separated by a river. First, select any point C from which both B and P are visible, and measure BC. Then take a point A on line with P and B, and run line AD' parallel to BC by the method described in Art. **39.** On the line AD', locate D on line with P and C. Finally measure the distances AB and AD. Then,

$$BP = \frac{AB \times BC}{AD - BC}$$

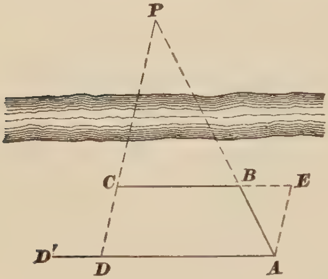


Fig. 22

The derivation of this equation is as follows: Draw AE parallel to DP and intersecting CB produced at E. Then, the angles of the triangle BCP being equal to those of the triangle ABE, these two triangles are similar; therefore, $\frac{BP}{AB} = \frac{BC}{BE}$, or

$$BP = \frac{AB \times BC}{BE}$$

Since $BE = AD - BC$,

$$BP = \frac{AB \times BC}{AD - BC}$$

Example.—Find the distance BP, Fig. 22, if $BC = 100$ feet, $AB = 52.4$ feet, and $AD = 124.2$ feet.

Solution.—By the formula,

$$BP = \frac{AB \times BC}{AD - BC} = \frac{52.4 \times 100}{124.2 - 100} = 216.5 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. If, in Fig. 18, AB and AC are each 100 feet and BC is 64.8 feet, what is the value of the angle DAE? Ans. 37° 48′

2. If the distances AB and AC, Fig. 18, are each 120 feet and the distance BC is 96.7 feet, what is the value of the angle DAE? Ans. 47° 32′

3. If the angle BAI, Fig. 19, is to be 56° 30′, and the distance AH, 80 feet, what should be the length of the perpendicular HI? Ans. 53.0 ft.

4. If an angle BAJ, Fig. 19, is to be constructed equal to 156° 15′, what must be the length of the perpendicular JK, when AK is equal to 100 feet? Ans. 44.0 ft.

5. If, in Fig. 21, $AD = 192.5$ feet and $BD = 12.6$ feet, what is the distance from A to B? Ans. 192.9 ft.

6. If the distances CB, AB, and AD, Fig. 22, are, respectively, 75, 42, and 103 feet, what is the distance BP?

Ans. 112.5 ft.

CHAIN SURVEY OF A CLOSED FIELD

FIELD WORK

45. Preliminary Examination.—The first step in the survey of a field is to find the marks and monuments at the corners. For this purpose the assistance of the owners of the field and of the neighboring property will be valuable. The tract should then be studied carefully with a view of finding the best method of making the survey.

46. Measurements.—In the survey of a field, the lengths of the sides are always required and the locations of important objects are usually necessary. The measurements for locating these objects from a side of the field are made at the same time as the side is being measured, as will be explained later.

If the boundaries are straight lines, the measurements are made by the methods previously described. In the case of an irregular boundary line, such as GNA, Fig. 23, which is the edge of a stream, one or more straight lines are run as close as possible to the boundary. From these auxiliary lines, perpendicular measurements, called *offsets*, are taken to those points on the boundary where any considerable change in direction occurs.

Thus, in Fig. 23, the straight survey line GA is run close to the shore line GNA. The distances GH, GJ, and GL and the offsets HI, JK, and LM are measured and recorded in the notebook while the line GA is being run. It will be seen that all distances along a main or auxiliary line for taking offsets are measured from the beginning of the line. For instance, instead of taking the distances *GH*, *HJ*, and *JL*, the distances *GH*, *GJ*, and *GL* are recorded. The parts *GI*, *IK*, *KM*, and *MA* of the boundary being nearly straight, the portion of the field between the straight line *GA* and the irregular line *GNA* is divided so as to form approximately the right triangles *GHI* and *LMA*, and the trapezoids *HIKJ* and *JKML*.

It is often difficult to measure directly along a line, as when a boundary is marked by a fence. In such cases, an offset of 3 or 4 feet is measured at each end of the line and the distance between the ends of these offsets is taken and recorded as the length of the line. The directions of such short offsets can be estimated by eye with sufficient accuracy.

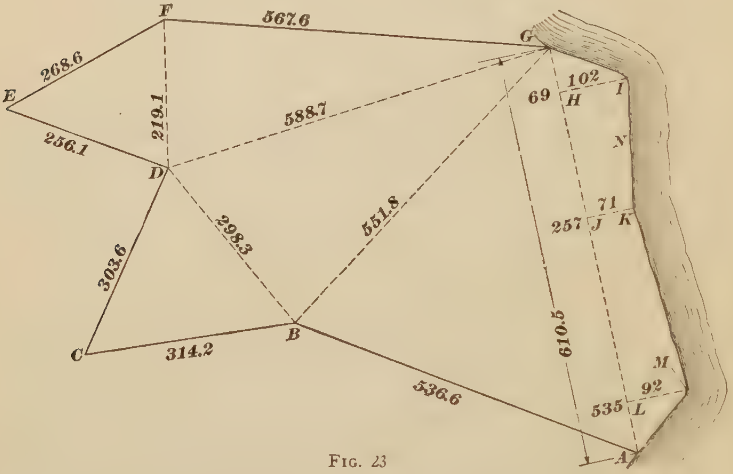


FIG. 23

47. Dividing a Field Into Triangles.—In order to make a plot of a field like that shown in Fig. 23 and to calculate its area, the field is divided into triangles by means of diagonals, which are measured on the ground. The surveyor should use his own judgment as to the best and most convenient diagonals to measure. He should avoid using diagonals that make triangles with very acute or very obtuse angles. Thus, in

Fig. 23, DG is a better diagonal to use than BF. Extra diagonals should be measured occasionally to check the work.

48. Tie-Lines.—Obstacles often make it impossible to measure directly the diagonals of a field. In such cases, their lengths may be determined by the methods described in Arts. **42, 43,** and **44,** or by the process illustrated in Fig. 24, which represents a field $ABCDE$ in which the diagonals BD and BE cross a pond, and cannot, therefore, be measured directly. If the sides BA and EA are produced to F and G, in such a manner that AF and AG are proportional to AB and AE, respectively, the triangle FAG is similar to BAE.

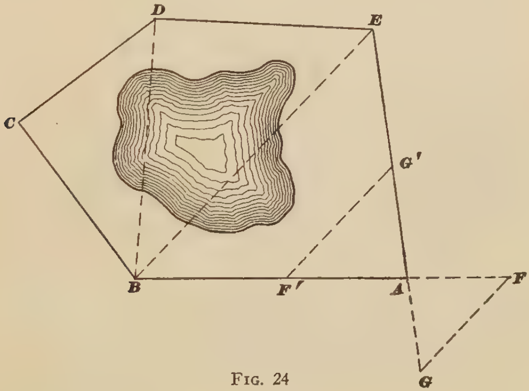


FIG. 24

Then the ratio of BE to FG is the same as that of AB to AF and of AE to AG. For convenience, the distances AF and AG should be made some simple fractional part of AB and AE, such as $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$. Then, BE will be 2, 3, or 4 times the distance FG.

For example, suppose that the sides BA and EA are 320 feet and 304 feet, respectively, and that AF, in the prolongation of BA, is made equal to $\frac{1}{4}$ AB, or $\frac{1}{4}\times 320 = 80$ feet. Then, the distance AG, in the prolongation of EA, must be laid off equal to $\frac{1}{4}$ AE, or $\frac{1}{4}\times 304 = 76$ feet. In this case, FG will be $\frac{1}{4}$ BE, or BE will be equal to 4 FG; if the length of FG is found to be 107 feet, the length of BE equals $4\times 107 = 428$ feet.

Instead of constructing a triangle such as AFG by producing the lines AB and AE, a triangle $AF'G'$ may be laid off inside the tract. If AF' and AG' are made proportional to AB and AE, respectively, the line $F'G'$ will be in the same proportion to EB as AF' is to AB, or as AG' is to AE.

Such lines as FG and $F'G'$ are called *tie-lines* because they tie the sides together. The length of the diagonal BD can be determined by means of tie-lines in a similar manner.

49. Location of Objects.—It is often desirable to locate important objects, such as buildings, roads, fences, etc., with reference to the main lines of a survey. Points very near the main line are most easily located by measurements along the line and perpendicular distances, or offsets, from the line. The points along the main line from which the offsets are to be measured can be determined by eye while the line is being run, and can be marked on the ground temporarily. The distances from the preceding corner and the lengths of the offsets should be measured and recorded. In Fig. 25, the point W may be located from AB by measuring the distance AO and the perpendicular offset OW.

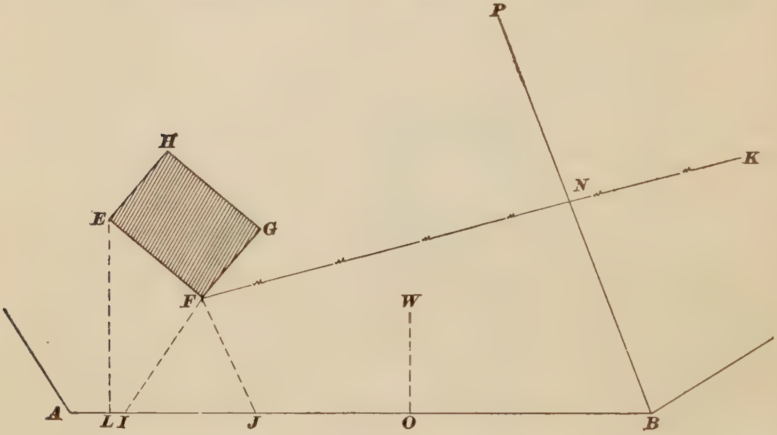


Fig. 25

When a point is so far from the main line that the direction of the perpendicular offset cannot be estimated accurately by eye, the method by a perpendicular offset is not convenient. In such a case, the point is located best by measuring the distances from two suitable points on the main line. For instance, in Fig. 25, the house corner F can be located by measuring the distances IF and JF. The directions of the sides of the house can be conveniently determined by taking the point I in the prolongation of FG.

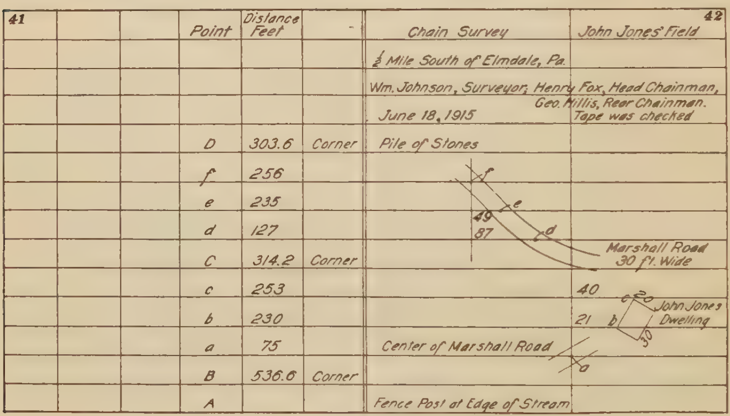
A rectangular object can be referenced by locating two corners and measuring the dimensions of the object. For the building $EFGH$ in Fig. 25 the point F can be located from $A B$, E can be located by distances from F and L, and G and H can be determined from E and F by measuring the dimensions of the building.

The direction of a straight line is determined by locating two points on it. For instance, a straight fence or road is located when two points on it are known. Thus, the fence FK, Fig. 25, may be referenced by locating the points F and N; F may be located from AB, as previously explained, and N may be located when the line $B P$ is being measured.

Corners of a field and other important stations should be *witnessed* or *referenced* by measurements to nearby objects, such as trees and corners of buildings, which are permanent and easily found. The purpose is to aid the surveyor to rerun his line, if necessary. He can thus either find or replace all important points.

50. Keeping Notes.—The notes, or record, of a chain survey are usually kept in an ordinary field book, commonly called a *transit book*, which fits in the pocket. The left-hand page of the notebook is ruled in six columns, as shown in Fig. 26. In one column are written the letters or station numbers by which the corners or important points are designated. Horizontally opposite each letter or number, in the next column, is recorded the distance of the point from the corner immediately preceding it. As only two columns of this page are needed, the fourth and fifth columns are used for this purpose, so as to bring them nearer the right-hand page, and to leave plenty of space to the left for remarks.

The right-hand page is ruled in blue with a vertical red line at the center. This page is used for sketches and remarks. The survey line is commonly represented by the red center line. In case more room is needed for sketching, the line being run may be drawn on one side of the center line of the page and parallel to it. On this page are noted also the date and location of the survey, the names and positions of the different members of the party, and any other remarks that the surveyor may deem necessary and useful. In sketching, it is better to face in the direction in which the line is being measured, and to represent the line as running from the bottom toward the top in the notebook. For this reason, nearly all surveying notes read upwards from the bottom of the page.



41			Point	Distance Feet	
			D	303.6	Corner
			f	256	
			e	235	
			d	127	
			C	314.2	Corner
			c	253	
			b	230	
			a	75	
			B	536.6	Corner
			A		

FIG. 26

The notes given in Fig. 26 are for part of the field the boundaries of which are represented in Fig. 23. The survey is supposed to have begun at *A*, from which the line was run to *B*, then to *C*, etc. The number 536.6 opposite *B*, denotes the distance from *A* to *B*; the distance from *B* to *C*, 314.2, is recorded opposite *C*; and so on for the other corners. The numbers between corners, as those between *B* and *C*, are the distances from the corner immediately preceding to the points directly opposite in the sketch. Thus, 230 and 253 are the distances from *B* to the points on *BC* where offsets were taken to the corners *b* and *c* of the dwelling indicated in the sketch. In locating a road, measurements are usually taken to the center line of the road instead of to one edge; thus, the distances 87 and 49 are to the center of Marshall road. On the right-hand page the survey line *CD* is represented by the vertical line to the left of, and parallel to, the line at the center of the page.

Notes should be full and plain, and should be kept as neatly as possible. The surveyor should keep his notes in such a manner that they can be readily understood, not only by himself, but by any one having a knowledge of surveying. This is especially necessary when the notes are not to be plotted by the same person who takes them. The pages of the notebook should be numbered. All corners and important stations should be fully described. If these points are mentioned in another survey, reference should be had to that survey.

OFFICE WORK

51. Plotting the Notes.—As a map shows the relative positions of points on the ground, the length of any line on the map is proportional to the length of the corresponding line on the ground. The distance on the ground represented by a unit distance on the map is called the *scale of the map*. For example, if 1 inch on the map represents 100 feet on the ground, the scale is said to be *100 feet to the inch*, or *1 inch = 100 feet;* if 1 inch on the map represents 500 feet on the ground, the scale is *1 inch = 500 feet*, etc. Since 100 feet equals 100 × 12, or 1,200, inches, a scale of 1 inch = 100 feet is the same as a scale of 1 inch = 1,200 inches; such a scale is, therefore, often called *1 to 1,200*. Similarly, a scale of 1 inch = 500 feet, or 6,000 inches, is sometimes called *1 to 6,000*.

For convenience in plotting distances to a given scale, a graduated rule, known as an *engineer's scale*, is commonly used by surveyors. These scales are usually made of boxwood, are 12 inches long, and are triangular in shape, as shown in Fig. 27. Flat scales, 6 inches long, are also made for convenience in carrying in the pocket. The scale shown in Fig. 27 has six systems of graduations, one on each side of each edge; it is, therefore, a combination of six scales. Each scale is so divided that the number of divisions in 1 inch is a multiple of ten, this number being indicated by large figures in the center of the scale; thus the numbers 10 and 50, Fig. 27, indicate that the scales are divided to tenths and fiftieths of an inch. Every tenth graduation mark is long, and the number of long graduations from the zero of the scale is shown by the figures. Hence, the number 32 on the 50-scale indicates a distance of 32×10, or 320, divisions from the zero of the scale; the actual distance in inches is unimportant.

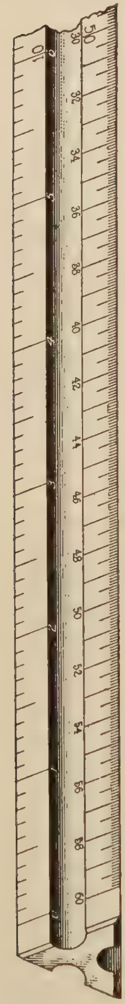


Fig. 27

52. The best scale to use in plotting a field is determined by the size of the field and the purpose of the map. If a scale of 1 inch = 10 feet, 1 inch = 100 feet, or 1 inch = 1,000 feet is chosen, the 10-scale will be most convenient, since then each small division will represent 1 foot, 10 feet, or 100 feet, respectively. For a scale of 1 inch = 40 feet, 1 inch = 400 feet, etc., the 40-scale is best; then each small division is 1 foot, 10 feet, etc. Readings to smaller values can readily be made by estimating parts of a division.

Suppose that a scale of 1 inch = 200 feet is used, and it is desired to lay off a line 283 feet long. Evidently, the 20-scale is best; then each small division represents 10 feet, and each long graduation indicates 100 feet. With the zero of the scale, *A* in Fig. 28 (*a*), opposite the beginning of the line, the graduation opposite the number 2 on the scale indicates 200 feet. The point *B* at the end of the measurement is then located 8.3 divisions beyond that long graduation, the decimal part of a division being estimated by eye. Suppose again that a line drawn to a scale of 1 inch = 600 feet is measured with the 60-scale; in this case, also, each division represents 10 feet, and each long graduation indicates 100 feet. With the zero of the scale, *A* in Fig. 28 (*b*), at the beginning of the line, the distance to the end of the line, at *B*, is found as follows: The numbered graduation just before *B* is 12 and the unnumbered long graduation between 12 and *B* is 13, which indicates 1,300 feet. Then, since the point *B* is exactly opposite the fourth division past the long graduation, the distance from that mark to *B* is 40 feet. Hence, the length of the line *AB* is 1,300+40 or 1,340 feet.

53. A plot of the field represented in Fig. 23 would be constructed in the following manner: Any line of the plot may be drawn first, but it should be so located that the map will come within the limits of the paper and be approximately in the center. The proper position can be determined by inspection of the sketch and the notes. In the present case it will be assumed that the line *FG* is drawn first; its length is made equal to 567.6 feet to the scale selected for the map. Then from *F* and *G* as centers, and with radii representing 219.1 and 588.7 feet, respectively, two arcs are described; the point of intersection locates point *D* on the map. Point *E* can be located by describing arcs from *F* and *D*; point *B* can be determined from *G* and *D*; and the other corners can be located in a similar manner from points previously found. Then the boundaries of the field are obtained by drawing straight lines between the points, or corners. The plot should be checked by the extra measurements made in the field and not used in locating the corners on the map.

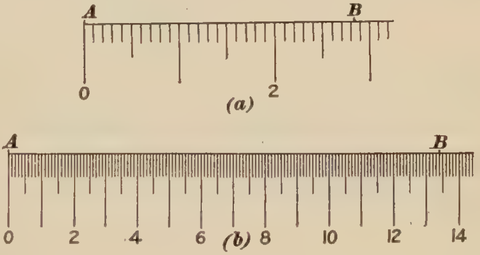


FIG. 28

After *GA* has been plotted, the points on the irregular boundary *GNA* are located by laying off distances *GH*, *GJ*, and *GL*, and erecting perpendiculars *HI*, *JK*, and *LM*, having the proper lengths. By drawing a freehand line through points *G*, *I*, *K*, *M*, and *A*, and making the parts between these points nearly straight, the irregular boundary is determined.

54. Calculating the Area.—The area of a field is obtained by calculating the areas of the triangles and trapezoids into which the field has been divided and then taking the sum of these partial areas.

When the lengths of the three sides of a triangle are known, its area can be found by the formula,

$$S=\sqrt{s(s-a)\,(s-b)\,(s-c)} \qquad (1)$$

in which S = area in square feet;

a, b, and c = lengths of sides, in feet;

and $s=\dfrac{a+b+c}{2}$

When the parallel bases of a trapezoid and the perpendicular distance between them are known, the area can be calculated by the formula

$$T=d\left(\frac{m+n}{2}\right) \qquad (2)$$

in which T = area in square feet;

m and n = parallel bases, in feet;

d = perpendicular distance between bases, in feet.

The area in square feet can be changed to acres by dividing by 43,560, since there are 43,560 square feet in an acre.

Example 1.—Find the area of the triangle *B C D* in Fig. 23.

Solution.—Here, the three sides are $a=303.6$ ft., $b=298.3$ ft., and $c=314.2$ ft. Then, $s=\frac{a+b+c}{2}=\frac{916.1}{2}=458.1$, $s-a=154.5$, $s-b=159.8$, and $s-c=143.9$. By formula **1**, the area of the triangle equals

$$S=\sqrt{458.1\times154.5\times159.8\times143.9}=40{,}340 \text{ sq. ft.} \quad \text{Ans.}$$

Note.—In computing an area from measurements which are not very accurate, four significant figures are sufficient.

Example 2.—Find the area of the trapezoid *H I K J* in Fig. 23.

Solution.—Here, $M = 102$ ft., $n = 71$ ft., and $d = 257 - 69 = 188$ ft., Then, by formula 2,

$$T = 188\left(\frac{102+71}{2}\right) = 16{,}262 \text{ sq. ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the area of the triangle *D E F* in Fig. 23. Ans. 26,070 sq. ft.

2. Find the area of the trapezoid *J K M L* in Fig. 23. Ans. 22,657 sq. ft.

LEVELING

DIRECT LEVELING

INTRODUCTION

DEFINITIONS

1. Vertical Lines.—A line in the direction of a plumb-line is vertical. For ordinary purposes it is convenient to assume that the earth is a true sphere with a smooth surface, and that a plumb-line held at any point on its surface is always directed toward the center of the sphere. Thus, if *O*, Fig. 1, is the center of the earth and *A* and *B* are two points on the earth's surface, then a vertical line has the direction *OA* at *A* and *OB* at *B*.

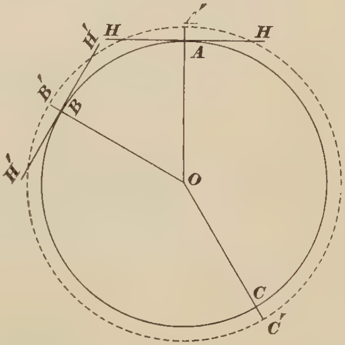


FIG. 1

2. Level Surfaces.—A surface, which at each point is at right angles to the direction of a plumb-line at the point, is a level surface; a line in such a surface is a level line. Every level surface is, therefore, assumed to be a part of a sphere having its center at the center of the earth. Thus, in Fig. 1, the circle *ABC*, representing the earth's surface, and the circle *A'B'C'*, representing a level

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surface, have the same center O. It should be kept in mind that a level surface is not a plane.

3. Elevations.—The vertical distance of any point above or below some level surface, adopted as a base for reference, is the elevation of the point with respect to the base. The base, or reference, surface is generally called a *datum*.

4. Leveling is the operation of determining the elevations of a series of points.

5. Horizontal Surfaces.—A horizontal surface at any point is a plane that is tangent to a level surface at that point; a line in such a surface is a horizontal line. A horizontal surface is perpendicular to a plumb-line at the point where it is tangent to a level surface. For example, in Fig. 1, HH represents a horizontal surface at the point A, and $H'H'$ represents a horizontal surface at B. Since the radius of curvature of the earth's surface is very great, a horizontal surface and a level surface will very nearly coincide for a considerable distance in every direction from the point of tangency. Hence, any reasonably short horizontal line may for ordinary purposes be considered to be a level line, and is commonly so considered. There are cases, however, in which the curvature of the earth's surface cannot be neglected.

6. Sea Level.—A datum may be an imaginary surface or the actual surface, commonly called sea level, which is the surface of the sea exactly midway between high and low tides. Sea level is the datum most generally used because it is the same at all points on the earth's surface and, therefore, furnishes a universal standard. The elevation of a point with respect to sea level is commonly termed its *altitude*.

METHODS OF LEVELING

7. Elevations are determined by three general methods, which differ with regard to the principles involved and the instruments and processes employed. They are: (1) *Direct leveling*, sometimes called *spirit leveling*, and also designated as *gravity leveling;* (2) *trigonometric leveling*, also known as *indirect leveling;* and (3) *barometric leveling.*

8. **Direct leveling** is the method of determining the elevations of points by measuring their vertical distances above or below a level line or a series of level lines. The device universally used for determining when a line is level is the *spirit level*, described later, from which fact this method of leveling is often called *spirit leveling.* The name *gravity leveling* is also sometimes employed because a level line is always at right angles to the direction of gravity. Direct leveling is the method used when a high degree of accuracy is required, and it is also employed when conditions make it more convenient than the other methods.

9. **Trigonometric leveling** is a method of determining the difference in elevation between two points by measuring the horizontal or the inclined distance between them and determining the angle between a horizontal line and the inclined line that joins the given points. The required difference in elevation is then one leg of a right triangle in which one acute angle and the other leg or the hypotenuse are known. It is a convenient method to use when the elevations of principal stations only are required. If carefully done, it is almost as accurate as direct leveling. Trigonometric leveling requires the measurement of angles, and is treated in the Section on *Transit Surveying.*

10. **Barometric leveling** is a method of determining the approximate difference in elevation between two points by measuring the difference between the atmospheric pressures at the points.

THE ENGINEERS' LEVEL

TYPES OF LEVELS

11. The instrument most extensively used in leveling is the *engineers' level*. It consists essentially of a telescope, having a very accurate spirit level attached longitudinally. The telescope is supported at the ends of a straight bar, which is firmly secured at the center to a perpendicular axis on which it revolves. The whole is supported on a *tripod*.

There are two general classes of engineers' levels: the *wye level*, also written **Y** *level*, in which the telescope rests in **Y**-shaped supports from which it can be removed; and the *dumpy level*, in which the telescope is rigidly attached to the bar supporting it.

The wye level is preferred by American engineers for ordinary work because of the ease with which it can be adjusted. The simple dumpy level is seldom used in America; but for very accurate work a form of the dumpy level, known as the *precise level*, has been found superior to the wye type because it has few movable parts and does not get out of adjustment easily.

THE WYE LEVEL

12. Description.—An engineers' wye level is shown in Fig. 2. The *telescope a b*, having attached to it an accurate and sensitive *spirit level c d*, rests in the **Y**-shaped supports *e* and *f*, in which it is held firmly by semicircular clasps, commonly called *clips*. The clips are hinged at one end, and, passing over the telescope, are held at the other end by small pins; the pins can be removed, and in order that they should not be lost, they are fastened to the supports by short cords. When the clips are open, the telescope can be turned in its supports so that the spirit level is no longer directly beneath the telescope. When the instrument is in use, however, the telescope must be prevented from turning. For this purpose, various devices are used; sometimes a small projection on

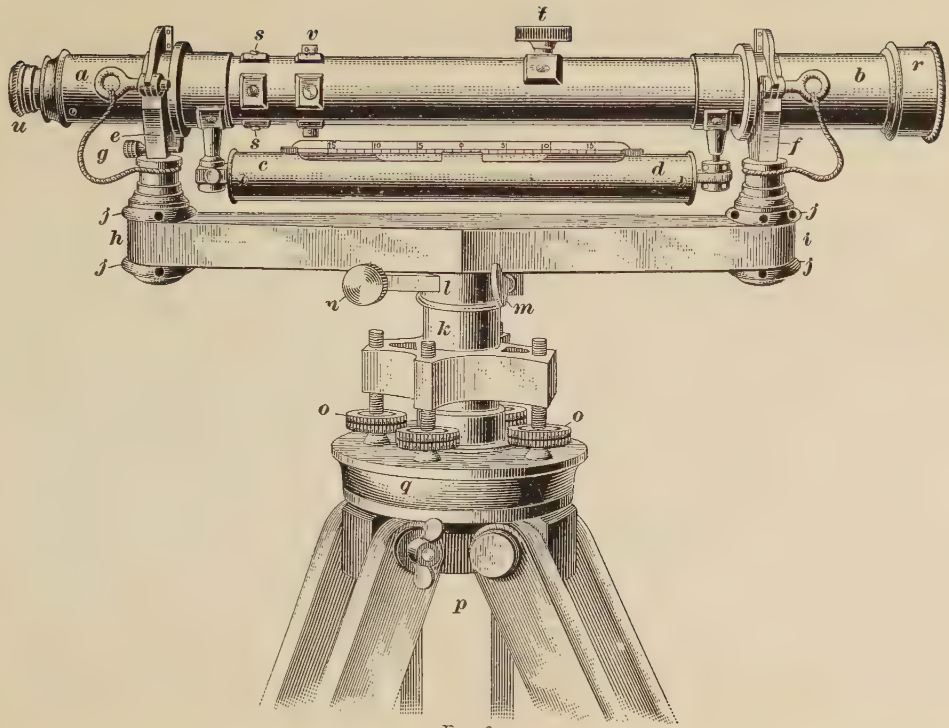

FIG. 2

the telescope bears against the stop-piece *g* but the best method is to have pins on the clips fit in holes in the top of the telescope.

The **Y**-shaped supports, or *wyes*, are distinguishing features of this form of level and from them the instrument derives the name *wye level*. The lower ends of the wyes pass through the ends of the horizontal bar *h i*, called the *level bar*, and are adjustable vertically by means of the capstan-pattern nuts *j*, which bear against the upper and lower surfaces of the bar. The bar *h i* is attached rigidly to a *center*, or *spindle*, which turns in the socket *k*. A collar *l*, which is connected to the level bar by means of a projection from the bar, revolves on the socket. When the clamp screw *m*, which is part of the collar, is loose, the telescope can be rotated in a horizontal plane. The instrument can be secured against rotation by tightening the screw *m*, which then holds the collar fixed on the socket. After the clamp *m* has been tightened, the telescope can be revolved slowly through a small angle by means of a screw *n*, known as a *tangent screw*. The projection from the level bar fits between the point of the tangent screw and a resisting spring. When the screw is tightened, its point pushes the projection, the spring is compressed, and the telescope rotates. When the tangent screw is loosened, the spring forces the projection from the level bar back against the point of the screw, and the telescope rotates in the opposite direction. The inclination of the socket *k* is controlled by the *leveling screws o*, which are four in number on some instruments and only three on others. The instrument is supported on a *tripod*, which consists of three legs shod with steel and connected by hinge joints to a metal *tripod head*. The upper part of the tripod is shown at *p* in Fig. 2. The tripod head is threaded in order that the plate *q* of the level can be screwed on.

13. Telescope.—A longitudinal section through the telescope *a b* is shown in Fig. 3. The essential parts are the *objective w*, the *eyepiece x x*, and the *cross-hairs*, or *cross-wires*, *y*.

The objective consists of a combination of *lenses* (pieces of glass with curved surfaces). When rays of light from the

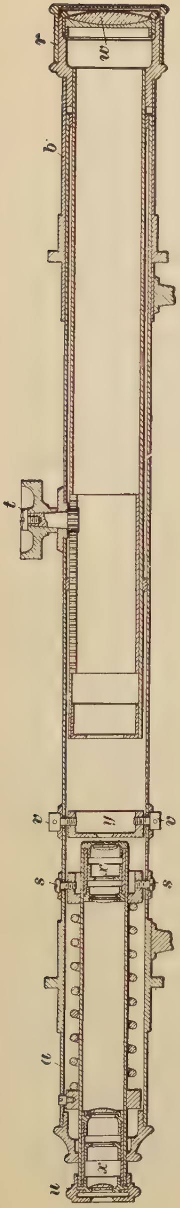

FIG. 3

object sighted at strike the objective, they are deflected to form a figure similar to the object, but inverted. This figure, called the *image*, is located in a definite plane. The cross-wires, described in the following article, should be placed in the telescope as near as possible to the image. Then, when the image and the cross-wires are viewed through the eyepiece, which is another combination of lenses that enlarges the image and the cross-wires, both will be seen clearly at the same time. Some eyepieces show the object upside down; others invert the image so that the object is seen in its natural position. Telescopes with eyepieces of the former class are called *inverting telescopes*, and those having eyepieces of the latter class are known as *erecting telescopes*. Although the eyepiece of an inverting telescope has fewer lenses and the lines of the object are, therefore, more clearly defined, most instruments have erecting telescopes.

The objective is shown covered by a metal cap *r*, Figs. 2 and 3, which protects it when the instrument is not in use. When a sight is taken through the telescope, this cap is removed and a thin metal tube, called a *sunshade*, is used to keep the glare of the sun from striking the objective and making the view of the object indistinct. A device for covering the eyepiece is also supplied.

The small screws at *s* are inserted to center the eyepiece in the telescope; they are necessary only with an erecting telescope, in which the eyepiece is very long.

14. The *cross-hairs*, or *cross-wires*, are two very fine platinum wires fastened at right angles to each other in a substantial brass ring or diaphragm. This ring is held in position in the telescope by four capstan-headed screws *v*, Figs. 2 and 3, that pass through holes in the telescope and screw into the ring, as shown in Fig. 4. The holes in the telescope are slightly larger than the screws so that the ring can be rotated in the telescope through a small angle. The ring can also be moved vertically or horizontally by turning the proper capstan screws. Formerly, the cross-hairs were made of clean spider web, but platinum wire has been found more satisfactory.

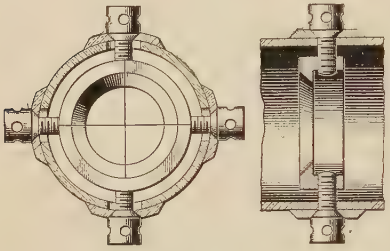

FIG. 4

15. Focusing.—As the distance from the telescope to the observed object varies, the location of the image changes. In order to bring the image in the plane of the cross-wires for any sight, the position of the objective in the telescope can be adjusted by the milled wheel *t*, Figs. 2 and 3, which is placed on the top of the telescope in this case, but is sometimes on the right-hand side. Changing the objective may make it necessary also to adjust the location of the eyepiece to obtain a distinct view of the object; this can be done by rotating the milled ring *u*. The operation of adjusting the eyepiece and the objective in order to see the cross-wires and the object distinctly at the same time is called focusing.

The objective is focused when the object sighted at appears clear and distinctly outlined. To focus the eyepiece, the telescope is pointed toward an open space and the eyepiece is moved by turning the ring *u* until the cross-wires appear sharp and distinct. When both the objective and the eyepiece are focused, the cross-wires will show no movement with respect to the observed object, no matter how the position of the eye may be changed.

16. Line of Sight and Line of Collimation.—A line of sight is a line joining any point with the eye of the observer. When a point is observed through a telescope, the line of sight is indicated by the line through the center of the objective and the intersection of the cross-wires. This line is sometimes called the *line of collimation*.

17. Spirit Level.—The spirit level *c d*, Fig. 2, is the part of the instrument on which accuracy chiefly depends. It consists of a sealed glass tube, curved slightly to correspond to the short upper arc of a large vertical circle, and so nearly filled with alcohol, or a mixture of alcohol and ether, as to leave only a small bubble of air. This tube, commonly called the *level tube*, is fastened securely, but not rigidly, in a metal case having a long, narrow longitudinal opening in its top through which the glass tube and air bubble can be seen. Each end of the metal case is attached to a stud projecting from the under side of the telescope tube, one end being adjustable vertically by means of capstan-pattern nuts and the other end adjustable laterally by means of capstan-headed screws, as shown in Fig. 2.

A line tangent to the upper surface of the level tube at its center is called the *axis of the bubble tube* or *axis of the level tube*. When the bubble is in the center of the tube, the axis of the bubble tube is horizontal. In order to show the position of the bubble with reference to the center of the tube, a graduated scale is marked either on the tube or on a small strip of silvered brass attached to the upper side of the tube. In Fig. 5 is shown the top view of the glass tube of a spirit level in which the graduations are marked on the tube and the bubble is represented as truly centered. In Fig. 2, the scale is marked on a metal strip; the bubble in this case is also centered.

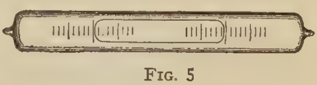

FIG. 5

18. Axis of Revolution.—The level has but one axis of revolution, which is the vertical axis of the instrument. In the use of the level, the only essential requisite for accurate work is that the line of sight shall be truly horizontal, and this condition should be indicated by the bubble's being in the center of the level tube. In other words, the line of sight should be parallel to the axis of the level tube. If the instrument is adjusted properly and is leveled up as described in the next article, the line of sight revolves on the vertical axis in a horizontal plane.

19. Setting Up.—The first step in making a level ready for use is to screw the instrument securely on the tripod head and to plant the tripod legs firmly in the ground in such positions that the plate *q*, Fig. 2, is nearly horizontal. If the instrument has four leveling screws, the telescope is rotated until it is over one pair of opposite screws and the bubble is brought to the center of the level tube by turning only these two screws. The screws should be held between the thumb and forefinger of each hand, and they should be turned in opposite directions; that is, the thumbs should move either toward each other or away from each other. Both screws should be turned at the same time and at about the same rate.

After the bubble has been brought to the center of the level tube for this position of the telescope, the telescope is turned on the vertical axis through an angle of 90° so that it is over the other pair of leveling screws. The bubble is then brought to the center of the tube by means of these two screws. The telescope is turned back to its first position, care being taken not to have it reversed end for end, to see if the bubble remains in the center. If it does, the instrument is leveled; if it does not, it is brought to the center over each pair of leveling screws alternately, until it stays in the center for both positions of the telescope.

If there are three leveling screws, the telescope is first placed parallel to the line through any two of them, and the bubble is brought to the center of the tube by means of these two screws. Then the telescope is revolved until it is over the third screw and the bubble is brought to the center by means of this screw alone. If the bubble remains in the center when the telescope is brought back to its first position, the instrument is leveled. Otherwise, the operations must be repeated until the bubble remains in the center for both positions of the telescope.

The expression *setting up the level* will be considered to include making the line of sight horizontal as well as merely placing the tripod legs in position, since the instrument must always be leveled before it can be used for determining elevations.

20. Care of Level.—The level should not be exposed to the sun, to rapid changes of temperature, to unequal temperatures on its different parts, to dust, or to rain when such exposure can be avoided. Changes of temperature disturb the adjustments, dust is injurious to the bearings and the lenses, while moisture obscures the lenses and is otherwise injurious to the instrument. When it is impossible to avoid working in the rain, wipe the lenses frequently and carefully with a soft linen cloth and, when the instrument is not in use, cover the eyepiece and put the cap on the objective. After returning to the office or camp, wipe the entire instrument very carefully and thoroughly, finishing with a piece of dry chamois skin; then, leave it in a moderately warm, dry place, so that every particle of moisture will be removed.

When a level is carried on its tripod in open country, the spindle should always be clamped slightly to prevent the wearing of the centers by swinging, and the instrument should be carried with the object end of the telescope down. In a wooded country where underbrush is dense, the level should be carried with the spindle unclamped, so that the telescope will turn freely on the spindle and yield readily to any pressure. A blow that would inflict no injury upon an unclamped instrument might seriously damage one clamped rigidly.

Care should be exercised not to use unnecessary force in screwing the instrument on the tripod, in tightening the clamp screw, or in turning the leveling screws. If the leveling screws bind, two adjacent screws should be loosened slightly. When the instrument is in use, all screws should bear firmly, but excess pressure is likely to cause damage.

ADJUSTMENTS OF WYE LEVEL

21. There are three important adjustments of the **Y** level, as follows:

1. To make the line of sight parallel to the line through the lowest points of the collars on the telescope, which rest in the wyes.
2. To make the axis of the level tube parallel to the line through the bases of the collars, and, consequently, parallel to the line of sight.
3. To make the axis of the level tube perpendicular to the vertical axis of the instrument, so that, when the instrument is leveled up, the bubble will remain centered while the telescope is revolved horizontally.

The first two adjustments are sufficient as far as accuracy of the instrument is concerned. The third adjustment affects only the rapidity with which the work can be performed. When this adjustment is made and the instrument is leveled up carefully, the bubble will remain centered in whatever direction the telescope is turned. But if this adjustment is poor, the bubble will have to be recentered by the leveling screws every time the telescope is pointed in a new direction.

The adjustments of the level should be tested frequently, as any defect will detract from the value of the work, from the rapidity with which it can be performed, or from both.

There is also a preliminary adjustment by which the cross-wires are made exactly horizontal and vertical when the instrument is leveled. This adjustment is not necessary, but it permits the use of any part of the horizontal cross-wire instead of the point of intersection of the cross-wires. It is also helpful in direct leveling to have a wire truly vertical.

22. Preliminary Adjustment.—The preliminary adjustment is made as follows: Set up the instrument at any convenient place and level the telescope over both pairs of opposite leveling screws. Then suspend a plumb-bob at some distance from the level and compare the direction of the vertical cross-wire with that of the plumb-line. If desired, the vertical edge of a building can be used instead of a plumb-line. If any deviation of the wire from the vertical line is visible, loosen two adjacent capstan-headed screws, *v* in Fig. 2, and rotate the cross-wire ring carefully by pressing against the heads of the screws or by tapping them lightly, until the wire coincides with the vertical line. If the telescope is inverting, the cross-wires are seen in their true positions; hence, the ring should be rotated in the direction in which it appears necessary. However, if the telescope is erecting, the wires are inverted and, therefore, the ring should be turned in the direction opposite to that in which the image of the wires is to be rotated. For instance, if *a b*, Fig. 6, shows the position of the plumb-line and *c d* that of the cross-wire, the ring should be rotated in the direction indicated by the upper arrow for an inverting telescope and in the opposite direction, as indicated by the lower arrow, for an erecting telescope. When the cross-wires are in the proper positions, the same two capstan-headed screws that were loosened should be tightened just enough to obtain a firm bearing on the shell of the telescope.

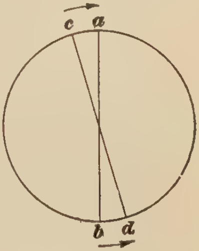

FIG. 6

23. First Adjustment, or Adjustment of Line of Sight. To make the first adjustment, plant the tripod firmly; but it is not necessary to level the telescope. Sight toward some distant vertical surface, such as a fence or building, clamp the instrument by means of the screw *m*, Fig. 2, and mark a point that coincides with the intersection of the cross-wires. Remove the pins, loosen the clips that hold the telescope in the wyes, and revolve the telescope in the wyes through one-half a revolution, that is, until the bottom side is up and the bubble tube is above the telescope. If the intersection of the cross-wires is still on the marked point, the line of sight is parallel to the line through the bottoms of the collars. If the intersection of the cross-wires is no longer on the point, mark the new position on the surface.

Suppose that the initial positions of the cross-wires are shown by the full lines in Fig. 7 and the point *a* at their intersection is marked. Suppose further that, after revolution of the telescope, the positions of the cross-wires are shown by the broken lines of short dashes, and the point of intersection *b* is marked on the surface near *a*. Then draw a straight line from *a* to *b* and mark its middle point *c*, which is the proper position of the point of intersection.

To bring the point of intersection to *c*, first move one of the cross-wires, say the horizontal, to *c* by means of the upper and lower capstan screws, and then set the vertical wire on *c* by means of the capstan screws on the sides. Both wires should not be moved at once since the cross-wire ring is liable to be rotated, and the preliminary adjustment to be spoiled. In moving a cross-wire, one capstan screw is always loosened first and then the opposite screw is tightened a corresponding amount. It is advisable to move the wire in short steps rather than large amounts at a time.

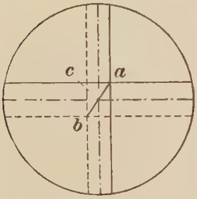

Fig. 7

In an inverting telescope, a wire is adjusted by loosening the screw away from which the image of the wire is to be moved and tightening the opposite screw. Thus, since the image of the horizontal wire in Fig. 7 should be moved upwards from *b* to *c*, the lower screw is loosened and the upper screw tightened; since the image of the vertical wire is to be moved to the right, the left-hand screw is loosened and the right-hand screw tightened.

To adjust a wire in an erecting telescope, loosen the screw toward which the image of the wire must be moved and tighten the opposite screw. In the case shown in Fig. 7, the upper screw is loosened and the lower one tightened for the horizontal wire, and the right screw loosened and the left tightened for the vertical wire.

24. Second Adjustment.—This adjustment is made in two parts, the second being somewhat dependent on the first. The first part is for adjusting the level tube laterally so that it can be adjusted accurately in a vertical direction. The work is done as follows:

Set up the instrument, clamp the spindle with the telescope directly over one pair of opposite leveling screws, and bring the bubble to the center of the tube by means of these two screws. Revolve the telescope in its wyes until the bubble tube is no longer directly under the telescope. If the bubble remains in the center of the tube, no lateral adjustment is necessary. But if the bubble leaves the center of the tube, bring it very nearly back to the center by means of the capstan-headed adjusting screws on the sides at one end of the level tube. Then revolve the telescope in its wyes back to its initial position, bring the bubble to the center of the level tube by means of the leveling screws, and test again to see whether it stays in the center when the telescope is revolved in the wyes. Repeat these operations as often as required.

Having adjusted the level tube laterally, bring the bubble exactly to the center of the tube by means of the leveling screws. Then lift the telescope out of the wyes and replace it with its ends reversed. In handling the telescope, take care not to disturb the position of the wyes. If the bubble is again in the center of the tube, the level tube is properly adjusted. However, if the bubble is not in the center, bring it half way back to the center by means of the capstan-pattern nuts at one end of the level tube. If the bubble is nearer the end of the tube with the nuts, lower that end by first loosening the lower nut and then tightening the upper nut. If the bubble is nearer the fixed end of the tube, raise the end with the nuts by first loosening the upper nut and then tightening the lower nut.

Again bring the bubble exactly to the center of the tube by means of the leveling screws and reverse the telescope in its wyes to see whether the bubble stays in the center. If it does not, repeat the operations until it does.

25. Third Adjustment.—To make the axis of the level tube perpendicular to the vertical axis of the instrument, level up the telescope first over one pair of opposite leveling screws and then over the other pair. From this second position, revolve the telescope through a half revolution on the vertical axis of the instrument so that it is over the same pair of screws but is reversed end for end. If the bubble stays in the center of the tube, the wyes do not need adjustment. But if the bubble leaves the center of the tube, bring it half way back by means of the capstan-pattern nuts at the ends of the level bar. Then center the bubble accurately by means of the leveling screws and test the adjustment by again turning the telescope through a half revolution. Repeat the operations until the bubble remains in the center for both positions of the telescope. As a check, try the reversal over the other pair of leveling screws. If the bubble is in adjustment over one pair of screws, it should be correct over the other pair also; any difference is the fault of the instrument and cannot be adjusted.

26. Tests for Adjustment of the Wye Level.—The adjustments of the wye level can be checked by the following two tests.

Set up the instrument at any convenient point and level the telescope over both pairs of opposite leveling screws.

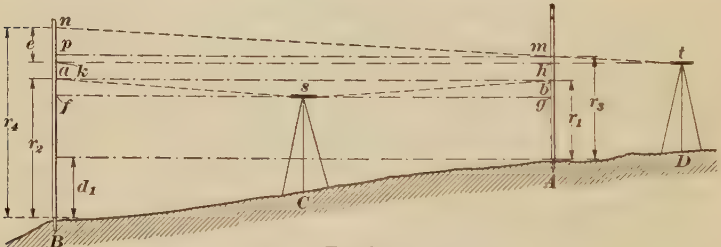

FIG. 8

Set a point at either end of the horizontal cross-wire and then slowly revolve the telescope on the vertical axis so that the point appears to move along the wire. If the point stays on the wire, the wire is horizontal. This test checks the preliminary adjustment described in Art. 22.

27. The following test is known as the *peg method* and indicates whether the line of sight is parallel to the axis of the level tube. There are several variations of the method, but only the one most commonly used is given. Select a stretch of nearly level ground and drive two pegs at *A* and *B*, Fig. 8, about 400 feet apart. Set up the instrument at *C*, midway between them and as nearly as possible on line with them; but great accuracy in line or distance is not required. With the bubble centered accurately, take readings on leveling rods held on the pegs at *A* and *B*. Leveling rods and the methods of taking these readings will be described later. If these readings are denoted by r_1 and r_2, the exact difference in elevation between the pegs is r_2-r_1, or d_1, no matter how much out of adjustment the instrument may be. If the level is in adjustment, the lines of sight will be horizontal along $s\,f$ and $s\,g$. If the instrument is not in adjustment, the lines of sight will be, say, along $s\,a$ and $s\,b$; but the angles $a\,s\,f$ and $b\,s\,g$ are equal. Since the distances $A\,C$ and $B\,C$ are very nearly equal, the differences on the rod, $a\,f$ and $b\,g$, can be considered equal; hence, $a\,b$ is parallel to $f\,g$ and is also horizontal. Therefore, the same difference in elevation between the pegs at *A* and *B* is obtained whether the lines of sight are along $s\,f$ and $s\,g$, or along $s\,a$ and $s\,b$.

Next, move the instrument to *D*, about 10 feet beyond *A* on line with *A* and *B* (a point beyond *B* would serve just as well), and again take rod readings on the pegs at *A* and *B*. If the readings are denoted by r_3 and r_4, the difference in elevation between the pegs, as determined by these readings, is r_4-r_3, or d_2. If the instrument is in adjustment, the values of d_1 and d_2 will be equal; that is, the line of sight is along the horizontal line $h\,k$. If the instrument is not in adjustment, the line of sight will be represented by some line, such as $m\,n$, which is not horizontal. Then d_1 and d_2 will not be equal, and the difference may be due to inaccuracy in the first adjustment, in the second adjustment, or perhaps in both. Therefore those adjustments should be repeated

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THE DUMPY LEVEL

28. Description.—A dumpy level is shown in Fig. 9. In its general construction it is similar to the wye level. However, there are two important differences: First, in the dumpy level, the telescope *a b* is attached rigidly to the level bar *c d*. Second, the level tube *e f* is attached to the level bar and is adjustable at one end and in a vertical direction only, while the other end is fastened permanently by a hinge; a lateral adjustment of the level tube is unnecessary. The telescope itself and the other parts of the instrument are the same as in a wye level and require no special description. The sunshade *g* is shown in position on the telescope.

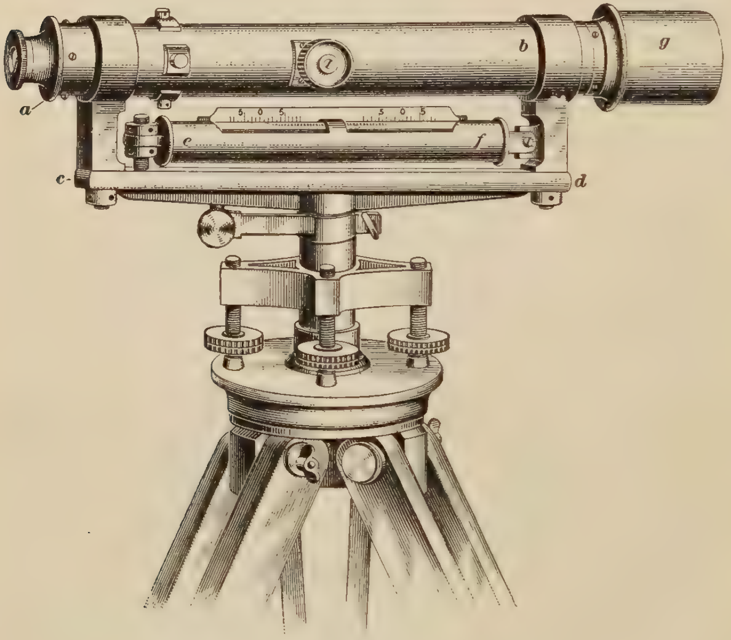


Fig. 9

29. Adjustments.—There are two adjustments of the dumpy level:

1. To make the axis of the level tube perpendicular to the vertical axis of rotation, so that, when the instrument is leveled up, the bubble will remain in the center of the level tube as the telescope is revolved.

2. To make the line of sight parallel to the axis of the level tube, so that the line of sight will be horizontal when the bubble stands in the center of the tube.

There is also the preliminary adjustment to make the cross-wires horizontal and vertical.

30. The preliminary adjustment is exactly the same as described for the wye level in Art. **22.** The first adjustment is performed in the same manner as the third adjustment of the wye level, explained in Art. **25.** However, in this case, the capstan screws, shown in Fig. 9 at the left-hand end of the level tube, are used instead of those at the ends of the level bar in Fig. 2.

For the second adjustment, the peg method is employed. The level is set up at *C*, Fig. 8, and rod readings r_1 and r_2 are taken on the pegs at *A* and *B*, as described in Art. **27.** Then the level is moved to *D* and rod readings r_3 and r_4 are taken on the pegs. If the horizontal cross-wire is properly centered, the values of r_2-r_1, or d_1, and r_4-r_3, or d_2, will be equal.

If d_1 and d_2 are not equal, the cross-wire is adjusted by the following method. Let $m\,p$ represent a horizontal line through m. Then the right triangles $n\,t\,k$ and $n\,m\,p$ are similar, since the acute angles at n are equal. Hence, the corresponding sides are proportional and $\frac{n\,k}{n\,p}=\frac{k\,t}{pm}$. From the figure, $n\,k=e$, $n\,p=r_4-r_3-d_1=d_2-d_1$, $k\,t=B\,D=A\,B+A\,D$, and $p\,m=A\,B$.

Therefore,
$$\frac{e}{d_2-d_1}=\frac{AB+AD}{AB}$$

or
$$e=\frac{AB+AD}{AB}(d_2-d_1)$$

In case d_2 is less than d_1, the difference d_1-d_2 is taken instead of d_2-d_1.

To make the adjustment, keep the level at *D* with the bubble centered in the tube. Then, move the horizontal wire by means of the capstan-headed screws until the reading on the rod at *B* is either r_4+e or r_4-e, according to the following rules:

Rule I.—*If* d_1 *is less than* d_2 *and* r_3 *is greater than* r_4, *or if* d_1 *is greater than* d_2 *and* r_3 *is less than* r_4, *add* e *to* r_4.

Rule II.—*If* d_1 *is greater than* d_2 *and* r_3 *is greater than* r_4, *or if* d_1 *is less than* d_2 *and* r_3 *is less than* r_4, *subtract* e *from* r_4.

It is to be remembered that r_3 is always the reading on the peg nearer the second set-up and r_4 is the reading on the peg farther from the set-up. Also, the correction e is always applied to r_4. If e is added to r_4, the upper capstan screw is loosened and the lower tightened for either an erecting or an inverting telescope. If e is subtracted from r_4, the lower screw is loosened and the upper screw tightened. The adjustment should be tested by taking new observations for r_3 and r_4; if r_4-r_3 is not equal to d_1, the wire must be readjusted.

When the difference between d_1 and d_2 is very small, say less than .02 foot, the following method of determining the reading at which the wire is to be set may be used. Make the rod reading at B equal to r_3+d_1 or r_3-d_1 according to whether r_4 is greater or less than r_3.

Example 1.—With the instrument at C, Fig. 8, rod readings on pegs at A and B, 450 feet apart, are, respectively, $r_1=3.718$ feet and $r_2=5.142$ feet. When the instrument is moved to D, 9 feet behind A, the rod readings on the pegs are $r_3=6.005$ feet at A, and $r_4=7.470$ feet at B. (*a*) Is the instrument in adjustment? (*b*) At what reading should the cross-wire be set to make the adjustment?

Solution.—(*a*) The true difference in elevation between the pegs is $d_1=r_2-r_1=5.142-3.718=1.424$ ft. The difference in elevation given by the readings from D is $d_2=r_4-r_3=7.470-6.005=1.465$ ft. Since d_1 and d_2 are not equal, the instrument is not in adjustment. Ans.

(*b*) To determine the correction e, substitute in the formula the values $A\ B=450$, $A\ D=9$, $d_1=1.424$, and $d_2=1.465$; then,

$$e=\frac{450+9}{450}(1.465-1.424)=.042 \text{ ft.}$$

Since d_1 is less than d_2 and r_3 is less than r_4, rule II applies. Hence, e is subtracted from r_4 and the required rod reading is $7.470-.042=7.428$ ft. Ans.

Example 2.—Suppose that, for the pegs in example 1, the level is set up 9 feet behind B and the rod readings are 4.836 feet at B and 3.422 feet at A. At what reading should the cross-wire be set to adjust the instrument?

SOLUTION.—As in example 1, $d_1 = 1.424$ ft. In this case, however, the peg at *B* is nearer the second set-up, and the peg at *A* is farther from the set-up; hence, r_3, which is the rod reading at *B*, is 4.836 ft. and r_4 at *A* is 3.422 ft. The difference d_2 is $4.836 - 3.422 = 1.414$ ft. and $d_1 - d_2 = 1.424 - 1.414 = .010$ ft., which is less than .02 ft.

Since r_4 is less than r_3, d_1 is subtracted from r_3 and the required rod reading is $r_3 - d_1 = 4.836 - 1.424 = 3.412$ ft. Ans.

EXAMPLES FOR PRACTICE

1. In adjusting a dumpy level, the instrument is set up midway between two pegs *A* and *B*, 400 feet apart, and the rod readings on the pegs are 4.172 feet at *A* and 2.065 feet at *B*. Then the level is set up 10 feet behind *B*, and the readings are 6.103 feet at *B* and 8.155 feet at *A*. What should be the reading at *A* for adjusting the cross-wire? Ans. 8.211 ft.

2. For the same two pegs as in example 1, the level is set up 8 feet behind *A*, and the rod readings are 5.555 feet at *A* and 3.440 feet at *B*. At what reading should the cross-wire be set? Ans. 3.448 ft.

LEVELING RODS

DESCRIPTION

31. Principle of Direct Leveling.—When an adjusted level is set up and leveled up, its line of sight is horizontal; and, since the line of sight is at right angles to the vertical axis of the instrument, it rotates about this axis in a horizontal plane, called the *plane of the instrument*. The elevation of this plane is the elevation of the instrument, usually called *height of instrument*, and every line lying in the plane is a horizontal line having the same elevation as the instrument. This elevation may be assumed arbitrarily or may be determined from the known elevation of some other point by measuring the vertical distance from the plane of the instrument, as defined by the line of sight, to the point of known elevation. A leveling rod is usually employed for this purpose.

The general method of direct leveling is illustrated in Fig. 10. It is assumed that the elevation of point *A* is 976 feet and it is desired to determine the elevation of point *B*. The level is set up at some convenient point so that the plane of the instrument is higher than both *A* and *B*. Then a leveling rod is held vertically at the point *A*, which may be the top of a stake, or some object, and the line of sight is directed toward the rod. The vertical distance from *A* to the line of sight can be read on the rod at the point cut by the horizontal cross-wire of the telescope. If the rod reading is 8 feet, the line of sight is 8 feet above the point *A* and the height of instrument is 976+8=984 feet. Then the leveling rod is held vertically at *B*, and the line of sight is directed toward the rod by revolving the telescope on its vertical axis. The vertical distance from *B* to the line of sight is given by the rod reading with which the cross-wire coincides. If the rod reading at *B* is 3 feet, it means that the point *B* is 3 feet below the line of sight and, therefore, the elevation of *B* is 984−3=981 feet.

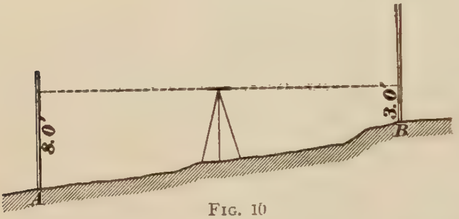


Fig. 10

32. Kinds of Leveling Rods.—In general, a leveling rod is a graduated wooden rod. There are several kinds of rods differing in constructive details but not in principle. The two types most commonly used are known as the *Philadelphia rod* and the *New York rod*, the important difference being in the method of marking the graduations.

33. Philadelphia Rods.—The Philadelphia rod, Fig. 11 or 12, is made in two sections, held together with brass sleeves *a* and *b*. The rear section slides with respect to the front section, and it can be held in any desired position by means of the clamp screw *c* on the upper sleeve *b*. In Fig. 13 (*a*) is shown a side view of part of a rod. The upper portion *A* of the rear section projects so that its face is in line with that of the front section *B*. When the projection rests on the lower section, as shown in Fig. 13 (*b*), the rod is said to be closed; a closed rod is sometimes called *short rod*. When the

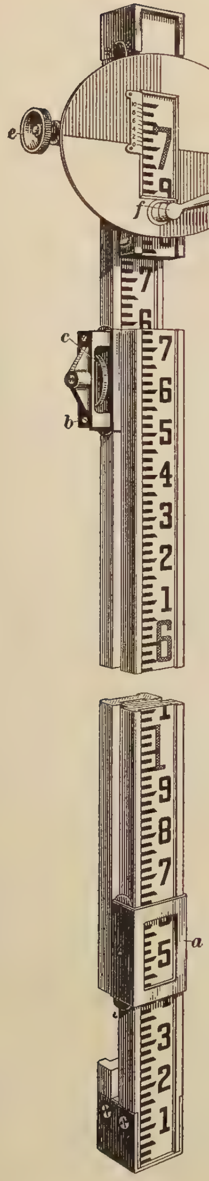


FIG. 11

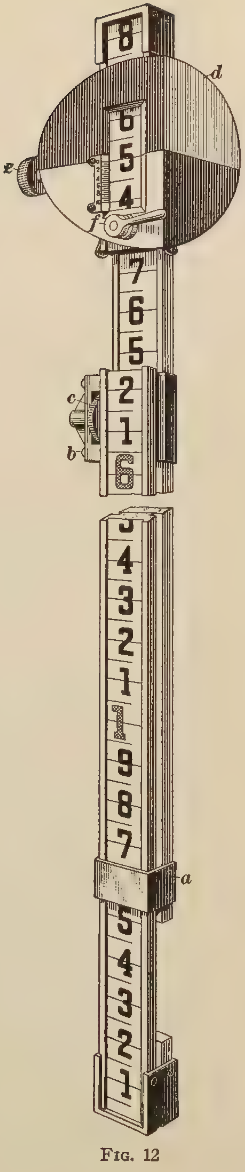
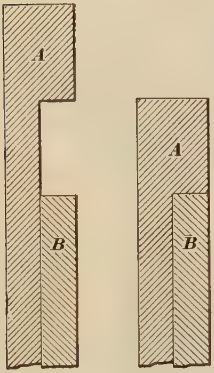


FIG. 12

rod is extended, no matter how much, it is known as *long rod*, or *high rod*.



(a) (b)

FIG. 13

In the type shown in Fig. 11, the divisions are alternate black and white spaces, each .01 foot high, painted on the rod. Each fifth hundredth is indicated by a longer graduation mark, so that an acute angle is formed at one corner of the black space of which the graduation is a part. The tenths are marked by large black figures, half above and half below the graduation mark, and the feet are shown in a similar manner by red figures (shaded in the illustrations). The graduations can be seen distinctly through the telescope of a level at a great distance, and, therefore, Philadelphia rods are sometimes called *self-reading rods* or *speaking rods.*

In the type of a Philadelphia rod shown in Fig. 12, the graduations indicate tenths of a foot and half-tenths. They are marked by single lines, and are numbered as in Fig. 11.

The projection at the top of the sliding portion of the rod is graduated upwards from the greatest value on the lower section so that, when the rod is closed, the graduations are continuous; they run to 7 feet for the type in Fig. 11 and somewhat above 6.5 feet for the rod in Fig. 12. The front of the sliding portion of the rod, which is hidden when the rod is closed, is also graduated upwards from the greatest value on the lower portion so that, when the rod is fully extended, the graduations are again continuous as shown in Fig. 14. The rod in Fig. 11 extends to 13 feet while the highest reading for the rod in Fig. 12 is 12 feet. The back of

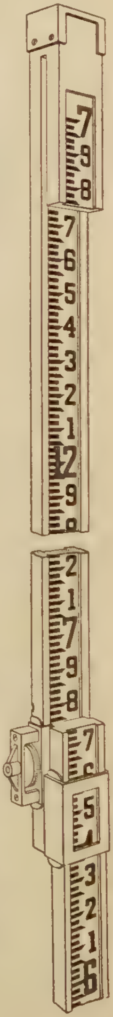


FIG. 14

the sliding portion of the rod is also graduated; the purpose and the arrangement of these graduations will be described later.

34. In case the graduations cannot be read directly from the telescope, the device shown at *d*, Fig. 11 or 12, called a *target*, is used. The target is a circular or elliptical metal plate divided into quadrants alternating red and white. The target in Fig. 11 is a plane surface but that in Fig. 12 consists of two plane surfaces bent at right angles to each other. When the rod is held vertical, one of the lines dividing the colors is horizontal and the other is vertical. There is an opening in the face of the target in order that the graduations on the face of the rod can be seen through it. One side of this opening is beveled to a thin edge, and a scale is marked along this edge so that it is close to the face of the rod. This scale is used for determining readings between graduation marks. One of its ends is exactly on the horizontal line dividing the colors on the target, and it extends either entirely above this line, as in Fig. 11, or entirely below, as in Fig. 12.

When the clamp screw *e* is loose, the target can be moved over the face of the rod until the line dividing the colors coincides with the horizontal cross-wire of the level. The target can then be fixed in that position by tightening the clamp *e*. The small lever *f* is used to move the target a short distance after the clamp *e* has been tightened. If the target is not needed, it can be removed from the rod.

35. When the target is used on a high rod, it is first set exactly at 7 feet on the extension part of the rod as shown in Fig. 11, or at 6.5 feet as in Fig. 12. Then the extension with the target is raised until the line dividing the colors coincides with the horizontal wire of the telescope, and the rod is held in that position by tightening the clamp *c*. In order to indicate the reading for a high rod, the graduations on the back of the sliding part and a scale on the back of the upper sleeve are used. The back of a rod of the type in Fig. 11 is shown in Fig. 15. The graduations begin at 7 feet at the top and increase

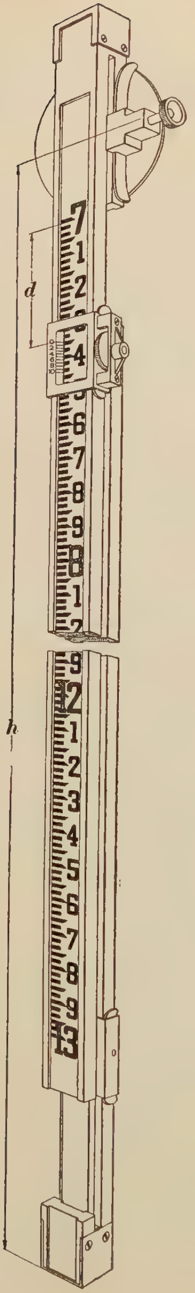


FIG. 15

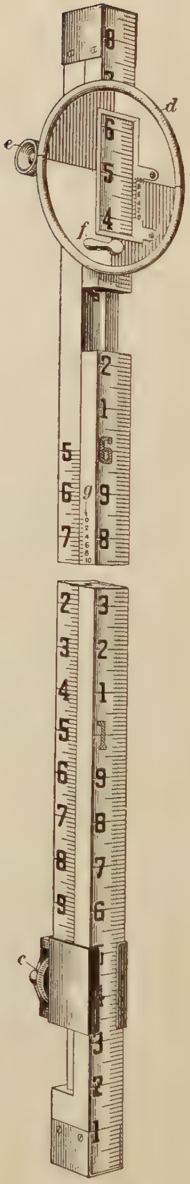


FIG. 16

downwards to 13 feet. The reason for this is given by the following explanation:

The reading of a high rod is the distance from the base of the rod to the target. Thus, for the position shown in Fig. 15, the rod reading represents the distance *h*. When the target is set at 7 feet on the extension part of the rod while the rod is closed, the reading of the rod is 7 feet and the 7-foot graduation on the back of the rod is opposite the zero of the scale on the sleeve. As the rod is raised, the 7-foot mark moves upwards, while the zero mark of the scale remains stationary since it is attached to the lower portion of the rod. The distance between these two marks, *d* in Fig. 15, therefore, increases as the rod is extended farther. Consequently, the distance *d* is equal to the amount by which the target is raised above 7 feet, and, for any high-rod setting, the distance *h* is equal to 7 feet plus the distance *d*. To obviate actual addition, the foot-graduations on the back of the rod are numbered downwards from 7 to 13; thus, when the rod is extended 1 foot, the reading on the back is $7+1=8$ feet, and so on. It is, therefore, seen that the numbers must increase downwards in order that the rod readings may become greater as the target is raised.

On the rod shown in Fig. 12, the graduations on the back of the rod begin at 6.5 feet at the top and increase downwards to 12 feet.

36. New York Rods.—The New York rod, Fig. 16, is in two sections that slide on each other by means of a tongue and groove; the upper section can be clamped in any position by the clamp screw *c*. The New York rod differs from the Philadelphia rod, first, in the character of the graduation marks, which are single lines stamped on the rod and blackened, much the same as on an ordinary rule. All New York rods are graduated to hundredths of a foot. Usually, the rod is graduated to 6.5 feet on the lower portion and extends to 12 feet. Another difference from the Philadelphia rod is that the inner face of the extending portion of a New York rod is not graduated. Therefore, the readings of an extended rod cannot be taken directly through the telescope. For determining the reading when the rod is extended, the rear section is graduated along one edge from 6.5 to 12 feet, increasing downwards, and an auxiliary scale *g* is marked on the edge of the lower portion. Since the graduations are not easily seen, a target is almost always necessary in using a New York rod.

37. Other Rods.—Sliding rods are also made in three sections, having a length of about 4.5 feet when closed and extending to 12 feet, or having a length of 5.5 feet when closed and extending to 15 feet. Sliding rods are objectionable because they sometimes stick or slip when extended. Therefore, some rods are made in one piece 10 or 12 feet long, and others are in two parts that are connected by a hinged joint and that fold together. All these rods are self-reading, and a target cannot be used on them. They are graduated only on the front face of each section, and when the rod is fully extended, the graduations are continuous.

For work in mines and tunnels and for other purposes, it is sometimes convenient to use a very short rod. Therefore, two-section rods are made which are 3 feet long when closed and extend to 5 feet.

READING THE RODS

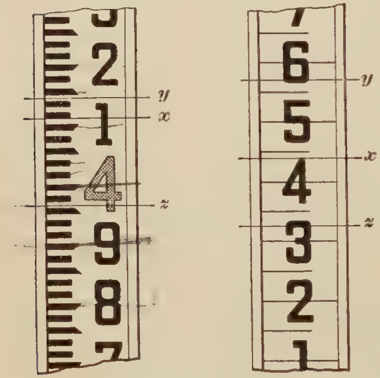
38. Rodman.—The man who carries the rod and holds it on the points whose elevations are to be taken is called a rodman. A good rodman is essential to accurate and rapid leveling. A man who is slow and inattentive to the work is not suitable for a rodman. In most localities, a line of levels of any considerable length will have enough rough places in it—that is to say, places where abrupt and considerable changes in elevation occur—to retard progress, however diligent the level party may be. The laziness or carelessness of an individual should never be allowed to delay the progress of the party.

39. Using the Rod.—A rod can be held on a point and carried more easily when it is closed; therefore, it is usually extended only when the reading exceeds the highest graduation on the lower section. Thus, with a rod like that shown in Fig.

11, short-rod readings may be taken up to 7 feet, and with the styles shown in Figs. 12 and 16, the highest reading on the short rod is 6.5 feet. The readings may be made either with or without the aid of the target *d*. In all cases, the rod is held with the front toward the level. When the target is not used, the levelman reads the position of the horizontal cross-wire directly from the telescope. If the target is used on a short rod, the rodman moves it up or down as indicated by word or signal from the levelman until the line separating the colors nearly coincides with the horizontal wire. The target is then clamped by tightening the screw *e*, and its dividing line is set exactly on the wire by means of the lever *f*. The reading of the rod is determined by the position of the end of the target scale which is on the line between the colors.

To make a high-rod reading without the target, the rod shown in Fig. 11 or 12 is extended to its full length. Then the graduations on the front appear continuous and the reading of the horizontal wire can be taken from the telescope. If the target is used on a high rod, it is first set exactly at 7 feet on the extension part of the rod shown in Fig. 11, or at 6.5 feet for the rods shown in Figs. 12 and 16. Then the rod is extended until the line dividing the colors on the target coincides with the horizontal wire, and is clamped in that position by tightening the screw *c*. The reading of the rod is indicated by the zero of the scale on the upper sleeve of the rod shown in Fig. 11 or 12, or on the side of the rod shown in Fig. 16.

40. Reading the Rod Directly.—If a target is not used, the reading of a rod is made directly from the telescope in the following manner. The number of feet is given by the shaded figure (red on the actual rod) below the horizontal cross-wire. The number of tenths is shown by the black figure directly below the wire. If the reading is required to the nearest hundredth on the rods shown in Figs. 11 and 16, the number of hundredths is found by counting the divisions between the last tenth and the graduation mark nearest to the wire. If thousandths of a foot are required, the number of hundredths is equal to the number of divisions between the last tenth and the graduation mark below the wire, and the number of thousandths is estimated by judgment. For example, the readings on the Philadelphia rod for the positions x, y, and z in Fig. 17 (*a*) are determined as follows: For x, the number of feet below is 4 and the number of tenths below is 1; the wire coincides with the first graduation above the tenth-mark and, consequently, the reading is 4.11 feet to the nearest hundredth, or 4.110 feet to the nearest thousandth. For y, the feet and tenths are again 4 and 1, respectively. The wire is just midway between the graduations indicating 4 and 5 hundredths, and, therefore, the reading to the nearest hundredth may be taken as either 4.14 or 4.15 feet; in determining the hundredths, it is convenient to observe that the wire is just below the acute-angle graduation denoting the fifth hundredth, and it is, therefore, unnecessary to count up from the tenth-graduation. If thousandths are required, the following method is used for finding the hundredths and thousandths: There are 4 divisions between the tenth-mark and the graduation below the wire; hence, the number of hundredths is 4. Since the wire is midway between the two graduation marks on the rod, and since the distance between graduations is 1 hundredth or 10 thousandths of a foot, the number of thousandths in the required reading is $\frac{1}{2} \times 10$, or 5; hence, the reading to the nearest thousandth is 4.145 feet. For z, the foot just above is 4 and, therefore, the foot below must be 3 (not shown in the figure); the number of tenths is evidently 9 and the number of hundredths 6; the distance to the wire from the hundredth mark below is about one-third of a graduation, as nearly as can be estimated, and, consequently, the number of thousandths is



(*a*) (*b*)

FIG. 17

$\frac{1}{3}\times 10$, or 3; hence, the reading to the nearest hundredth is 3.96 feet and, to the nearest thousandth, 3.963 feet. A New York rod may be read in the same way.

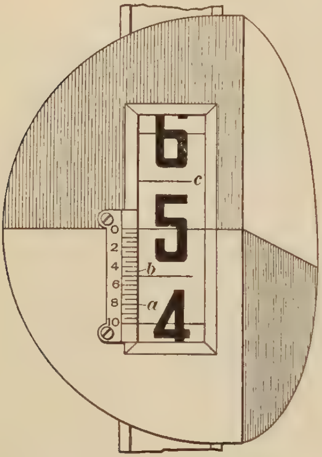
41. For the Philadelphia rod shown in Fig. 12, the feet and tenths are obtained as just explained for the other rod, and the hundredths and half-hundredths may be estimated with the aid of the half-tenth graduations. Thus, the readings of the cross-wire for the positions x, y, and z in Fig. 17 (b) are found as follows: Assume that the foot mark below the part of the rod shown is 5. For x, the number of tenths is 4, and the number of hundredths is less than 5 since the wire is below the half-tenth (.05) graduation; the distance above the tenth-mark is estimated to be about $\frac{4}{5}$ of the distance between graduations and the number of hundredths is, therefore, $\frac{4}{5}\times 5 = 4$; hence, the reading is 5.44 feet. For y, the number of tenths is 5 and the number of hundredths is more than 5; the distance above the half-tenth graduation is estimated to be $\frac{2}{5}$ of the distance to the tenth-mark and, consequently, the number of hundredths is $5+\frac{2}{5}\times 5=7$; the reading is, therefore, 5.57 feet. For z, the wire is midway between the tenth and half-tenth graduations and its distance above the mark that indicates 3 tenths is $\frac{1}{2}\times .05 = .025$; hence, the rod reading is 5.325 feet.

Direct readings on a long rod are made in the same way as on a short rod because the rod is fully extended and the graduations appear continuous.

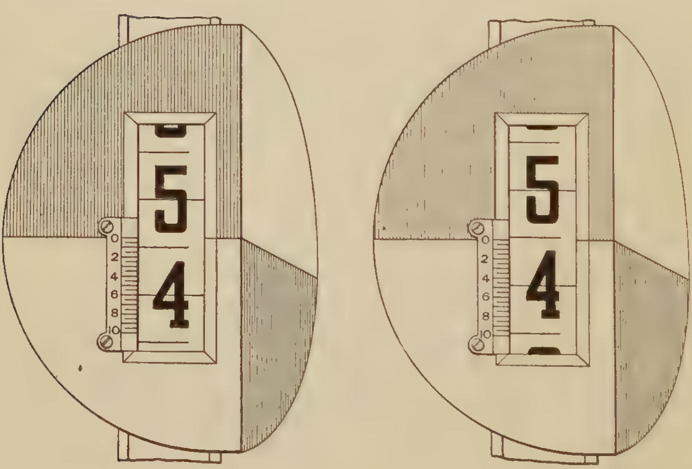
42. Scales.—In order to aid in reading a rod, a target is sometimes used. When the rod is graduated only to tenths or half-tenths of a foot, the scales on the target and on the upper sleeve are made exactly .1 foot long and are divided into 10 or 20 parts—that is, to hundredths or half-hundredths of a foot—so that hundredths or half-hundredths can be read directly and thousandths can be estimated with a fair degree of accuracy.

In Fig. 18 are shown settings of the target on the rod illustrated in Fig. 12; the zero point of the scale coincides with the line dividing the colors on the target and indicates the position where the cross-wire cuts the rod. The length of the scale is

.1 foot and, since it is divided into 20 parts, each small division is $\frac{1}{20} \times .1 = .005$ foot. The numbers on the scale indicate hundredths of a foot, the distance from 0 to 2 being .02 foot, from 0 to 4, .04 foot, etc. Thus, in (*a*), the distance of the point *a* below the zero mark is .08 foot and the distance of the point *b* below zero is .045 foot. It will be observed that the numbers on the scale increase downwards. Since the zero of the scale coincides with the cross-wire of the telescope, the reading of the rod consists of two parts: The feet and tenths are given on the rod by the nearest numbers below the zero of the scale; the remainder of the reading is the distance measured on the scale from the zero



(*a*)



(*b*) (*c*)

Fig. 18

of the scale to the nearest lower tenth-graduation on the rod. It should be noticed that, when the target is used, the half-tenth graduations on the rod, as at *c*, are disregarded.

For the setting in (*b*), the zero of the scale is above the graduation 4 on the rod; if the next lower foot on the rod is assumed to be 5, the feet and tenths for the reading are 5 and 4, respectively. The distance from the graduation 4 on the rod to the zero of the scale is shown to be .06 foot, since graduation number 6 on the scale is opposite that graduation on the rod. Hence, the reading is $5.4+.06=5.46$ feet.

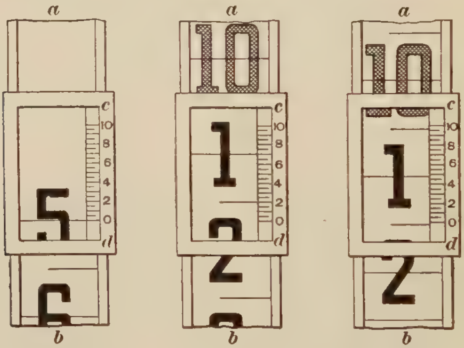
In case no graduation of the scale coincides exactly with a graduation on the rod, the thousandths are estimated by judgment. Thus, in Fig. 18 (*c*), the graduation 4 on the rod is slightly below a point midway between the marks on the scale indicating .045 and .05, and the distance from the graduation 4 on the rod to the zero point on the scale is estimated to be .048. Hence, the reading of the target is taken as 5.448 feet.

43. When a target is used on a high rod, the reading is made by means of the scale on the back of the upper sleeve or on the side of the rod according to the type of the rod. In Fig. 19, *a b* represents part of the back of the rod shown in Fig. 12, and *c d* is the scale on the upper sleeve. When the rod is closed, the zero of the scale coincides with the graduation on the rod indicating 6.5 feet, as shown in (*a*), because the target is set at 6.5 feet when the rod is extended. As the upper section of the rod is raised, the scale remains stationary, since it is attached to the lower section, but the graduations on the back of the rod move. The numbers of these graduations increase downwards in order that the reading should become greater as the target is raised. The numbers on the scale, therefore, increase upwards.

The method of reading a long rod is similar to that for a short rod, the difference being that the feet and tenths are given by the numbers above the zero of the scale instead of below it; for instance, in Fig. 19 (*b*) the number of feet is 10 and the number of tenths is 1. The hundredths and thousandths are found from the position of the scale with respect to the tenth-graduation on the rod just as for a short rod; since the mark indicating 7 hundredths coincides with the graduation 1 on the rod, the reading is 10.17 feet.

In (*c*), the feet and tenths are again 10 and 1, respectively, but no graduation on the scale coincides with the graduation on the rod. The graduation 1 is between the graduations on the scale indicating .045 and .05 and the distance from 1 on the rod to zero on the scale is estimated to be .047 foot; hence, the reading is taken as 10.147 feet.

44. Principle of the Vernier.—When the rods in Figs. 11 and 16 are read directly from the telescope, it is necessary to estimate thousandths of a foot. When the target is used, thousandths may be read accurately by means of the scale on the target, which is called a vernier. A vernier is a movable auxiliary scale used for the purpose of measuring accurately, on another scale, fractional parts of the smallest subdivisions of the main scale. Thus, if the main scale is divided into feet and tenths of a foot, the vernier may be used to measure on the main scale hundredths of a foot; or if the main scale is divided into tenths and hundredths of a foot, the vernier may be used to measure thousandths of a foot. As verniers are used extensively on surveying instruments, the principles



(*a*) (*b*) (*c*)

FIG. 19

on which their construction is based should be thoroughly understood.

To illustrate the fundamental principle of a vernier, suppose that the bar *a b*, Fig. 20, is to be measured with the scale *c d*, which is divided in inches. The scale is placed against the bar so that the zero of the scale coincides with one end of the bar, and the position of the other end of the bar is observed. In this case, the bar measures 3 inches plus the distance between the graduation mark 3 and the point *e* which is opposite the end of the bar. Suppose now that it is desired to measure the distance 3-*e* in eighths of an inch. For this purpose a vernier *f g*, Fig. 21, is prepared, which has a total length of 7 inches and is divided into eight equal parts. Each division is thus equal to $\frac{7}{8}$ inch and the difference between one division of the main scale and one division of the vernier is $1 - \frac{7}{8} = \frac{1}{8}$ inch. Although distances in surveying are generally measured in feet and decimals of a foot rather than in feet, inches, and fractions of an inch, inches and eighths are used here to demonstrate the general principle of a vernier.

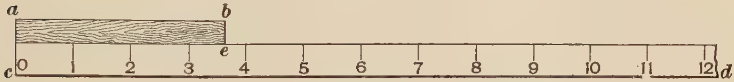


FIG. 20

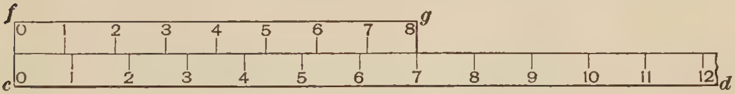


FIG. 21

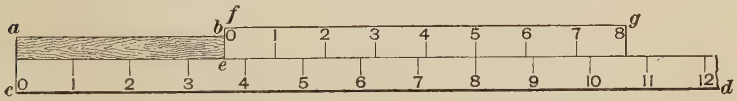


FIG. 22

To measure the distance 3-*e* with the vernier, place the vernier against the main scale, as shown in Fig. 22, so that the zero of the vernier coincides with the point *e*. To find the number of eighths in the distance 3-*e*, look for a graduation mark on the vernier that coincides with some graduation on the main scale. In this case it will be noticed that graduation number 5 of the vernier coincides with a mark on the main scale; thus, the number of eighths in 3-*e* is 5, and the distance 3-*e* is, therefore, $\frac{5}{8}$ inch. The explanation of this is as follows.

Imagine that the vernier is placed against the main scale so that the zero mark of the vernier coincides with line 3 of the main scale, and then the vernier is slid along until its zero reaches the point *e*. The distance covered by the zero mark of the vernier during this motion will measure the distance 3-*e* on the bar. When, in this motion, the vernier is in the position shown in Fig. 23 (*a*), the graduation mark 1 coincides with a graduation on the main scale; and the zero mark of the vernier has covered a distance equal to the difference between one division of the main scale and one division of the vernier, or $\frac{1}{8}$ inch. When the vernier is in the position shown in (*b*), point 2 coincides with a point on the main scale; the zero of the vernier has, therefore, moved a distance equal to the difference between two divisions of the main scale and two divisions of the vernier, which is $\frac{2}{8} = \frac{1}{4}$ inch. When in the position shown in (*c*), the zero of the vernier has moved $\frac{3}{8}$ inch; and, finally, when the zero point of the vernier has moved to the point *e*, Fig. 22, it has covered the distance equal to the difference between 5

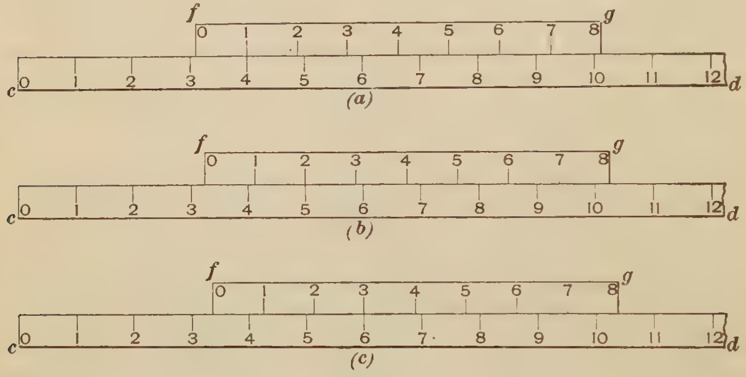


FIG. 23

divisions of the main scale and 5 divisions of the vernier, or $\frac{5}{8}$ inch.

The same conclusion can be drawn directly by analyzing the position shown in Fig. 22. In this position the distance between point 4 on the vernier and point 7 on the main scale equals the difference between one division of the main scale and one division on the vernier; the distance from point 3 on the vernier to point 6 on the main scale equals the difference between two divisions on the main scale and two divisions on the vernier, and so on; finally, the distance between point 3 on the main scale and the zero mark of the vernier, which is opposite point *e*, is equal to the difference between five divisions of the main scale and five divisions on the vernier, or $\frac{5}{8}$ inch.

45. In general, the length of a vernier is made equal to a certain number of the smallest divisions of the main scale. In the example just considered the smallest division of the main scale is 1 inch and the number of divisions covered by the vernier is 7; the total length of the vernier is, therefore, 7 inches. This length is divided into a number of parts which is one more than the selected number of main-scale divisions. In the preceding example the vernier is, therefore, divided into $7+1=8$ equal parts, each being $\frac{7}{8}$ inch long.

The difference between one division of the main scale and one division of the vernier is called the *least reading of the vernier*. It is equal to the length of one smallest division of the main scale divided by the number of parts into which the vernier is divided. In the example just considered, the smallest division of the main scale is 1 inch and the vernier is divided into 8 parts; therefore, the least reading of the vernier is in this case $\frac{1}{8}$ inch.

46. To measure the length of a line with a scale and a vernier, the zero mark of the scale is placed at one end of the line and the vernier is slid along the scale until its zero mark is at the other end of the line. Then the length consists of two parts: first, the distance from the zero mark of the main scale to the graduation of the main scale preceding the zero mark of the vernier, which is indicated by the number of that graduation; second, the distance from the zero mark of the vernier to the preceding main-scale graduation, which is determined by multiplying the least reading of the vernier by the number of the vernier graduation that coincides with a scale graduation. The first distance is sometimes called the *reading of the scale* and the second the *reading of the vernier;* but generally the term *reading of the vernier* is understood to mean the entire distance from the zero mark of the scale to the zero mark of the vernier. For instance, in the case shown in Fig. 22, the distance on the main scale from the zero mark to the graduation preceding the zero mark of the vernier is 3 inches, as indicated by the number 3 at that graduation; the fractional part of a division is $\frac{5}{8}$ inch, which is the product of the least reading of the vernier, or $\frac{1}{8}$ inch, and the number of the vernier graduation that coincides with a scale graduation, or 5; hence, the total length of the bar is $3+\frac{5}{8}=3\frac{5}{8}$ inches.

It should be noticed particularly that in reading the vernier the number of the main-scale graduation, with which the vernier graduation coincides, is of no importance and is not observed. It should also be noticed that the numbers on the vernier increase in the same direction as do those on the main scale.

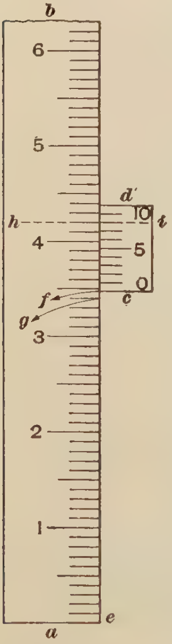


FIG. 24

47. Vernier for Level Rod.—In Fig. 24 is shown part of a level rod *a b* with a vernier *c d*. The divisions marked 1, 2, etc., on the rod are tenths of a foot, and each of these is divided into ten equal parts, or hundredths of a foot. The vernier covers nine of the smallest subdivisions of the scale and is, therefore, .09 foot long; it is divided into $9+1=10$ parts, each space being one-tenth of .09, or .009 foot. The difference between one division on the rod and one on the vernier is $.01-.009=.001$ foot, which is the least reading of the vernier.

The least reading can also be obtained by dividing the value of a rod division by the number of parts in the vernier; thus, $\frac{.01}{10} = .001$ foot.

The reading of a leveling rod is the distance from the base of the rod to the line dividing the colors on the target, or to the zero of the vernier. This distance consists of two parts, one from the base of the rod to the graduation preceding the zero of the vernier, and the other from that rod graduation to the zero of the vernier. The first distance is found by the same method as for a direct reading, except that the zero of the vernier takes the place of the cross-wire. The reading of the vernier, which is added, is found by multiplying the least reading of the vernier by the number of the vernier graduation that coincides with a rod graduation.

In Fig. 24, it is assumed that the base of the rod is at *e* and the reading of the rod is the distance *e f*, which is made up of the two parts *e g* and *g f*. The first of these is $.3 + .04 = .34$ foot. To find the distance *g f*, it will be noticed that the eighth graduation mark of the vernier coincides with a graduation mark of the rod (it is not necessary to note which mark). This coincidence is indicated by the dotted line *h i*. Since one division of the rod is greater by .001 than one division of the vernier, the seventh mark of the vernier is .001 foot from the rod graduation immediately below it; the sixth mark of the vernier is .002 foot from the mark on the rod immediately below it, etc. In this manner it is found that the zero mark of the vernier is .008 foot from the rod graduation *g* immediately below it. The distance *g f* is, therefore, .008 foot. According to the principle explained in the preceding article, this result may be obtained by multiplying the least reading of the vernier, .001, by 8; thus, $.001 \times 8 = .008$ foot.

The reading of the rod for the target setting in Fig. 25 (*a*) is determined as follows: The number of feet is given by the red figure on the rod *a b* below the zero of the vernier *c d*; no foot graduation being shown in the diagram, it will be assumed as 4. The number of tenths is shown by the black figure below the zero of the vernier; in this case, it is 6. The

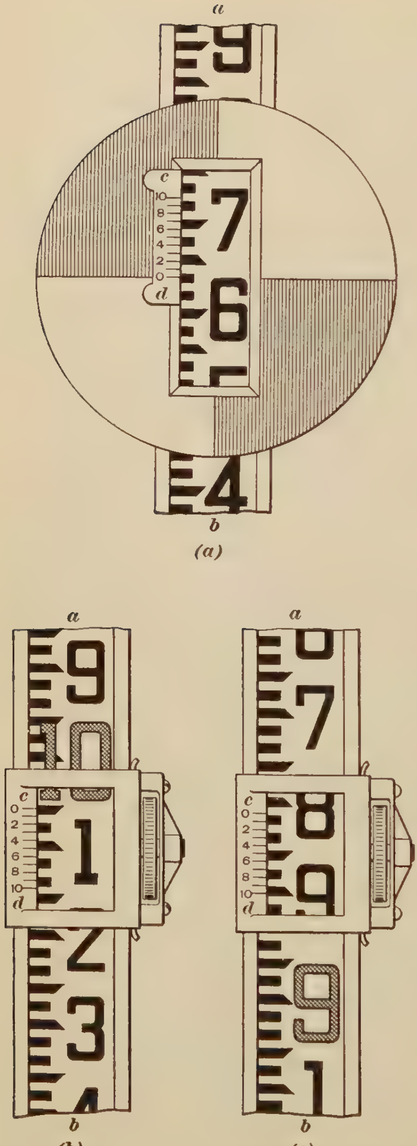


FIG. 25

number of hundredths is found by counting the graduations on the rod between the tenth below and the zero of the vernier; the zero of the vernier is above the graduation of the rod indicating .03 foot. Here, the fifth vernier graduation coincides with a rod graduation, and, consequently, the number of thousandths is 5. The reading of the target is, therefore, 4.635 feet.

48. When the rear section of the rod is extended, the reading is determined by means of the vernier on the upper sleeve through which the section slides. In Fig. 25 (*b*), *a b* is a portion of the rear section of the rod and *c d* is the vernier on the sleeve. The method of determining the reading is similar to that for the front of the rod, but since the numbers on the rear of the rod increase downwards, the foot, tenth, and hundredth are above the zero of the vernier; the numbers on the vernier also increase downwards to correspond to the direction in which the numbers on the rod increase. In this case, the foot above the zero of the vernier is 10, the tenth above it is zero, and the hundredth is 5; since the number of the vernier graduation that coincides with a rod graduation is 3, the number of thousandths is also 3; the rod reading is, therefore, 10.053 feet.

In (*c*), the foot below the zero of the vernier is shown as 9 and the foot above will, therefore, be 8; the tenth above the zero of the vernier is 8; the hundredth above the zero of the vernier is 0; and since the seventh graduation of the vernier coincides with a graduation mark on the rod, the number of thousandths is 7. Hence, the reading is 8.807 feet.

The scale on the target and the scale *g* on the side of the rod shown in Fig. 16 are both verniers; and the method of reading the rod is the same as that explained for the rod shown in Fig. 11.

49. Formulas Relating to Verniers.—In the following formulas, let

s = length of one of the smallest subdivisions of the scale;
n = number of equal parts into which the vernier is divided;
r = least reading of the vernier;
l = total length of the vernier;
m = number of the vernier graduation coinciding with a graduation of the scale;
R = reading of the vernier.

Then, from the principles explained in the preceding articles,

$$r = \frac{s}{n} \qquad (1)$$

$$n = \frac{s}{r} \qquad (2)$$

$$l = (n-1)s \qquad (3)$$

$$R = r\,m \qquad (4)$$

For example, in Fig. 22, the value s of a scale division is 1 inch; the number n of vernier divisions is 8; the total length l of the vernier is $(n-1)\,s=(8-1)\,1=7$ inches; the least reading r of the vernier is $\frac{s}{n}=\frac{1}{8}$ inch; m is 5, since the fifth vernier graduation coincides with a graduation on the scale; and the reading R of the vernier is $r\,m=\frac{1}{8}\times 5=\frac{5}{8}$ inch.

Example 1.—A scale is divided into inches and half inches; the vernier is divided into eight equal parts, which cover seven of the half-inch divisions of the scale. (*a*) What is the least reading of the vernier? (*b*) What is the reading of the vernier when its third graduation coincides with a graduation of the scale?

Solution.—(*a*) Here $s=\frac{1}{2}$ in., and $n=8$. Therefore, by formula **1**,

$$r=\frac{\frac{1}{2}}{8}=\frac{1}{16}\text{ in.}\quad\text{Ans.}$$

(*b*) Here $r=\frac{1}{16}$ in. and $m=3$. Therefore, by formula **4**,

$$R=\frac{1}{16}\times 3=\frac{3}{16}\text{ in.}\quad\text{Ans.}$$

Example 2.—The scale of a barometer is divided into inches and fiftieths. (*a*) What must be the length of a vernier, and how must the vernier be divided, that its least reading may be .002 inch? (*b*) What is the reading of the vernier when its seventh mark coincides with a graduation on the scale?

Solution.—(*a*) In this case, $s=\frac{1}{50}=.02$ in., and $r=.002$ in. Then, by formula **2**,

$$n=\frac{.02}{.002}=10$$

The vernier must, therefore, be divided into ten equal parts covering nine subdivisions of the scale. Its length will be $\frac{9}{50}$ in. Ans.

(*b*) Formula **4** is applied, in which $r=.002$ in. and $m=7$. Hence,

$$R=.002\times 7=.014\text{ in.}\quad\text{Ans.}$$

50. It will be readily seen that a vernier on a leveling rod is practical only when the value s is relatively small, because then the vernier will be of reasonable length. If it were required to use a vernier on the rod in Fig. 12 with a least reading of the vernier of .005 foot, the number of parts of the vernier would be $\frac{.05}{.005}=10$. Then the length of the vernier would be (10 − 1) .05 = .45 foot, which is about $5\frac{1}{2}$ inches. Since the target would have to be about 12 inches in diameter, a vernier in this case would be impractical.

OPERATIONS OF DIRECT LEVELING

GENERAL METHOD

51. Running a Line of Levels.—The fundamental principle of direct leveling was illustrated in Fig. 10, in which case the elevation of point *B* was determined from that of point *A* by setting up the level between the two points and taking rod readings at *A* and *B*. In Fig. 26, rods at *A* and *K* cannot be seen from the same position of the level. Therefore, if it is required to find the elevation of point *K* from that of *A*, it is necessary to set up the level several times and to establish intermediate points such as *C*, *E*, and *G*. These are the conditions commonly met with in practice, and may serve as an illustration of the general methods of direct leveling.

Let the elevation of *A* be assumed as 20.00 feet. The level is first set up at *B* so that a rod held at *A* will be visible through the telescope; and the reading of the rod is found to be 8.42 feet. The height of instrument, abbreviated *H. I.*, is found by adding this reading to the elevation of point *A*; thus, 20.00 + 8.42 = 28.42 feet = *H. I.* The rod reading at *A* is taken by directing the line of sight back toward the start of the line and is, therefore, called a *backsight reading*, or simply a *backsight;* backsight is abbreviated *B. S.* A better definition of a backsight, however, is a rod reading which is taken on a point of known elevation to find the height of instrument. Since a backsight is usually added to the elevation of the point on which the rod is held, a backsight is often called a *plus sight*, written +*S*.

After the *H. I.* has been determined by a backsight on *A*, a point *C* is selected which is slightly below the line of sight, and the reading is taken on a rod held at *C*. If this reading is 1.20 feet, the point *C* is 1.20 feet below the line of sight, and

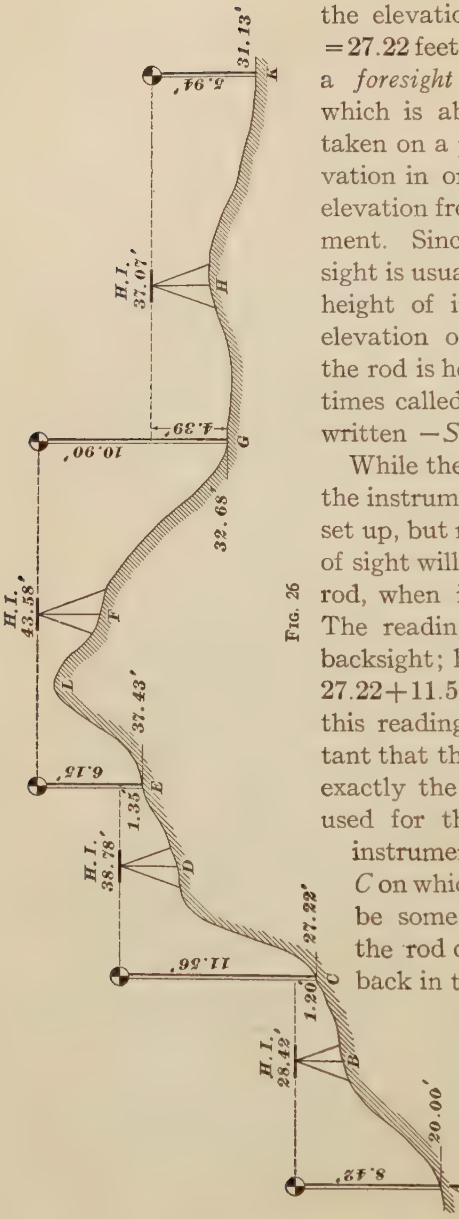


FIG. 26

the elevation of *C* is 28.42 − 1.20 = 27.22 feet. This reading is called a *foresight reading*, or *foresight*, which is abbreviated *F. S.*; it is taken on a point of unknown elevation in order to determine that elevation from the height of instrument. Since the reading for a foresight is usually subtracted from the height of instrument to get the elevation of the point on which the rod is held, a foresight is sometimes called a *minus sight* and is written −*S*.

While the rodman remains at *C*, the instrument is moved to *D* and set up, but not so high that the line of sight will go over the top of the rod, when it is again held at *C*. The reading 11.56 is taken as a backsight; hence, the *H. I.* at *D* is 27.22 + 11.56 = 38.78 feet. When this reading is taken, it is important that the rod should be held on exactly the same point that was used for the foresight when the instrument was at *B*. The point *C* on which the rod is held should be some stable object, so that the rod can be removed and put back in the same place as many times as may be necessary. For this purpose, a sharp-pointed rock, or a well-defined projection on some permanent object, is preferable; but if nothing better is available, a stake or peg can be driven firmly in the ground and the rod held on top of it. Such a point as C, on which both a foresight and a backsight are taken, is called a *turning point*, abbreviated $T. P.$

When the $H. I.$ at D is known, another turning point E is chosen and a foresight of 1.35 feet is obtained. Then the elevation of E is $38.78-1.35=37.43$ feet. The instrument is set up at F and the backsight on E is 6.15 feet; the new $H. I.$ is $37.43+6.15=43.58$ feet. The instrument at F should be high enough to have the line of sight clear the ground at L. The rod is held on a turning point at G, and since the foresight is 10.90 feet, the elevation of G is $43.58-10.90=32.68$ feet. When the instrument is moved to H, the backsight on G is 4.39 feet and the $H. I.$ is $32.68+4.39=37.07$ feet. For the rod held at K, the foresight is 5.94 feet; the elevation of K, therefore, is $37.07-5.94=31.13$ feet. The difference in elevation between A and K is $31.13-20.00=11.13$ feet; that is, point K is 11.13 feet higher than point A.

The process of determining the elevations of any series of points is called *running a line of levels*. All the points whose elevations may be determined by a line of levels need not be turning points as is the case in Fig. 26. At each position of the instrument, foresights on any number of points may be taken for the purpose of determining their elevations, either before or after the foresight on the turning point is taken. Points on which only foresights are taken are called *intermediate points*. Thus, the distinction between a turning point and an intermediate point is that both a backsight and a foresight are taken on a turning point, whereas on an intermediate point only a foresight is taken.

EXAMPLE.—The backsight reading on a turning point is 5.28 feet and the foresight reading on the next turning point is 3.25 feet. If the elevation of the first point is 142.00 feet, what are (*a*) the height of instrument and (*b*) the elevation of the second turning point?

SOLUTION.—(*a*) The *H. I.*, which is equal to the elevation of the first point plus the backsight on it, is $142.00+5.28=147.28$ ft. Ans.

(*b*) The elevation of the second turning point is equal to the height of instrument minus the foresight on the turning point; in this case, it is $147.28-3.25=144.03$ ft. Ans.

EXAMPLES FOR PRACTICE

1. The backsight reading on a point is 7.36 feet and the foresight reading on a second point is 2.84 feet. If the elevation of the first point is 200.00 feet, what are (*a*) the height of instrument and (*b*) the elevation of the second point?
Ans. (*a*) 207.36 ft.; (*b*) 204.52 ft.

2. If, when the instrument is set up in a new position, the backsight on the second point mentioned in the preceding example is 11.32 feet, what is the height of instrument? Ans. 215.84 ft.

3. The height of instrument is 125.32 feet and the foresight on a turning point is 4.33 feet. After the instrument has been moved to a new position, the backsight on the turning point is 8.57 feet. What is the elevation of a station, on which the rod reading is 9.20 feet?
Ans. 120.36 ft.

4. The height of instrument is 233.06 feet and the foresight on a turning point is 6.32 feet. After the instrument has been set up in a new position, the backsight on the turning point is 9.58 feet. What are the elevations, to the nearest tenth of a foot, of three stations, on which the rod readings are 5.2 feet, 6.3 feet, and 7.5 feet, respectively?
Ans. 231.1 ft.; 230.0 ft.; 228.8 ft.

PRACTICAL CONSIDERATIONS

52. Plumbing the Rod.—In order to get the correct vertical distance from the line of sight to the point on which the rod is placed, the rod must be vertical or plumb. When there is not much wind, the rodman can hold the rod practically plumb by balancing it as nearly as possible. Usually, the levelman considers that the rod is plumb in the direction across the line of sight when the edge of the rod is parallel to the vertical cross-wire, and the rodman plumbs the rod in the direction of the line of sight by standing to one side of the rod and judging if it is plumb, or, if possible, comparing it with the vertical edge of a near-by building.

The following method is commonly used for getting an accurate reading especially in a strong wind: The rodman slowly tips the rod backwards and forwards, in the direction of the line of sight and the levelman takes the least reading of the rod as the correct reading. If the target is used, it must be in such a position that the division line never goes above the horizontal cross-wire but just touches the wire at one position.

The form of a target shown on the rod in Fig. 12 is helpful in plumbing the rod. When the target is properly set and the rod is plumb, the horizontal line dividing the colors on the two faces will appear straight and will coincide with the horizontal cross-wire; while if the rod is not plumb, this line will appear broken. For very accurate work, the rod is plumbed by means of a *rod level*, which carries two spirit levels at right angles to each other, and can be attached to the rod.

53. Lengths of Sights.—The most advantageous lengths of sights will depend on the telescope of the instrument, the character of the surface of the country, the sensitiveness of the level bubble, and the degree of accuracy required. In the interests of speed and accuracy, it is always best to have the sights of moderate length; extremely long or short sights should be avoided. For reasonably accurate work, the sights should not usually exceed 400 feet. Where the country is level and time is of great importance, while only an ordinary degree of accuracy is required, quite long sights may be taken. Where the surface rises or falls rapidly, short sights become necessary.

54. The most valuable and reliable safeguard against errors due to imperfect adjustment of the instrument is obtained by equal lengths of sights for backsights and foresights on turning points; that is, the distances over which the sights for the backsight and the foresight from the same position of the level are taken should be approximately equal. Should any inequality of length occur in one such pair of sights, it should be balanced in the next pair, or as soon as possible. For example, should the foresight in one pair of sights be taken over a greater distance than the backsight, then in the next pair of sights the distance for the backsight should be made correspondingly longer than that for the foresight. On a hill, the sights on turning points will necessarily be much shorter in one direction than in the other if the level is set nearly in line with the turning points. In order to make the lengths of sights for the backsight and foresight from a set-up approximately equal, the level may be set to one side of the line. It is not necessary to measure the lengths of the sights accurately; they can be determined closely enough by counting steps in walking.

55. Bench Marks.—A permanent point whose elevation is determined in running a line of levels, and which is properly witnessed and recorded for reference, is called a *bench mark*. Any well-defined and easily identified point on a permanent object, such as the door-sill of a building, a stone or concrete monument, the projecting root of a tree, or a point on a large rock, will serve for a bench mark. If possible, any line of levels should be started from a bench mark, so that the elevations of the points will be obtained with reference to a definite datum. Bench marks should be established at intervals of from 1,000 to 2,000 feet, depending on the character of the ground and the purpose for which the levels are taken.

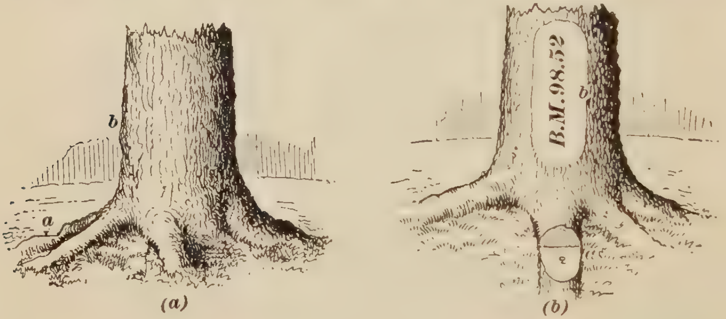


FIG. 27

To set a bench mark on the root of a tree, a broad notch is cut in the root, as shown at *a* in Fig. 27 (*a*) and (*b*); and a nail is driven almost flush with the surface thus formed. To witness the bench mark, the tree is *blazed* on the side facing the bench mark as shown at *b*. In the blazed space, the letters *B. M.* and the elevation of the bench mark are plainly written with keel, which is a kind of crayon. The rod is held on the nail, which marks a definite point that does not change in elevation. The elevation of each bench mark is always recorded in the notebook with sufficient description to identify the bench mark unmistakably.

The datum to which the elevation of the bench mark refers may be any imaginary level surface, or it may be sea level, as explained in Art. **6.** Many bench marks whose elevations refer to sea level have been established by the United States Coast and Geodetic Surveys, by the engineers of the various railroad companies, and by other engineers. Whenever practical, a line of levels should be started from such a bench mark in order to refer the elevations of the points to sea level. If the elevation is not written on the bench mark or on its witness, it can usually be obtained from the railroad company or from the engineer who set it. When the elevation of the starting point is assumed, its value should be so much above the datum that in running the line of levels none of the elevations taken will be less than zero. It is customary to make the assumed elevation some multiple of 10 feet, usually 100 feet.

56. Care in Reading the Rod.—The rod is always read more closely on bench marks and turning points than on intermediate points or stations. This is because an error in the rod reading on a turning point will affect all subsequent elevations in the line of levels, and an error in the rod reading on a bench mark will affect all later levels that may be referred to the bench mark as a starting point, whereas an error in the rod reading on an intermediate point will affect only the elevation of that point. Consequently, the rod is usually read on an intermediate point only as closely as it is desired to determine the elevation of that point. For grading earth roadways and work of a similar character, the rod is usually read to the nearest tenth of a foot on intermediate points and to the nearest hundredth of a foot on turning points and bench marks; while for work requiring a higher degree of accuracy, such as the surface of a finished pavement, the rod is usually read to the nearest hundredth of a foot on intermediate points and to the nearest thousandth of a foot on turning points and bench marks.

57. Signals and Calls.—In running a line of levels it is necessary for the levelman and the rodman to be in almost constant communication with each other. As a means of communication, certain convenient signals and calls are employed. It is important that the levelman and rodman understand these in order to avoid mistakes. When a target reading is taken, the target is set by the rodman in the proper position on the rod, according to signals given by the levelman. An upward movement, or raising, of the hand is the signal for raising the target; a downward movement, or lowering, of the hand is the signal for lowering it; a circle described by the hand is the signal for clamping the target; and a wave with both hands indicates that the target is set properly, or all right. Other signals, such as that for plumbing the rod, are arranged by the members of the party.

The rodman should then read the position of the target on the rod and call out the reading to the levelman, who records it; he should call first the number of feet, or, if the reading is less than 1 foot, he should call naught (not ought); then, after pausing a moment, he should give the decimal part of the reading. Thus, if the rod is being read to hundredths of a foot only, the number 8.40 is called *eight, four, naught;* a reading of 8.04 is called *eight, naught, four.* If the rod is being read to thousandths, the number 8.401 is called *eight, four, naught, one;* 8.410 is called *eight, four, one, naught.* The levelman should always repeat the reading to the rodman in order to insure clear understanding before he records it in the notebook.

When a self-reading rod is used, the rodman keeps the rod extended to full length, unless the country is very level, and the levelman reads the rod on all intermediate points. Then his signal for all right is usually an outward wave of the hand. Sights on turning points or bench marks are often read more closely by use of the target. In many cases, however, the levelman reads the rod on turning points as well as on intermediate points.

In accurate work it is the common practice for the levelman to read the rod without using the target, and to call the reading to the rodman who then sets the target at that value as a check. In this way, mistakes in feet, which are liable to occur, are avoided, and much time is saved by making it unnecessary to move the target up and down several times before the correct position is found.

FORMS FOR LEVEL NOTES

58. Forms of keeping level notes differ somewhat in detail, according to the individual preferences of the engineer, but all are based on the height of instrument. As previously explained, the height of instrument is found by adding a backsight reading to the known elevation of a bench mark or turning point, and the elevation of any point is determined by subtracting a foresight reading from the height of instrument.

The notebook for field notes is ruled with six columns on each page. Three forms of notes in common use will be illustrated and explained.

59. In Fig. 28 is shown the method of keeping level notes which is most used. The title or purpose of the survey, the location, the date, and the names and positions of the members of the party should be given. The pages should be numbered. The notes may read downwards from the top of the page, or upwards from the bottom of the page. In the first column of each page are recorded the points, or stations, at which the rod is held; in the second column, the backsight readings are given; in the third column, the heights of instrument; in the fourth column, the foresight readings; and in the fifth column, the elevations of the stations. The sixth column is left for other purposes, such as the description of bench marks, turning points, and important stations. Usually the entire right-hand page is left for remarks.

In the column for stations, the bench marks and turning points are designated *B. M.* and *T. P.*, respectively, and the intermediate points are identified by either letters or numbers. An elevation between two heights of instrument is found by subtracting the corresponding foresight from the *H. I.* preceding it in the notes. It will be observed that the readings and elevations for bench marks and turning points are given to hundredths of a foot while those on intermediate points are to the nearest tenth. The backsights and foresights must be recorded in the field but the heights of instrument and the elevations can be determined at any convenient time.

Level Notes

16
Profile for Siding—P. R. R.
Near Greensburg, Pa.
For Location Notes, See Field Book 62, Page 57.

17
Oct. 16, 1925
J. Scott—Leveler
A. Barnes—Rodman

Sta.	B. S.	H. I.	F. S.	Elev.		Remarks
B. M.				100.00		Spike on root of white oak stump 60 feet to left of Sta. 0.
	2.17	102.17				
0			4.8	97.4		
1			6.2	96.0		
2			9.1	93.1		
2+50			8.2	94.0		
3			7.6	94.6		
4			7.4	94.8		
T. P.			8.54	93.63		Top of stake near Sta. 4.
	4.58	98.21				
4+60			11.9	86.3		Spring Brook.
5			0.5	97.7		
T. P.			2.67	95.54		Nail in notch in stump of beech tree opposite Sta. 5+30.
	10.32	105.86				
5+75			2.4	103.5		
6			2.1	103.8		
7			6.4	99.5		
8			7.7	98.2		
9			6.5	99.4		
10			8.7	97.2		
T. P.			10.17	95.69		On rock near Sta. 10.
	2.44	98.13				
11			2.4	95.7		
12			7.2	90.9		
13			8.8	89.3		
B. M.			11.29	86.84		Spike on root of large maple tree 50 ft. to the right of Sta. 13+80.

Fig. 28

60. In Fig. 29 is shown a modification of the level notes shown in Fig. 28, these notes referring to the same survey.

LEVEL NOTES

16
Profile for Siding—P. R. R.
Near Greensburg, Pa.
For Location Notes, See Field Book 62, Page 57.

17
Oct. 16, 1925
J. Scott—Leveler
A. Barnes—Rodman

Sta.	B. S.	H. I.	F. S.	I. S.	Elev.	Remarks
B. M.	2.17	102.17			100.00	Spike on root of white oak stump 60 ft. to left of Sta. 0.
0				4.8	97.4	
1				6.2	96.0	
2				9.1	93.1	
2+50				8.2	94.0	
3				7.6	94.6	
4				7.4	94.8	
T. P.	4.58	98.21	8.54		93.63	Top of stake near Sta 4.
4+60				11.9	86.3	Spring Brook.
5				0.5	97.7	
T. P.	10.32	105.86	2.67		95.54	Nail in notch in stump of beech tree opposite Sta. 5+30
5+75				2.4	103.5	
6				2.1	103.8	
7				6.4	99.5	
8				7.7	98.2	
9				6.5	99.4	
10				8.7	97.2	
T. P.	2.44	98.13	10.17		95.69	On rock near Sta. 10.
11				2.4	95.7	
12				7.2	90.9	
13				8.8	89.3	
B. M.			11.29		86.84	Spike on root of large maple tree 50 ft. to the right of Sta. 13+80.
	19.51		32.67			
						100.00
						19.51
						119.51
						32.67
						86.84

FIG. 29

One difference is that the foresights on turning points and the foresights on intermediate points are kept in separate columns, those on turning points being designated by *F. S.*, and those on the intermediate points, by *I. S.* By this arrangement the foresights on the turning points can be added conveniently for checking the notes in the manner described in a following article. Another difference from the notes in Fig. 28 is that the backsight and the height of instrument are placed on the same line with the foresight and the elevation for the turning point.

61. In Fig. 30 is shown another method of keeping level notes that is in quite general use. The distinguishing feature of this method is a single column for all rod readings, the column being headed Rod. Since the backsights are added and the foresights subtracted, they are indicated by the signs + and −, respectively. The column of rod readings is placed between the columns of heights of instrument and elevations for convenience in performing the operations of addition and subtraction.

These notes refer to the same survey as do the notes in Figs. 28 and 29, but more accurate values are used; the rod readings and elevations for bench marks and turning points are given to thousandths of a foot and the values for intermediate points are taken to the nearest hundredth.

62. How to Check Level Notes.—A common method of checking level notes affords a reliable check on the elevations of turning points and heights of instrument, which in a general way is a check on the line of levels as a whole, since all other elevations are deduced from these. The method is based on the fact that all the backsights are additive or + quantities and all the foresights are subtractive or − quantities. Therefore, if the sum of all the backsights in a line of levels, or any portion of it, is added to the elevation of the starting point, and from the sum thus obtained the sum of all the foresights on turning points in the same portion is subtracted, the remainder is the last height of instrument or the elevation of the last turning point, according as the last sight included is a backsight or a foresight.

The application of this method of checking is shown in the notes given in Fig. 29. The elevation of the bench mark near

Station 0 is 100.00 feet. The sum of the backsights, determined by adding the values in the column headed B. S., is

LEVEL NOTES

16

Profile for Siding—P. R. R.
Near Greensburg, Pa.
For Location Notes, See Field Book 62, Page 57.

17

Oct. 16, 1925
J. Scott—Leveler
A. Barnes—Rodman

Sta.	H. I.	Rod	Elev.			Remarks
B. M.			100.000			Spike on root of white oak stump 60 ft. to left of Sta. 0.
	102.172	+ 2.172				
0		− 4.75	97.42			
1		− 6.14	96.03			
2		− 9.03	93.14			
2+50		− 8.19	93.98			
3		− 7.58	94.59			
4		− 7.36	94.81			
T. P.		− 8.543	93.629			Top of stake near Sta. 4.
	98.212	+ 4.583				
4+60		−11.93	86.28			Spring Brook
5		− 0.47	97.74			
T. P.		− 2.674	95.538			Nail in notch in stump of beech tree opposite Sta. 5+30.
	105.862	+10.324				
5+75		− 2.39	103.47			
6		− 2.04	103.82			
7		− 6.38	99.48			
8		− 7.67	98.19			
9		− 6.43	99.43			
10		− 8.70	97.16			
T. P.		−10.171	95.691			On rock near Sta. 10.
	98.134	+ 2.443				
11		− 2.45	95.68			
12		− 7.20	90.93			
13		− 8.85	89.28			
B. M.		−11.292	86.842			Spike on root of large maple tree 50 ft. to the right of Sta. 13 +80.

FIG. 30

19.51 feet; this is added to the elevation of the starting point, the sum being 100.00+19.51=119.51 feet. The sum of the foresights is 32.67 feet; this is subtracted from the result just obtained, and the difference, which is 119.51 − 32.67 = 86.84 feet, is the elevation of the bench mark near Station 13+80. When the foresights are added, care must be taken to exclude all readings on intermediate points in case the form of notes is similar to that shown in Fig. 28 or in Fig. 30.

The leveler should check each page of notes, placing a check-mark (√) at the last height of instrument or elevation checked. This should preferably be done as soon as the page is filled, but if there is not time in the field, each day's work should be checked the same night.

CONDITIONS AFFECTING ACCURACY OF DIRECT LEVELING

CURVATURE AND REFRACTION

63. Curvature.—As has been explained, a level line is a curved line and the line of sight is a horizontal line, tangent to a level line at the instrument. Consequently, the reading of a rod held on a point is always greater than the difference in elevation between the horizontal cross-wire and the point because the line of sight cuts the rod at an elevation that is higher than the elevation of the wire. The error in the rod reading due to the curvature of the earth's surface may be computed in the following manner:

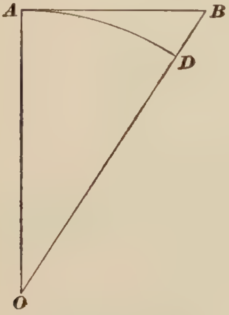


FIG. 31

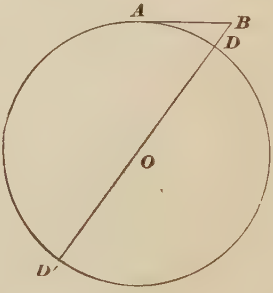


FIG. 32

In Fig. 31, let O represent the center of the earth, AB a horizontal line, and AD a level line through the cross-wire of an instrument at A. Then the error due to curvature is BD. Since the earth's surface is assumed to be spherical, the level line AD is a circular arc whose radius OD or OA is equal to the earth's radius. In order to show the conditions clearly, the distances in the illustration are shown very much out of proportion; OA is in fact about 20,890,000 feet and for ordinary sights AB is about 400 feet. In geometry it is shown that:

If from a point without a circle a tangent to the circle and a secant are drawn, the tangent is a mean proportional between the whole secant and its external segment.

Thus, in Fig. 32, the tangent AB is a mean proportional between the secant BD' and its external segment BD, or $\overline{AB}^2 = BD \times BD'$. But DD' is a diameter and is equal to twice the radius OD. Hence, in Fig. 32, $\overline{AB}^2 = BD \times (BD + 2\ OD)$. As BD is exceedingly small compared with the diameter of the earth, $2\ OD$, it may be dropped from the quantity within the parenthesis without appreciable error.

Let e_c = error due to curvature, in feet;
d = length of sight, in feet;
r = radius of earth, in feet.

Then, the preceding expression becomes $d^2 = 2\,r\,e_c$, from which

$$e_c = \frac{d^2}{2r}$$

64. Atmospheric Refraction.—It is a well-established law of physics that a ray of light in passing from a rarer to a denser medium is bent in a direction toward the denser medium, that is, so that its path will be concave on the side toward the denser medium. This bending of a ray of light is called refraction. Since the atmosphere is densest at the surface of the earth and becomes rarer as the distance from the earth's surface increases, it follows that a ray of light in passing from a higher to a lower elevation by an inclined path will be bent, or refracted, toward the surface of the earth, that is, so that its path will curve in the same general direction as the earth's surface.

Owing to refraction, a ray of light, that apparently is the straight line BA in Fig. 33, actually follows the curved path

CA because *B* is farther from the earth's surface than *A*; that is, the point observed through the level at *A* appears to be point *B* but is really point *C*. Then the error due to atmospheric refraction, which is represented by *BC* in Fig. 33, is given by the formula

$$e_r = m\frac{d^2}{r}$$

in which e_r = error due to refraction, in feet;
m = numerical coefficient;
d = length of sight, in feet;
r = radius of earth, in feet.

The coefficient m is called the *coefficient of refraction*. Its value varies somewhat for different elevations, but is about .072. The value of r is approximately 20,890,000 feet.

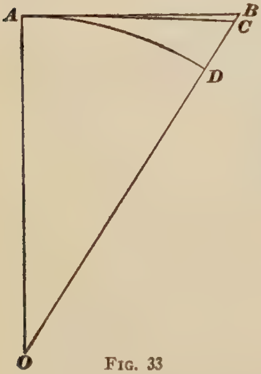


FIG. 33

65. Combined Error.—The combined effect of curvature and refraction causes an error represented by the distance *CD* in Fig. 33, which is evidently equal to the error due to curvature minus that due to refraction. If the combined error in feet is denoted by e, then

$$e = e_c - e_r = \frac{d^2}{2r} - m\frac{d^2}{r} = (.5 - m)\frac{d^2}{r}$$

The values of the correction e in decimals of a foot for various values of d expressed in feet are given in Table I; the corrections in this table are computed for $m = .0719$ and $r = 20{,}890{,}000$. For example, if the length of sight is 600 feet, the error due to curvature and refraction is found to be .007 foot. For a length between 300 and 5,280 feet, not listed in the column of distances in Table I, the value of the correction may be found by interpolation. It should be remembered that the correction for curvature and refraction is necessary only in very accurate work and where the difference in lengths of sights for backsights and foresights is great.

TABLE I

CORRECTION FOR THE COMBINED EFFECT OF CURVATURE AND REFRACTION

d	*e*	*d*	*e*	*d*	*e*	*d*	*e*
300	.002	1,600	.052	2,900	.172	4,200	.362
400	.003	1,700	.059	3,000	.184	4,300	.379
500	.005	1,800	.066	3,100	.197	4,400	.397
600	.007	1,900	.074	3,200	.210	4,500	.415
700	.010	2,000	.082	3,300	.223	4,600	.434
800	.013	2,100	.090	3,400	.237	4,700	.453
900	.017	2,200	.099	3,500	.251	4,800	.472
1,000	.020	2,300	.108	3,600	.266	4,900	.492
1,100	.025	2,400	.118	3,700	.281	5,000	.512
1,200	.030	2,500	.128	3,800	.296	5,100	.533
1,300	.035	2,600	.139	3,900	.312	5,200	.554
1,400	.040	2,700	.149	4,000	.328	5,280	.571
1,500	.046	2,800	.161	4,100	.345	10,560	2.285

ERRORS IN LEVELING

66. Sources of Error.—In leveling, the principal sources of error are defects of adjustment of the level, failure of the rodman to hold the rod plumb, and mistakes in reading the rod or recording the readings. The error due to curvature and refraction may become considerable when the difference between the sums of the lengths of the sights for backsights and foresights is great, even though the instrument is in perfect adjustment. If lengths of sights for the backsight and the foresight on turning points from the same position of the level are equal, errors due to imperfect adjustment and also those due to curvature and refraction are balanced, since the error in the foresight is equal to that in the backsight. However, since it is impractical to make the lengths of sights for backsights and foresights exactly equal, it is desirable to avoid errors as far as possible by adjusting the parts of the level by the methods previously explained. The methods of determining when the rod is plumb, also, have been described; and it is apparent that errors due to faulty plumbing can be easily guarded against. Mistakes in determining and recording rod readings can only be avoided by exercising great care in all operations and by checking whenever possible.

The rays of the sun shining directly on the object glass render the field of view indistinct and the sighting of the telescope uncertain. To prevent this, most instruments are provided with a sunshade, which fits the end of the telescope and projects over the object glass. If the sunshade is lacking, the levelman can hold his hat so as to shade the object glass.

Wind is also a source of error; it sometimes causes the instrument to vibrate, thus preventing the accurate setting of the target and it frequently exerts sufficient pressure against the instrument to cause the bubble to leave the center of its tube. As the pressure is fluctuating, the accurate centering of the bubble is rendered almost impossible. Wind also makes plumbing of the rod more difficult. Under such conditions, the levelman should wait for a lull in the wind, during which, if the rodman is alert, he can usually get a reasonably accurate reading. At a second lull, he can check the value and feel safe regarding it.

Other possible causes of error are the moving of the level due to the settling of the tripod legs in soft or thawing ground or due to their sliding on a smooth surface when a passing train or car produces vibration. It is also important to guard against moving of the turning point or failure of the rodman to hold the rod on the same point for the foresight and the backsight. Sliding of the tripod legs due to vibration can be prevented by proper choice of the set-up and by care in placing the metal points of the legs. Moving of a turning point can be avoided by choosing good permanent points. To avoid holding different points for the backsight and the foresight, turning points should be well defined and well marked when there is a chance of confusion.

67. Personal Equation.—There is also what is known as the personal error, sometimes called the personal equation, which is recognized as a defect of vision peculiar to the individual. By reason of this peculiarity of vision, two persons may observe the reading of a rod, or set the target, on the same sight and under precisely the same conditions, and obtain somewhat different readings. But as this personal equation, or error, is constant for the same person and affects all his observations in the same manner, it does not materially detract from the accuracy of work. Haste is also a fruitful source of error and is little if any aid to progress. Rapid and accurate work can be performed without haste, but work done in a hurry is not usually performed either rapidly or accurately.

68. Required Degree of Accuracy.—It has been found from experience that small errors occur much more frequently than large ones, and that those of an accidental character tend to balance each other. A line of levels 20 miles long, in which the rod is read directly to the nearest hundredth of a foot, will give nearly the same results on intermediate points, and nearly the same difference of elevation between the points at its extremities, as a line of precise levels taken with exact target readings. That painful degree of exactness termed hair-splitting is of no advantage in ordinary leveling; it represents very little actual gain in the accuracy of the results, and a very considerable increase in the cost of the work. The degree of accuracy required in each case should be ascertained and the levels taken with sufficient care to obtain that accuracy; greater exactness is an unnecessary waste of time.

If a line of levels is run from any point, completes a circuit, and comes back to the same point, the sum of all the backsights should equal the sum of all the foresights. The difference between these sums is called the *error of closure*. It is a reasonably well-established principle that the total or final value of the accidental errors of direct leveling increases with the length of the circuit and is proportional to the square root of that length. Hence, the permissible error of closure in a line of levels may be expressed by the general formula

$$E = c\sqrt{L}$$

in which E = permissible error of closure;
c = numerical coefficient;
L = length of circuit.

The value of c depends on the character of the survey and on the units in which the values of E and L are expressed. In practice, this formula has taken the following forms, which may be regarded as representative of the respective degrees of accuracy required in the leveling work in the various surveys named:

1.	Chicago Sanitary District	$E=.012\sqrt{M}$
2.	New York State Barge Canal	$E=.016\sqrt{M}$
3.	Missouri River Commission	$E=.018\sqrt{M}$
4.	Mississippi River Commission	$E=.018\sqrt{M}$
5.	United States Coast Survey	$E=.029\sqrt{M}$
6.	United States Lake Survey	$E=.042\sqrt{M}$
7.	United States Geological Survey	$E=.05\sqrt{M}$
8.	Good average work of ordinary character	$E=.05\sqrt{M}$
9.	Preliminary railroad surveys	$E=.1\sqrt{M}$

In these formulas E denotes the permissible error of closure in feet, and M the length of the circuit in miles. For example, in a circuit 100 miles long, in the United States Geological Survey, the permissible error of closure is found by substituting 100 for M in the formula for class 7; thus, $E=.05\sqrt{100}=0.5$ foot.

69. Check-Levels.—Before the elevations obtained by a line of levels which does not form a closed circuit are finally adopted as a basis for construction work, their accuracy should be verified by another line of levels over the most important and permanent points whose elevations were taken by the former line. This second line then completes the circuit. Levels for the purpose of verifying previous work are called *check-levels*, or *test levels*. In running check-levels, the most common practice is to take only the elevations of bench marks and important permanent points with such turning points as may be necessary in order to cover the distance; nearly all the intermediate points are omitted. The check-levels should always be run in the direction opposite to that of the original line in order to eliminate the effect of unavoidable errors. If the variation in the elevation of a point as given by the two sets of levels is less than the permissible value of E, the true elevation of the point is determined as follows: The difference in elevation between the starting point and the point in question is computed from each line of levels independently. Then, the average difference is added to, or subtracted from, the elevation of the starting point. But if the variation is greater than the allowable value of E, the levels should be run again over that portion of the line in which the variation occurs, in order to determine which of the elevations is correct. Where the true elevation of a bench mark is found to differ from the elevation marked on it, the value should be corrected.

EXAMPLES FOR PRACTICE

1. In good average work of the ordinary character, what would be the permissible error of closure in a line of levels of which the length is 18 miles?
Ans. .212 ft.

2. In the United States Geological Survey, what is the permissible error of closure in a line of levels 50 miles long? Ans. .354 ft.

3. In the United States Lake Survey, what is the permissible error of closure in running a line of levels over a distance of 30 miles?
Ans. .230 ft.

PROFILES

70. Definition.—A *profile* is a representation of the vertical section along a survey line. It shows the horizontal and vertical distances between points on the line as if the line were straight; that is, in a vertical plane. The actual line may be partly curved as in the case of a railroad or highway, or broken as in a sewer or water line.

The vertical distances on a profile are usually represented to a larger scale than the horizontal distances in order to make more distinct the irregularities of the surface along the line of survey. Therefore, a profile is seldom a true section along the line, except in geological work, where the horizontal and vertical distances are represented to the same scale. For railroad work, profiles are commonly constructed to a horizontal scale of 400 feet to the inch and a vertical scale of 20 feet to the inch. For municipal work, it is common to use a horizontal scale of 40 feet to the inch and a vertical scale of

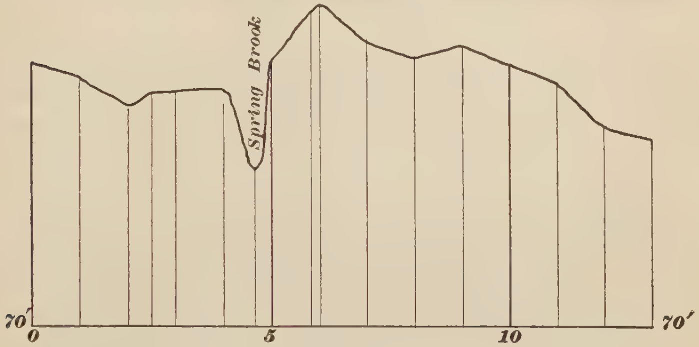
4 feet to the inch. Other scales are also used according to the character and requirements of the work.

71. Construction.—A profile can be constructed on plain paper in the following manner:

First, draw a reference line near the lower edge of the paper. For convenience, it is customary usually to make the elevation of the reference line slightly less than the lowest point on the survey, so that all points to be located will be above this line. Then, on the reference line, lay off to the horizontal scale the distances from the starting point of the survey to the stations whose elevations have been taken. Number these station points at intervals for identification, and from each point draw upwards in pencil an indefinite vertical line. On each vertical line locate a point representing the ground surface, by laying off, to the vertical scale, the difference in elevation between the reference line and the ground surface at the station. A heavy ink line connecting these points will constitute the profile of the survey line. The profile line should be drawn freehand to correspond to the natural irregularities of the earth's surface. As a guide in drawing the ink line when the points are far apart, they may first be connected by light pencil lines drawn with a straightedge. After the ground line has been drawn, the vertical lines at the stations are usually inked in between the reference line and the ground line.

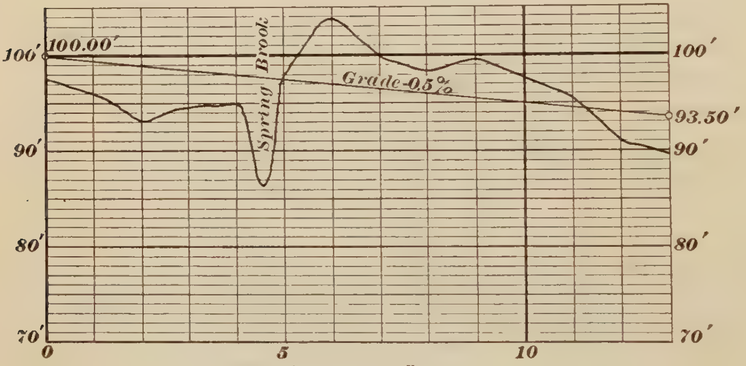
In Fig. 34 is shown the profile that would be drawn from the level notes given in Fig. 28. The horizontal scale is 1 inch equals 400 feet, and the vertical scale is 1 inch equals 20 feet. The elevation of the reference line is taken as 70 feet, this number being marked at each end of the line; on long lines the elevation would be marked at several intermediate points. The station numbers along the reference line in Fig. 34 are numbered every 500 feet, though often they are numbered at every 100 feet or at every 1,000 feet. The calculations for drawing this profile are as follows:

The elevation of Station 0, as given in Fig. 28, is 97.4; therefore, its height above the reference line is $97.4-70=27.4$ feet. This distance laid off on the vertical line at Station 0 locates the position of the surface at that place. The elevation of Station 1 is 96.0; therefore, a distance of $96.0-70=26.0$ feet is laid off on the vertical line at Station 1, thus locating the surface at that point. To locate the surface at Station 2, the distance to be laid off on the vertical line at that station is $93.1-70.0=23.1$ feet. The heights of the remaining stations above the reference line are laid off in a similar manner on the vertical lines at the corresponding stations. Finally, the surface line is drawn between the points as already described.



Scales: Horiz. 1″ = 400 ft.
Vert. 1″ = 20 ft.
FIG. 34

72. Profile Paper.—In order to facilitate the construction of profiles, paper prepared especially for the purpose is commonly used; this has horizontal and vertical lines in pale green, blue, or orange, so spaced as to represent certain distances to the horizontal and vertical scales. Such paper is called profile paper. The most common form of profile paper is divided into $\frac{1}{4}$-inch squares. Then the space between each two horizontal lines is divided into 5 equal parts by lighter horizontal lines, the distance between these light lines, therefore, being $\frac{1}{20}$ inch. Consequently, with the scales commonly used for railroad profiles, the light horizontal lines are 1 foot apart and the vertical lines are 100 feet apart. Of course, any desired values can be assumed for the spaces according to the requirements of the work. In order to aid in estimating distances, each tenth line in both directions is made extra heavy. A piece of profile paper showing the profile for the level notes given in Fig. 28 is illustrated in Fig. 35. The method of locating the points is similar to that explained in the preceding article but the distances can be found from the lines on the paper without actually measuring them. The elevation of the extra-heavy line is assumed to be 100 feet, and each division between light horizontal lines represents 1 foot. Then the ground at Station 0 is located 2.6 spaces below the extra-heavy line, and the ground at Station 1 is 4 spaces below. At Station 4+60, the elevation is 86.3, and the ground is 13.7 spaces below the extra-heavy line; since each heavy line represents 5 spaces, the point can be located by counting 2 heavy lines and then 3.7 spaces additional. The ground at Station 4+60 can also be located by first determining the heavy line at elevation 85.0 and then proceeding upwards 1.3 spaces.



Scales: Horiz. 1" = 400 ft.
Vert. 1" = 20 ft.

FIG. 35

73. Grade Lines.—In important engineering work, before the actual construction is begun, it is customary to decide on the position of some prominent line in the completed work and to adopt it as a line of reference; this line is called a *grade line*. The elevation of the grade line at any point is known as the *grade* at that point. When planning a railroad, the grade line represents the proposed position of the base of the rail; and for a street or highway, the grade line is the finished surface at the center line. In constructing a railroad or a highway, however, the proposed surface of the ground, which is called the *subgrade*, is taken as the grade line. Grade lines are usually straight for considerable distances. The grade line is shown with the profile of the present ground so that the distance of the present ground above or below grade can be readily seen. If a point on the profile of the original ground is above the subgrade, material must be excavated; while if a point on the profile of the ground is below the subgrade, it is necessary to fill in material. The process of excavating and filling to make the surface of the ground correspond to the proposed subgrade is called *grading*.

The principal purpose for which a profile is constructed is to enable the engineer to establish the grade line. For a railroad or a highway, the subgrade should, when possible, be located in such a position that the excavation and embankment will be nearly the same in amount. The position of the grade line having been determined, it is drawn on the profile in red ink.

74. Rate of Grade.—The inclination of a grade line to the horizontal may be expressed by the relation between the rise or fall of the line and the corresponding horizontal distance. The amount by which the grade line rises or falls in a unit horizontal distance is called the *rate of grade* or the *gradient*. The rate of grade is usually expressed as a percentage; that is, as the rise or fall in a horizontal distance of 100 feet. For instance, if the grade line rises 2 feet in 100 feet, it has an *ascending grade of 2 per cent.*, which is written $+2\%$. If the grade line falls 1.83 feet in 100 feet, it has a *descending grade of 1.83 per cent.*, which is written -1.83%. The sign $+$ indicates a rising grade line and the sign $-$ indicates a falling grade line. Sometimes the sign % is omitted.

The rate of grade is written along the grade line on the profile, and the elevation of grade is written at the extremities of the line and at each point where the rate of grade changes.

It is common practice to enclose in small circles the points on the profile where the rate of grade changes.

75. All computations relating to rates of grade can be performed by applying the following rules:

Rule I.—*The rate of grade, in per cent., is equal to the total rise or fall in any horizontal distance divided by the horizontal distance and multiplied by 100.*

Rule II.—*The total rise or fall of a grade line in any given horizontal distance is equal to the rate of grade, in per cent., multiplied by the horizontal distance and divided by 100.*

Rule III.—*The horizontal distance in which a given grade line will rise or fall a certain amount is equal to the amount of rise or fall divided by the rate of grade, in per cent., and multiplied by 100.*

EXAMPLE.—The total rise of a certain grade line is 66 feet in a horizontal distance of 1 mile. (*a*) What is the rate of grade? (*b*) If the elevation of the grade at Station 2+00 is 150.00 feet, what is the elevation of the grade at Station 13+00?

SOLUTION.—(*a*) In 1 mile there are 5,280 feet; then, by rule I, the rate of grade is

$$\frac{66\times 100}{5{,}280}=1.25 \text{ per cent.} \quad \text{Ans.}$$

(*b*) The horizontal distance between Station 2+00 and Station 13+00 is 1,100 feet. The total rise of the grade line between these two stations is, according to rule II,

$$\frac{1{,}100\times 1.25}{100}=13.75 \text{ ft.}$$

The elevation of the grade at Station 2 is 150.00 feet; therefore, the elevation of the grade at Station 13 is 150.00+13.75=163.75 ft. Ans.

76. Cut and Fill.—The subject of leveling does not properly include cut and fill, but since grading is associated very closely with the subjects of profiles and grade lines, the following explanation is given here. The vertical distance of the subgrade below the surface line at any point, as shown on the profile, will be the depth of excavation, or cutting, necessary to bring the surface of the ground to the established grade at that point. Likewise the vertical distance of the subgrade above the surface line at any point will represent the depth of embankment, or filling, necessary to bring the surface of the ground to the proposed grade. The depth of excavation and

GRADING NOTES

Sta.	H. I.	Rod	Elev. of Surf.	Elev. of Grade	Cut or Fill	Remarks
B. M.			100.000			Spike on root of white oak stump 60 ft. to left of Sta. 0.
	102.172	+ 2.172				
0		− 4.75	97.42	100.00	F. 2.58	
1		− 6.14	96.03	99.50	F. 3.47	
2		− 9.03	93.14	99.00	F. 5.86	
2+50		− 8.19	93.98	98.75	F. 4.77	
3		− 7.58	94.59	98.50	F. 3.91	
4		− 7.36	94.81	98.00	F. 3.19	
T. P.		− 8.543	93.629			Top of stake near Sta. 4.
	98.212	+ 4.583				
4+60		−11.93	86.28	97.70	F. 11.42	Spring Brook.
5		− 0.47	97.74	97.50	C. 0.24	
T. P.		− 2.674	95.538			Nail in notch in stump of beech tree opposite Sta. 5+30.
	105.862	+10.324				
5+75		− 2.39	103.47	97.12	C. 6.35	
6		− 2.04	103.82	97.00	C. 6.82	
7		− 6.38	99.48	96.50	C. 2.98	
8		− 7.67	98.19	96.00	C. 2.19	
9		− 6.43	99.43	95.50	C. 3.93	
10		− 8.70	97.16	95.00	C. 2.16	
T. P.		−10.171	95.691			On rock near Sta. 10.
	98.134	+ 2.443				
11		− 2.45	95.68	94.50	C. 1.18	
12		− 7.20	90.93	94.00	F. 3.07	Spike on root of large maple tree 50 ft. to the right of Sta. 13+80.
13		− 8.85	89.28	93.50	F. 4.22	
T. P.		−11.292	86.842			

FIG. 36

the depth of embankment are, for short, commonly spoken of as the *cut* and the *fill*, respectively.

The cut or the fill is usually calculated for each station in connection with the level notes and recorded in the notebook.

At each station where the elevation of the surface exceeds the elevation of the grade, the difference will be a cut. At each station where the elevation of the grade exceeds the elevation of the surface, the difference will be a fill. The cuts are designated either by the letter *C.* or by the sign +, and the fills are indicated either by *F.* or by −.

77. Calculation of Cut and Fill.—The grading notes given in Fig. 36 are a repetition of the typical level notes of Fig. 30, to which are added the elevation of grade and the cut or the fill at each station. This form of notes is convenient since the six columns on the left-hand page of the notebook are sufficient.

The elevation of grade at Station 0 is established at 100.0 feet, and the rate of grade is taken as −0.5. Hence, the elevation of grade at Station 1 is $100.0-.5=99.5$ feet; the elevation of grade at Station 2 is $99.5-.5=99.0$ feet. The elevation of grade at each succeeding station is determined in a similar manner and written in the column headed Elev. of Grade. The elevation of grade at Station 4+60 is $98.0-\frac{.5\times60}{100}=97.7$ feet, and the elevation of grade at Station 5+75 is $97.5-\frac{.5\times75}{100}=97.12$ feet.

The difference between the elevation of grade and the surface elevation at Station 0 is $100.00-97.42=2.58$ feet; since the elevation of grade exceeds the elevation of the present ground, a fill is necessary as indicated in the notes. At Station 5, the difference between the present surface and the proposed grade is $97.74-97.50=0.24$ feet; since the present surface is higher, material must be cut.

EXAMPLES FOR PRACTICE

1. Between Stations 10+00 and 25+00 of a certain survey there is a grade of +2.00 per cent. If the elevation of the grade at Station 10+00 is 48.00 feet, what is the elevation of the grade (*a*) at Station 15+00, (*b*) at Station 18+75, and (*c*) at Station 23+67?

Ans. $\left\{\begin{array}{l}(a)\ 58.00\text{ ft.}\\(b)\ 65.50\text{ ft.}\\(c)\ 75.34\text{ ft.}\end{array}\right.$

2. If the elevation of the grade at Station 0 is 150.10 feet and that of the grade at Station 15+80 is 67.15 feet, what is the rate of grade?
Ans. −5.25%

3. What is the total rise of a +3.75 per cent. grade in a distance of 2,640 feet? Ans. 99.00 ft.

4. In what horizontal distance will a grade of +4.2 per cent. effect a rise of 94.50 feet? Ans. 2,250 ft.

BAROMETRIC LEVELING

THE BAROMETER

78. Air Pressure.—A body submerged in water is subjected to a pressure caused by the weight of the water above it; and the deeper the body is below the surface of the water, the greater is the pressure. The air surrounding the earth, known as the *atmosphere*, also has weight and, therefore, exerts a pressure on bodies, which is called *atmospheric pressure*. This pressure depends on the distance below the surface of the atmosphere, or on the distance above or below sea level, which is the usual reference surface. The atmospheric pressure is greater at sea level than at higher points, and decreases as the altitude increases. The difference between the atmospheric pressure at two points may, therefore, serve as a measure of the difference in elevation between the points.

79. Barometers.—Instruments for measuring atmospheric pressure are known as barometers. Barometers are of two general kinds. In one kind, called the *mercurial barometer*, the atmospheric pressure is indicated by the height of a column of mercury. In the other kind, called the *aneroid barometer*, the pressure is indicated by the deflection of the sides of a metal box from which the air has been removed.

Mercurial barometers are more accurate than aneroid barometers, but, as they are not so portable, their use is confined almost exclusively to laboratory work. Aneroid barometers are now made sufficiently accurate for ordinary purposes; and the ease with which they can be carried makes them well suited for leveling where only approximate elevations are required.

80. Mercurial Barometer.—A mercurial barometer, illustrated in Fig. 37, is constructed as follows: A glass tube, about 36 inches long and closed at one end, is filled with mercury. Then, while the open end of the tube is covered to prevent the escape of the mercury, the tube is inverted and the open end is submerged in mercury contained in a cup. When the open end of the tube is uncovered, and the mercury allowed to come to rest, the surface of the mercury in the tube will be about 30 inches vertically above that in the cup.

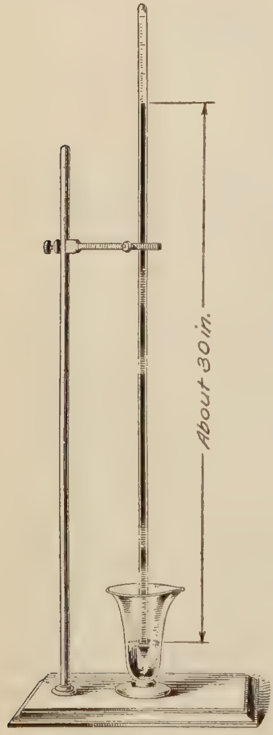


Fig. 37

The reason why the surface of the mercury does not have the same elevation in the tube and in the cup is that there is no air above the mercury in the tube, but the mercury in the cup is exposed to the atmospheric pressure. Therefore, the atmospheric pressure balances a column of mercury about 30 inches high, the weight of which can be determined. The weight of a cubic inch of mercury is .49 pound; and if the tube is assumed to have an area of 1 square inch, the volume of the mercury column is $30 \times 1 = 30$ cubic inches, and its weight is $.49 \times 30 = 14.7$ pounds. Since the weight acts on an area of 1 square inch, the pressure due to this weight is 14.7 pounds per square inch. This pressure is balanced by the atmospheric pressure; hence, the atmospheric pressure at sea level for a temperature of 60° Fahrenheit is 14.7 pounds per square inch. Such a pressure is often called a pressure of 1 atmosphere and is the standard commonly used for the atmospheric pressure when no other is given. It must be remembered that the atmospheric pressure is affected by other conditions besides altitude, and that the barometer reading fluctuates as the weather changes.

The temperature of the air also affects the pressure somewhat, cold air being heavier than hot air. The reading of a barometer, therefore, depends on the temperature as well as on the altitude. For this reason the temperature should be observed whenever the pressure is read.

81. Atmospheric pressure is often expressed by the height of a mercury column which it supports. For instance, if the mercury in the tube of a barometer is 29 inches above the mercury in the cup, the pressure is said to be 29 inches; similarly, a pressure of 31 inches means that the mercury in the tube of a barometer is 31 inches above the mercury in the cup. When the atmospheric pressure varies, the height of the mercury column changes; an increase in pressure causes the surface of the mercury in the tube to rise, and a decrease in pressure allows the surface to drop. The higher the altitude, the lower is the pressure, and, consequently, the shorter is the column of mercury. The atmospheric pressure at sea level is about 30.0 inches; at an elevation of 1,000 feet above sea level, it is about 28.9 inches; at 2,000 feet, 27.8 inches.

82. Aneroid Barometer.—There are many types of aneroid barometers, but all are essentially the same in action. An aneroid barometer consists of a metal case with a glass face; and in the case is a flat cylindrical chamber from which the air has been exhausted. The circular sides of this chamber, corresponding to the ends of a cylinder, consist of thin corrugated metal plates reinforced at the center by strong metal disks. The variation in the atmospheric pressure is indicated by changes in the deflection of the plate. As the atmospheric pressure increases, the difference between the pressures inside and outside the chamber becomes greater; consequently, the plate is deflected more. Conversely, as the atmospheric pressure decreases, the difference between the pressures outside and inside the chamber becomes less, and the elastic resistance of the metal reduces the deflection. The motion of the plate, greatly multiplied, is transmitted by means of a system of levers to a pointer that moves over graduated scales; the pointer and scales are visible through the glass face of the instrument, as shown in Fig. 38. Most aneroid barometers have two scales as shown in the figure. The inner, called the *mercury scale*, indicates the pressure in inches of mercury at the time of observation; readings on the outer, or *altitude scale*, may be used for determining differences in elevation directly.

83. Altitude and Mercury Scales.—Some aneroid barometers, like that in Fig. 38, show altitudes above sea level only; others read both above and below sea level. Also the limits of the altitudes vary considerably. The barometer shown in Fig. 38 measures up to 10,000 feet above sea level, but similar instruments are made that measure to 20,000 feet.

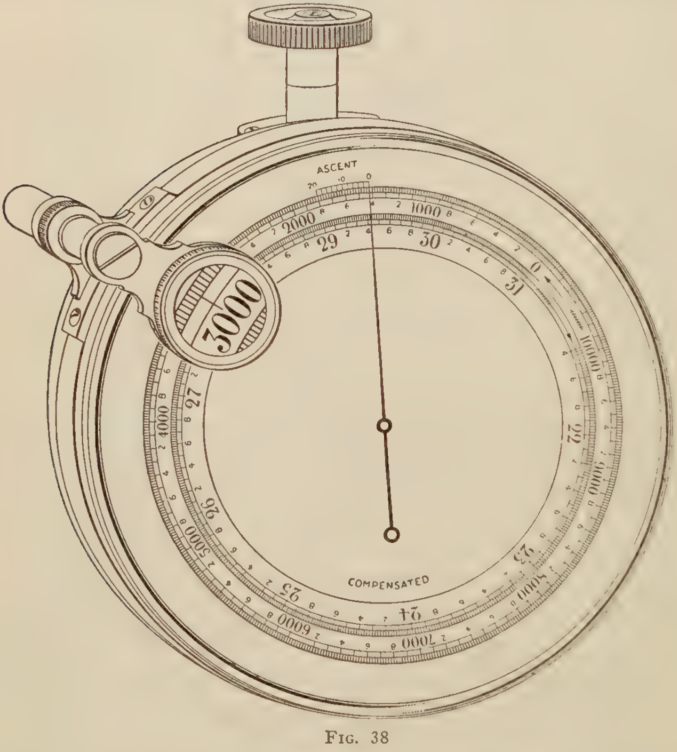


FIG. 38

There are various methods of graduating the scales. On the mercury scale in Fig. 38, the inches are numbered with large figures, and every alternate tenth of an inch is numbered with a smaller figure. Each tenth of an inch is divided into 5 parts, and, therefore, each very small division represents $\frac{1}{5}\times .1 = .02$ inch. In case the pointer is between graduations, the nearest half-division, that is, the nearest hundredth of an inch, can be easily estimated. For the position shown, the pointer reads about 29.43 inches.

On the altitude scale in Fig. 38, each 1,000 feet is numbered and the smaller figures indicate the hundreds. Each 100 feet is divided into 5 parts and each small division, therefore, is $\frac{1}{5}\times 100 = 20$ feet. By means of the vernier, which is shown near the 1,400-foot graduation, the altitude can be read to the nearest foot.

The vernier can be set at any point on the scale by turning the milled screw shown at the top of the case. To assist in reading the scale and vernier, a movable magnifying glass, shown at the 3,000-foot graduation in Fig. 38, is provided.

84. The mercury scale on a barometer records accurately the atmospheric pressure, but the pressure at any altitude varies for different locations, and at the same place it is continually changing as the temperature changes. Furthermore, as shown in Fig. 38, the zero of the altitude scale generally coincides with 31 inches on the mercury scale, which is seldom the value of the atmospheric pressure at sea level. The elevation indicated on the altitude scale of a barometer may, therefore, differ considerably from the true value at the point of observation. However, if observations are taken at two points, the difference in elevation between the points can be determined with sufficient accuracy for ordinary purposes.

Barometers are made in which the zero of the altitude scale can be moved to coincide with any graduation on the mercury scale. But, for good work, zero of the altitude scale should be kept at the mercury reading that was assumed to correspond to sea level in graduating the altitude scale.

85. Compensation for Temperature.—Unless a barometer is specially designed, variation in temperature so affects its parts that different readings are obtained for the same actual pressure. Therefore, some instruments are so constructed that this effect of temperature changes is automatically corrected. Such barometers are marked, "Compensated," as shown in Fig. 38.

86. Care of Aneroid Barometer.—An aneroid barometer is necessarily of delicate construction. Therefore, great care should be exercised, when the instrument is being used and carried, to protect it from shocks and jars, from moisture, from the direct rays of the sun, and from the heat of the body or an artificially warmed room. Each aneroid barometer is provided with a leather case in which it should always be carried.

DETERMINATION OF ELEVATIONS WITH A BAROMETER

INTRODUCTION

87. Variations in Pressure.—If the pressure for a given altitude were constant, a certain barometer reading would always indicate a certain altitude. Unfortunately, however, the pressure of the air for a given elevation varies. The three principal factors in the variation are time, location, and temperature.

The pressure of the air changes, as the weather changes, from day to day and from hour to hour; in dry settled weather it is about from 30.5 to 31 inches of mercury at sea level, while in stormy weather it is about 1.5 inches lower. It is also a matter of observation that at the same time the weather may be different in different places; consequently, the atmospheric pressure depends on the locality. Then, too, a given volume of cool air weighs more than an equal amount of warm air; hence, the pressure of the air varies with the temperature.

88. Taking Readings.—When a barometer reading is taken, the instrument should have as nearly as possible the temperature of the surrounding air, which temperature should be observed at the same time. A mercurial barometer should be kept vertical and the surface of the mercury in the cup should be at the elevation of the point the height of which is to be determined. An aneroid barometer should be horizontal and in the open air. The case should be tapped gently before the reading is taken.

89. Formulas for Difference in Elevation.—Since the pressure of the air varies with the altitude, it follows that the difference in elevation between two points is measured by the difference between the pressures at the points. It has also been found that the pressure of the air is affected by its temperature. When the pressures and temperatures at two points are known, the approximate difference in elevation between the points may be found by one of the following formulas. These formulas are derived from the data of many experiments, and the results are sufficiently accurate for practical use.

When the pressures in inches of mercury are observed, the formula for the difference in elevation is

$$z = 58.4 \frac{p_1 - p_2}{p_1 + p_2}(836 + t_1 + t_2) \qquad (1)$$

in which z = difference in elevation between two stations, in feet;

p_1 = pressure at lower station, in inches of mercury;
p_2 = pressure at higher station, in inches of mercury;
t_1 = temperature at lower station, in degrees Fahrenheit.
t_2 = temperature at higher station, in degrees Fahrenheit;

When the altitude scale on the barometer is read, the difference in elevation may be found by the formula

$$z = (h_2 - h_1)\frac{900 + t_1 + t_2}{1{,}000} \qquad (2)$$

in which z, t_1, and t_2 have the same meanings as in formula **1**;

h_2 = altitude reading at upper station, in feet;
h_1 = altitude reading at lower station, in feet.

Since the results obtained by the use of these formulas are only approximate, they may be taken to the nearest 10 feet.

Example 1.—Suppose the barometer at one station reads 26.25 inches with the temperature at 72° F., and at a second station it reads 24.95 inches with the temperature at 46° F. What is the difference in elevation?

Solution.—In formula **1**, substitute the values $p_1 = 26.25$ in., $p_2 = 24.95$ in., $t_1 = 72°$, and $t_2 = 46°$. Then,

$$z = 58.4 \times \frac{26.25 - 24.95}{26.25 + 24.95}(836 + 72 + 46) = 1{,}410 \text{ ft.} \quad \text{Ans.}$$

Example 2.—The reading on the altitude scale at a station is 437 feet and the temperature is 60° F. At another station, the barometer reading is 1,118 feet and the temperature 50° F. Find the difference in elevation between the stations.

Solution.—In formula **2**, substitute the values $h_1 = 437$ ft., $h_2 = 1{,}118$ ft., $t_1 = 60°$, and $t_2 = 50°$. Then,

$$z = (1{,}118 - 437)\frac{900 + 60 + 50}{1{,}000} = 690 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The reading of a barometer at a station is 28.44 inches and the temperature is 60°. At another station the barometer reading is 24.33 inches and the temperature is 40°. Calculate the difference in elevation between the two stations. Ans. 4,260 ft.

2. The readings of the altitude scale at two stations are 4,526 feet and 5,910 feet; and the respective temperatures are 42° and 34°. What is the difference in elevation? Ans. 1,350 ft.

3. The barometer readings at two points are 2,350 feet and 6,600 feet. If the temperatures are, respectively, 58° and 42°, what is the difference in elevation? Ans. 4,250 ft.

FIELD OBSERVATIONS

90. Method With One Barometer.—The field observations in barometric leveling may be made with either one barometer or two. If one instrument is used, it is placed first at the point of known elevation and then is moved as rapidly as possible to the other stations whose elevations are required. At each point, the time, the temperature, and the barometer reading are observed. The difference in elevation between the starting point and each other station can be computed by one of the formulas of Art. **89**. The elevation of any station is then found from the elevation of the reference point by adding or subtracting the difference in elevation for the station in question.

In the preceding method large errors are likely to be introduced on account of the time which elapses between observations at the different points. Better results are obtained if the observations are repeated, the points being visited in the reverse order, and the average of the two values for each station taken.

91. Method With Two Barometers.—The best way to determine differences of elevation from air pressures is to use two barometers. One instrument is kept at a point of known elevation. At intervals, the time, the temperature, and the barometer reading are observed. The other barometer is moved from station to station, and at each, the time, the temperature, and the barometer reading are recorded. The same scale must be read on both barometers. The data for the reference point for any time can be obtained from the observed values by interpolation, the variation in conditions being assumed to be uniform between observations.

Observations at Station of Known Elevation			Observations with Moving Barometer			
Elev. of $a=604$						
Time A. M.	*Barom. In.*	*Temp. Deg. F.*	*Sta.*	*Time A. M.*	*Barom. In.*	*Temp. Deg. F.*
8:30	29.20	63°				
9:00	29.36	65°	*b*	9:00	28.65	60°
9:30	29.55	68°				
10:00	29.65	72°	*c*	9:55	30.00	58°
10:30	29.68	75°	*d*	10:22	28.06	54°
11:00	29.62	77°	*e*	10:55	29.80	50°

FIG. 39

Typical field notes for leveling with two barometers are shown in Fig. 39.

The elevations at the stations are determined in the following manner. Since observations at a and b were taken at the same time, the difference in elevation is found simply by applying formula **1** of Art. **89.** Thus the vertical distance between a and b is $z=58.4\times\frac{29.36-28.65}{29.36+28.65}(836+65+60)=690$ feet.

Station *b* is higher than *a* because the barometer reading at *b* is lower. The elevation of *b* is, therefore, $604+690=1{,}294$ feet.

In order to find the elevation at *c*, it is necessary to determine the barometer reading and the temperature at *a* at 9:55 A.M. The data for the reference point *a* at any time can be obtained from the observed values by interpolation, as the variation in conditions is assumed to be uniform between observations. The difference in time between 9:30 and 10:00 is 30 minutes; the change in pressure at *a* for this interval is $29.65-29.55=.10$ inch and the change in temperature is $72°-68°=4°$. The interval between 9:30 and 9:55 is 25 minutes. The variation in pressure at *a* from 9:30 to 9:55, therefore, is $\frac{25}{30}\times .10=.08$ in., and the variation in temperature is $\frac{25}{30}\times 4=3°$. The values of the pressure and temperature at *a* at 9:55 equal, respectively, $29.55+.08=29.63$ in. and $68+3=71°$. Then the difference in elevation between *a* and *c* is $z=58.4\times\frac{30.00-29.63}{30.00+29.63}(836+58+71)=350$ feet, and since *c* is lower than *a* the elevation of *c* is $604-350=254$ feet. The elevations of *d* and *e* are obtained in a similar manner.

92. Accuracy of Barometric Leveling.—The differences between the altitudes as determined by direct leveling and those computed from barometer readings are often considerable. It is impossible to give any precise rule. First, because of the inaccuracy of the barometer itself, an error of 5 or 10 feet may be expected. Then the results depend on many varying quantities, such as the weather, the vertical and horizontal distances between the stations, the method of conducting the work, and the care of the observer. As an indication of the value of barometric leveling, it may be stated that a variation of 20 feet may be expected; and when the points are far apart, one of 50 feet is possible even in good work.

EXAMPLES FOR PRACTICE

1. From the notes in Fig. 39, determine the elevation of Station *d*. Ans. 2,174 ft.

2. Find the elevation of Station *e*. Ans. 444 ft.

COMPASS SURVEYING

THE COMPASS

PRELIMINARY EXPLANATIONS

INTRODUCTION

1. Compass and Transit.—The main instruments used for measuring angles in surveying are the *compass* and the *transit*. Formerly, the compass was used extensively in surveying, but it has been largely superseded by the transit, which is a more accurate instrument and is more suitable for most kinds of surveying work. At present, the compass is used chiefly for relocating lines of old surveys. It is also employed to a limited extent in preliminary work on railroads and occasionally in new land surveys where the property is of little value.

2. One important feature that distinguishes the method of measuring angles with the compass from that with the transit is that by means of a transit the angle formed by any two lines can be measured directly, whereas the compass can serve only for measuring the angle that a given line makes with the magnetic meridian or magnetic needle. To determine the angle between two lines by means of the compass, the angle that each line makes with the magnetic needle must first be measured; and from these values, the required angle may then be calculated as follows:

Let AB, Fig. 1, be the direction of the magnetic needle. To determine the angle CAD by means of the compass, the angles

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BAC and BAD that AC and AD make with the magnetic needle must first be measured. Then in the case shown in (a),

$$\text{angle } CAD = \text{angle } BAC + \text{angle } BAD$$

and in the case shown in (b),

$$\text{angle } CAD = \text{angle } BAD - \text{angle } BAC$$

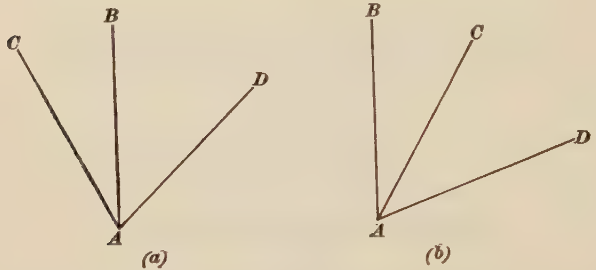


FIG. 1

3. Meridians.—The directions of the lines of a survey are usually given with respect to some fixed reference line, which is called a *meridian*. At each point on the earth's surface, there are two definite lines, the *true meridian* and the *magnetic meridian*, which are generally used in surveying.

4. True Meridian.—The axis on which the earth rotates is an imaginary line cutting the earth's surface in two points known as the *north geographic pole* and the *south geographic pole*. The line passing through a point on the earth's surface and directed toward the geographic poles is called the *true meridian* at the given point.

5. Magnetic Meridian.—If a magnetized steel bar is allowed to rotate on a pivot near its center, the bar will always take very nearly the same direction at any place. The line thus indicated is called the *magnetic meridian* at the given point; it has the general direction of the true meridian. A magnetized bar is acted upon by two magnetic forces, which are assumed to be at two points, called *magnetic poles*, on opposite sides of the earth's surface and near the ends of the earth's axis. The magnetic pole near the north geographic pole is the *north magnetic pole*, and the other, which is near the south geographic pole, is the *south magnetic pole*. The same end of a freely-suspended magnetized bar will always point toward the north and this end is known as the *north end* of the bar; the other end of the bar is its *south end.*

Meridians are circles on the earth's surface and meet at the poles. But, for the purposes of ordinary surveying where relatively small areas are considered, meridians are treated as parallel straight lines that lie in a horizontal plane.

6. Azimuths.—The *azimuth* of a line is the angle between a meridian and the line, and is always measured from the meridian in a clockwise direction from 0° to 360°: in surveying, azimuths are generally measured from the north point, but sometimes the south point is used. Unless otherwise stated, azimuths will be considered as measured from the north. In Fig. 2, NS represents a meridian with N toward the north; the azimuth of OA is 115°, that of OB is 246°, and that of OC is 300°. Azimuths are called *true azimuths* when measured from a true meridian, and *magnetic azimuths* when measured from a magnetic meridian.

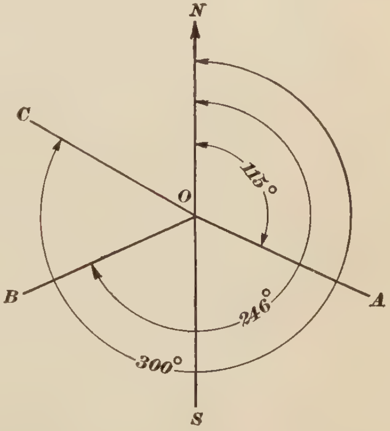


FIG. 2

7. Bearings.—The angle between a meridian and a line may be given by the *bearing* of the line instead of by the azimuth. In determining bearings, the plane around a point is divided into four quadrants by two lines, of which one is in the direction of the meridian and the other is at right angles to the meridian. Bearings are reckoned from 0° to 90° in each quadrant, the zero points being in the meridian and the 90° points in the east-and-west line. It is thus seen that there are four lines, one in each quadrant, which make the same angle with the meridian. In order to distinguish between these lines, the letters *N*, *E*, *S*, and *W*, indicating north, east, south, and west, respectively, are used to show the quadrant. Thus, if a line is in the northeast quadrant, its bearing is written with the letter *N* preceding the value of the angle, and the letter *E* following the angle. For a line in the southeast quadrant, the letter *S* precedes the angle and *E* follows. Lines in the other quadrants are indicated by the corresponding letters in a similar manner. For example, the bearing of the line OP_1 in Fig. 3 is written N 60° E, called *north 60° east*, because the line OP_1 lies in the quadrant between north and east and the angle with the meridian is 60°. The bearing of the line OP_2 is N 42° W, called *north 42° west*, because OP_2 is in the quadrant between north and west and the angle with the meridian is 42°. Similarly the bearing of the line OP_3 is S 70° W (south 70° west) and the bearing of the line OP_4 is S 50° E (south 50° east).

The angle is always measured from the north or south, and never from the east or west; and the letter *N* or *S* always precedes the angle, while *E* or *W* follows. If the line is in the direction *ON*, its bearing is said to be *due north;* similarly, the bearings of the lines *OE*, *OS*, and *OW* are *due east*, *due south*, and *due west*, respectively. Obviously, a bearing can never be greater than 90°. If *NS* is the magnetic meridian, the bearings are *magnetic bearings*, while if *NS* represents the true meridian, the bearings are *true bearings*.

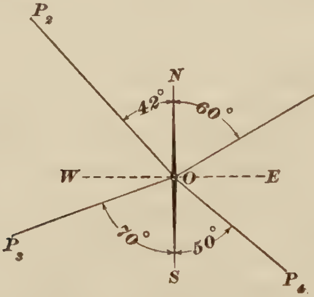


FIG. 3

8. Angle Between Line and Meridian.—Lines on the earth's surface are seldom horizontal; but when the direction of a line is considered, the horizontal angle between the meridian and the line is understood. Thus, let AB, Fig. 4 (*a*), be a line on the ground; AN, a meridian through A; BH, a vertical line through B; and AH, a horizontal line passing through A and intersecting BH at H. Then the direction of the line AB is represented by the angle G between the meridian, which is horizontal, and AH. Any line, as AC, AD, or AE, Fig. 4 (*b*), through A and a point on the vertical line through B, has the same direction as AB.

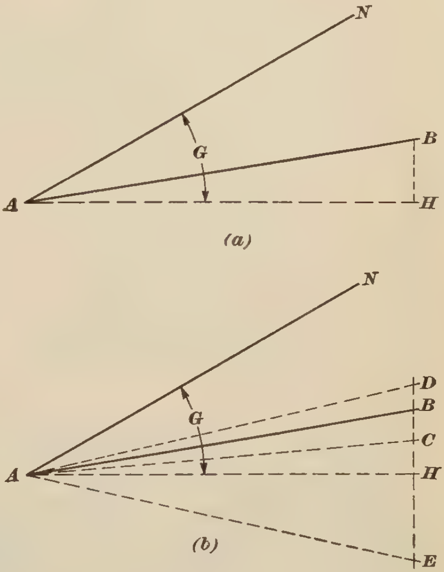


FIG. 4

DESCRIPTION OF COMPASS

9. General Construction.—The compass used in surveying, which is known as a *surveyors' compass*, is shown in Fig. 5. Its essential parts are a magnetized steel bar *a*, called a *magnetic needle*, which is supported freely on a pivot *b* at the center of a horizontal graduated circle *c*; and a pair of sights *d* attached to this circle.

The needle and the graduated circle are enclosed in a brass case *e*, called the *compass box*, which has a glass cover. The compass box is secured to a plate *f*, at the ends of which the sights *d* are attached by means of milled-headed screws *g* and *h*. The sights are brass bars with narrow vertical slits *i* having small holes at the tops and bottoms; in sighting, a slit in one sight is viewed through a hole in the other sight. Often one of the slits in each sight is replaced by a wider opening with a very fine vertical wire to mark the line of sight. The sights either are removable, as shown in Fig. 5, or are hinged so that they can be folded down over the compass box; thus, the instrument can be placed in a flat box when not in use. Sometimes one of the sights is graduated on the side so that angles in a vertical plane can be measured; in Fig. 5, the right-hand sight is so graduated.

In order to indicate when the plate *f* is horizontal, two *spirit levels j*, one parallel to the line of sight and the other at right angles to it, are screwed to the plate. Each level consists of a glass tube nearly filled with alcohol or a mixture of alcohol and ether, the remaining space being occupied by a bubble of air. The tube is mounted in a brass case, which is attached to the plate.

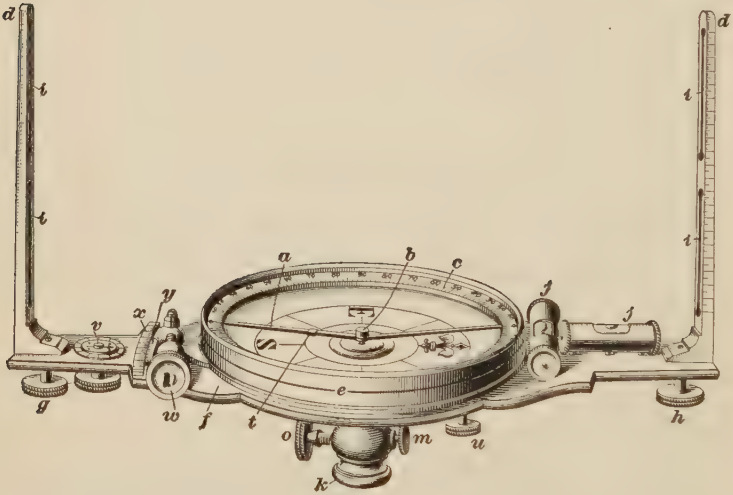


Fig. 5

10. Tripod and Mounting.—The compass is usually supported on a *tripod*, Fig. 6, which consists of three legs *a* shod with steel points and connected by hinge joints to a metal *tripod head b*. Occasionally a single straight pole about 4½ feet long, called a *Jacob staff*, is used instead of a tripod. This staff has a pointed metal shoe which can be stuck in the ground. With both types of supports, a special mounting, such as that shown in Fig. 7, is provided at the top for attaching the compass. The socket *k*, Fig. 5, which fits over the spindle *l*, Fig. 7, is either permanently attached or screwed to the plate *f*, Fig. 5, and is held in place on the spindle by the lock screw *m*, Fig. 5, which drops into the small groove *n*, Fig. 7. The socket turns freely on the spindle, but can be secured in any position by the clamp screw *o*, Fig. 5.

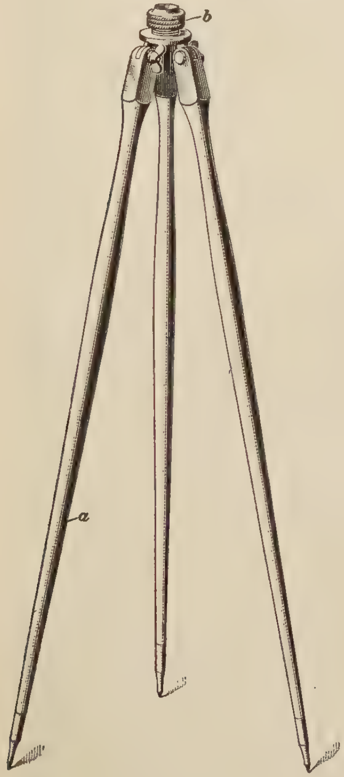


FIG. 6

In Fig. 8 is shown a sectional view of the mounting with the socket *k*. On the lower end of the spindle *l* is a carefully turned ball *p*; this rests in a spherical socket *q* in the top of the sleeve *r*, which screws on the tripod head or staff head. The ball is held in the socket by a ring *s*, which is screwed on the sleeve *r* and which is ground to fit the ball. When the ring *s* is loosened, the ball can be moved easily to level the instrument; and by tightening the ring, the ball can be held securely in the desired position. Such a joint is called a *ball-and-socket joint.*

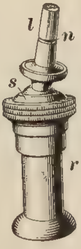


FIG. 7

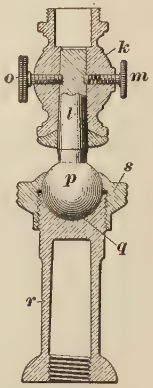


FIG. 8

Instead of the mounting shown in Fig. 7, an arrangement consisting of a plate and four leveling screws can be used for leveling the compass. However, as the compass is never used in work requiring great accuracy, the simple ball-and-socket joint is generally employed.

11. Needle.—In Fig. 9 is shown a plan of the compass box. The needle *a* is mounted on a hard steel pivot *b*, ground to a fine point and secured in position at the center of the graduated circle *c*. The setting where the needle rests on the pivot usually consists of a jewel placed in a small metal cap on the needle. In the northern hemisphere, the south end of the needle carries a small sleeve or coil of wire *t*, Figs. 5 and 9. This coil serves to distinguish the north end from the south end, but the chief reason for its use is explained in the next article.

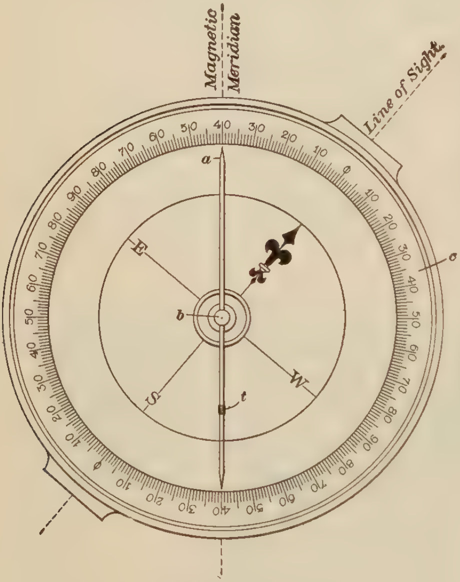


FIG. 9

By means of a lever, operated by the thumbscrew *u* shown on the under side of the compass plate in Fig. 5, the needle may be lifted off the pivot and pressed against the glass cover of the compass box. This prevents the pivot from becoming blunted when the compass is being carried.

12. Dip of Needle.—In the northern hemisphere, the needle is nearer to the north magnetic pole than to the south magnetic pole, and the north end of the needle is attracted more strongly than the south end. Therefore, when a needle is freely suspended, its north end is lower than the south end. The angle of inclination with the horizontal is called the *dip of the needle*.

In the southern hemisphere, the needle tends to dip with its south end lower than the north end. In order to keep the needle horizontal, the coil of wire previously mentioned is placed on the south end in the northern hemisphere and on the north end in the southern hemisphere. Since the dip of the needle varies with the latitude, the wire coil is made free to slide along the needle.

13. Needle Circle.—The graduated circle *c*, Figs. 5 and 9, is called the *needle circle* or *compass circle*. It is divided into quadrants by two lines, one in line with the sights and the other at right angles to the first. The graduations in line with the sights are marked 0; one of them is called the *north point*, and the other the *south point*. The sight near the north point is called the *north sight* and that near the south point is the *south sight*. The north point is sometimes indicated as in Figs. 5 and 9, but the letter *N*, or some other symbol, is often used; the south point is indicated by the letter *S*. The graduations at the other quadrant points are numbered 90, and are marked by the letters *E* and *W*, indicating east and west, respectively. The circle is sometimes divided into degrees, as shown here, but more often it is graduated in half degrees; each tenth degree is numbered.

The fact that the terms north and south are applied to the zero points does not mean that the line through these points is always in a north-and-south direction; these names are used for convenience in reading bearings, as will be explained later. For the same reason, the positions of the east and west points are interchanged. When one faces north, east is on the right and west on the left; but on the needle circle, *E* is placed on the left and *W* on the right.

14. Outkeeper.—The small dial *v*, Fig. 5, which is called an outkeeper, is used for counting tallies in chaining; it is turned by the milled-headed screw shown just beneath it. This is not an essential part of the instrument, and is not found on all compasses.

ADJUSTMENTS OF COMPASS

15. Conditions of Adjustment.—A new compass, made by a good maker, is always in adjustment when it leaves the factory, but rough usage, a fall, or a hard blow may throw it out of adjustment. Besides several conditions that are taken care of in the construction of the instrument, the following are necessary for accurate work:

1. The bubbles should remain in the centers of the tubes throughout a complete revolution of the compass plate on the spindle.
2. The ends of the needle and the pivot must be in the same vertical plane.
3. The point of the pivot must be at the center of the needle circle.

To ascertain whether these conditions obtain, three tests are performed. If corrections are necessary, adjustments should be made. The methods of making these tests and adjustments will now be described.

16. To Adjust Plate Levels.—Set the tripod legs or the Jacob staff firmly in the ground, and bring the bubbles to the centers of the level tubes by moving the plate carefully by means of the ball-and-socket joint. Then revolve the compass on the spindle through 180°; that is, turn it end for end. If the bubbles remain in the centers of the tubes, the levels are in adjustment. But if turning the compass end for end causes either bubble to run toward one end of the tube, lower that end or raise the opposite end sufficiently to bring the bubble halfway back toward the center by means of the small screws that attach the ends of the tube to the plate. Then bring the bubbles to the centers by moving the plate by means of the joint. Repeat the operation until both bubbles remain in the centers of the tubes in both positions of the compass. This completes the first adjustment.

17. To Straighten Needle.—After the plate levels have been adjusted, bring the bubbles to the centers, release the needle, and, when it comes to rest, turn the compass so that the north end of the needle is exactly opposite some prominent graduation mark of the needle circle; also, observe the exact reading of the south end of the needle. Then remove the glass and rotate the needle without turning the plate, so that the south end of the needle is exactly at the former reading of the north end. This can be done best by pushing the needle with a match or a small piece of wood, which does not attract the needle. With the needle in this position, observe the new reading of the north end. If the north end reads the same as the south end did before the needle was rotated, the needle is straight. If the first reading of the south end and the second reading of the north end are not the same, remove the needle from the pivot and bend the needle carefully to correct one-half the difference. Suppose that, in the first position, the north end of the compass is set at N 20° E and the south end reads S 19° 30′ W. Then when the south end is set at N 20° E, assume that the north end reads S 20° W. If the needle were straight, the north end in the second position would read S 19° 30′ W. To straighten the needle, make the north end read half-way between S 19° 30′ W and S 20° W, or S 19° 45′ W, when the south end reads N 20° E. Check the adjustment by repeating the operation, using different graduation marks.

It should be noticed that the two ends of the needle do not necessarily read alike for the same position of the needle; and if they do read alike, it does not indicate that the needle is straight. Thus, in the second position in the example just given, the reading of both ends of the needle is 20° but the needle was not straight. If in the second position the north end of the needle had read S 19° 30′ W to agree with the reading of the south end in the first position, no adjustment would have been necessary.

18. To Center Needle Pivot.—Having, if necessary, straightened the needle, turn the compass in several positions and observe if the readings of the north and south ends of the needle agree for each position. If they do, the point of the pivot is at the center of the needle circle. If the readings of the ends of the needle do not agree, find the position of the plate which shows the greatest difference and clamp the plate securely. Determine the distance that one end of the needle would have to be moved to make the readings of the two ends alike. Then remove the needle from the pivot and bend the pivot carefully at right angles to the direction of the needle so that the point moves one-half that distance. Repeat the operations until the readings of the two ends of the needle agree in all positions of the plate.

SURVEYING WITH A COMPASS

FUNDAMENTAL PRINCIPLES

19. Object of Compass Measurements.—The compass is used primarily for measuring the magnetic bearings of lines. As previously explained, the angle between two lines may be determined from the bearings of the lines and, therefore, angles may be measured indirectly by means of the compass.

20. Declination.—Except in relatively few places, the magnetic meridian through a point on the earth's surface does not coincide with the true meridian at the point. In other words, the needle of the compass does not point toward the geographic poles of the earth. The angle between the magnetic meridian and the true meridian, or, what is the same thing, the angle that the compass needle makes with the true meridian, is called the *magnetic declination* or the *declination of the needle*. For some points on the earth, the north end of the needle is deflected west of true north, and for other points, it is east of true north. In the first case, the declination is said to be west, and in the second case, the declination is east, to correspond with the direction in which the north end of the needle is deflected from the true north. The amount of the declination of the needle is different in different localities, and also varies noticeably from year to year in the same locality. The differences and variations in the declination are not regular, though in a general way they follow a more or less definite system.

21. Local Attraction.—The compass needle may be deflected from the magnetic meridian by the attraction of an electric current or any near-by body of iron or steel, such as a pile of steel on the ground, the rails of a railway, a gas or water pipe, the tape or chain, keys or a knife on the observer, etc. Such a disturbing influence is called *local attraction*, although avoidable attractions caused by objects not fixed to the place are not usually included by this term.

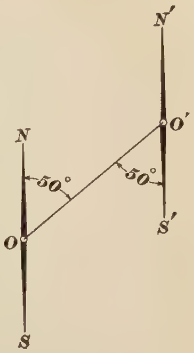


FIG. 10

22. Forward Bearings and Back Bearings.—In Fig. 10, O and O' are any two points and NS and $N'S'$ are meridians at O and O', respectively. Since NS and $N'S'$ are parallel, the angles NOO' and $S'O'O$ are equal; thus, if the bearing of the line from O to O' is N 50° E, it follows that the bearing of the line from O' to O is S 50° W. The bearing of a line in one direction is called the *forward bearing;* while the bearing of the same line in the opposite direction is its *back bearing*. The angle is the same in the forward bearing and in the back bearing, but the quadrant is diagonally opposite. For example, if the forward bearing of a line is S 30° E, its back bearing is N 30° W. In every case, the back bearing is determined from the forward bearing by simply changing the letter N to S or S to N, and by changing E to W or W to E. The bearing of the line from O to O' is called the bearing of OO'; the bearing from O' to O is the bearing of $O'O$. Hence, in determining the bearing of a line, the order of the letters identifying the line is important.

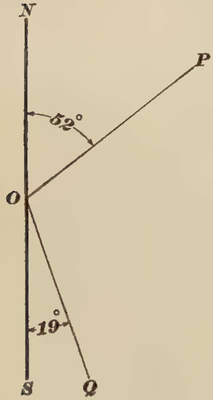


FIG. 11

23. Angle Between Two Lines Whose Bearings Are Known.—If an angle is turned from OP to OQ, Fig. 11, the angle is called POQ; if the angle is turned from OQ to OP, it is

QOP. Since it is possible to turn an angle either clockwise or counter-clockwise, it is important to state in which direction the angle is measured. Thus, the angle *POQ* turned directly is called *POQ clockwise*, or *POQ to the right;* whereas, the angle *POQ* turned through *N* and *S* is *POQ counter-clockwise*, or *POQ to the left.*

When two lines are in the same quadrant, that is, both bearings have the same letters, the angle between the lines is equal to the difference between the two bearings. For example, if the bearings of two lines are N 66° 30′ E and N 39° 15′ E, the angle between the lines is 66° 30′ − 39° 15′ = 27° 15′.

When the angle is required between two lines which are in different quadrants, it is usually advisable to make a rough diagram and to calculate the angle by inspection of the figure. First, a line is drawn to represent the meridian; then from any point on this line, the two given lines are drawn about in the directions indicated by the bearings. Thus, in Fig. 11, *NS* represents the meridian, *N* being the north point; the bearing of *OP* is N 52° E; and that of *OQ* is S 19° E. From the figure, it is seen that *POQ* is equal to 180° − 52° − 19° = 109°. In Fig. 12, the bearing of *OP* is N 12° W and the bearing of *OQ* is S 43° E. Then angle *POS* clockwise is 180° + 12° = 192°

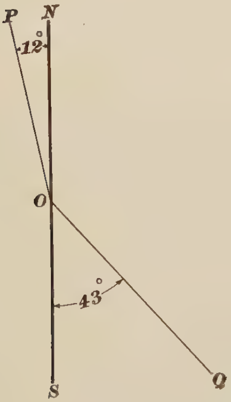


FIG. 12

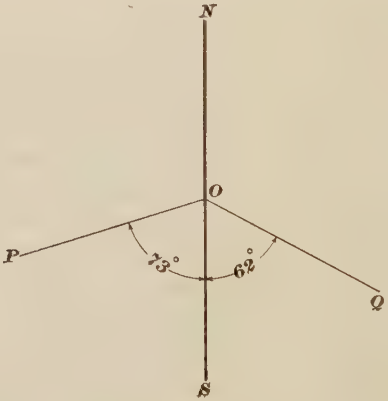


FIG. 13

and *POQ* is 192° − 43° = 149°. In Fig. 13, the bearing of *OP* is S 73° W and that of *OQ* is S 62° E; in this case, angle *POQ* is obviously equal to 73° + 62° = 135°.

In determining the angle between two lines from their bearings, it is important to take the bearing of each line away from the point of intersection of the lines. For example, suppose that in Fig. 14, *AB* and *BC* are two lines of a survey, the bearing of *AB* being N 42° E and the bearing of *BC* being S 75° E; let it be required to find the angle *ABC*. The desired angle is, in this case, the angle between *BA* and *BC*, the bearing of *BA* being equal to the back bearing of *AB*, or S 42° W. If the meridian *NS* is drawn through *B*, it is seen that the angle *ABC* is 42° + 75° = 117°. If the bearing of *AB* had been used, as shown by the dotted line *BA'*, the angle obtained would be that between *BC* and *AB* produced, or *A'BC*, which is equal to 180° − 42° − 75° = 63°.

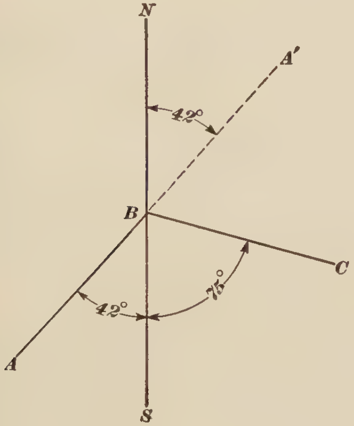


Fig. 14

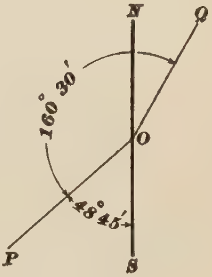


Fig. 15

24. Bearing of Line From Bearing of Another Line and Angle Between Lines.—When the angle between two lines and the bearing of one of the lines are known, the bearing of the other line may be found. As in the preceding article, it is advisable to make a rough diagram of the conditions. Suppose, for instance, that the bearing of the line *PO*, Fig. 15, is N 48°

45′ E and it is desired to find the bearing of another line OQ, making an angle POQ equal to 160° 30′ clockwise. The bearing of OP is equal to the back bearing of PO, or S 48° 45′ W. If NS is the meridian through O, angle NOP is 180° − 48° 45′ = 131° 15′, and NOQ is 160° 30′ − 131° 15′ = 29° 15′. Hence, the bearing of OQ is N 29° 15′ E.

If a given line PO, Fig. 16, has a bearing of N 30° 45′ E, and the angle between this line produced and another line OQ is 70° 15′ to the right, the angle NOQ is $NOC + COQ$ = 30° 45′ + 70° 15′ = 101°. Since this is greater than 90°, the line OQ is in the southeast quadrant and the angle SOQ is 180° − 101° = 79°. Hence, the bearing of OQ is S 79° E. If the line OP, Fig. 17, has a bearing of S 48° 15′ W and the angle between OP and OQ is 88° 30′ to the left, the angle SOQ is 88° 30′ − 48° 15′ = 40° 15′ and the bearing of OQ is S 40° 15′ E.

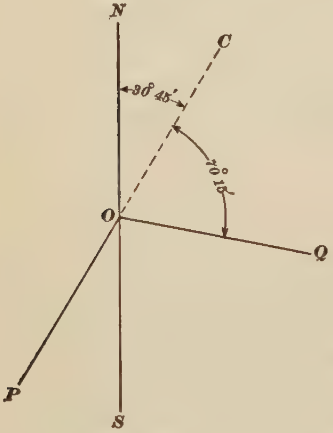


FIG. 16

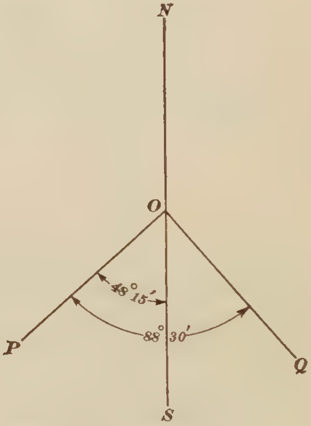


FIG. 17

EXAMPLES FOR PRACTICE

1. Find the angle between two lines OA and OB, whose bearings are N 32° 15′ E and N 42° 30′ W. Ans. 74° 45′

2. What is the angle between lines OA and OB, if the bearing of OA is N 15° 30′ E and that of OB is S 46° 45′ W? Ans. 148° 45′

3. The bearing of a line AO is N 48° 15′ W and the bearing of OB is N 76° 30′ W. Find the angle $A\,O\,B$. Ans. 151° 45′

4. The bearing of a line $O\,A$ is N 47° 30′ W. What is the bearing of a line $O\,B$ if angle $A\,O\,B$ is 138° 15′ to the left? Ans. S 5° 45′ E

5. If the bearing of a line AO is S 20° 30′ W and the angle AOB is 130° 45′ to the right, what is the bearing of $O\,B$? Ans. S 28° 45′ E

6. The bearing of line $A\,O$ is S 20° 15′ E. Find the bearing of the line OB, which makes an angle with $A\,O$ produced equal to 60° 45′ to the right. Ans. S 40° 30′ W

FIELD PROBLEMS

25. Setting Up.—In setting up a compass mounted on a Jacob staff, the staff is stuck as nearly as possible in a vertical position at the point over which the compass is to be set. If a tripod is used, the compass can be placed accurately by means of a plumb-bob suspended from a small ring directly under the center of the needle circle. However, in compass work, it is a waste of time to center the compass exactly over the station. With a little practice, it is possible to set up the compass almost over the point by judgment. If desired, the position of the compass can be determined by dropping a small pebble from below the center of the circle and observing how close to the marked point the pebble strikes.

After the compass is properly placed, the plate *f*, Fig. 5, is brought to a horizontal position, as shown by the spirit levels, by moving it on the ball-and-socket joint.

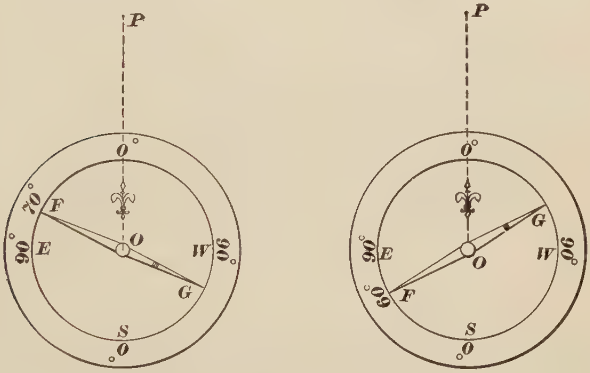
26. Practical Suggestions.—In leveling the compass by means of a ball-and-socket joint, do not grasp the sights, but take hold of the compass plate near the needle box.

When the compass is not in use or is being carried, keep the needle off the pivot point so that the point will not be dulled and the sensitiveness of the needle thus affected. After setting up in a new place, bring the needle as near as possible in the magnetic meridian before releasing it. Then the needle will come to rest more quickly and, also, the vibration being thus reduced, the sharpness of the pivot point will be preserved. In case the needle swings more than about 10° on each side of the meridian, its progress can be checked by raising it off its pivot when near the center of its swing; when the needle is released again, the swing will be reduced. It is advisable to allow the needle to swing through about 3° before coming to rest, as it will then assume a more nearly correct position than if it is released very nearly in the meridian. When the compass is put away, or is allowed to remain in one position for a long time, permit the needle to assume its position in the meridian and then raise it off the pivot; if it is not in the meridian, it is liable to lose some of its magnetic strength.

27. Taking Bearings.—To find the bearing of a line, the compass is set up at some point on the line, preferably at one extremity. Then, a range pole is held vertically at some other point on the line, as far as possible from the compass. With his eye behind the south sight, the surveyor revolves the compass horizontally on the spindle until the range pole is approximately on the line through the sights. He then looks through the slits, his eye being at the south slit, and turns the plate until the range pole is exactly on the line of sight. The line of sight should be directed as nearly as possible to the bottom of the range pole in order to diminish the error due to any deviation of the pole from the vertical. The plate is then clamped in position, and the reading of the graduated circle opposite the north end of the needle is observed as explained in the following article. This is the bearing of the line.

In finding the bearing of an important line, it is always advisable to take a forward bearing and a back bearing. As previously stated, the angles should be equal. If the observed values agree closely, say within ½ degree or 1 degree according to the purpose of the survey, it is customary to take the average as the correct value. If the observed angles differ greatly, either the position of the needle was observed incorrectly or there was local attraction. The observations should be checked carefully; if no error is found, the cause of the difference must be local attraction. The method of correcting for local attraction will be treated later.

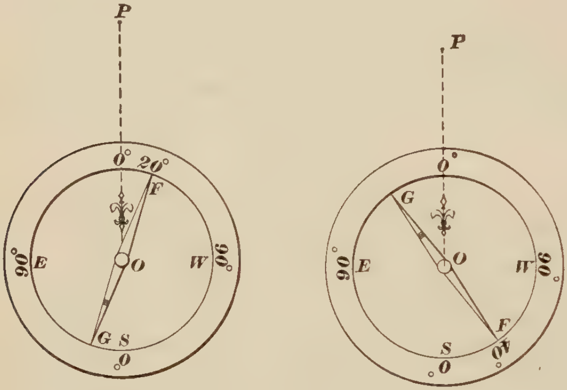
28. Reading Needle Circle.—In determining the bearing of a line, the plate is revolved until the sights are on the given line, with the north sight forward. The needle remains fixed in the magnetic meridian and, for magnetic bearings, the zero points of the needle circle are in the line of sight. Hence, the magnetic bearing of the line of sight is the angle between the needle and the line through the zero points of the needle circle. Let *OP*, Fig. 18 (*a*) or (*b*), be a line whose magnetic bearing is required. The compass is set at *O* and the sights are brought in the line *O P*. Then the angle between the needle *FG* (north end at *F*) and the line of sight, which is measured by the arc between the north end of the needle and a zero point of the circle, is the required bearing. If the bearing of *OP* is northeast, as in (*a*), or southeast, as in (*b*), the north end of the needle is to the left of the line of sight when the observer faces toward the north point of the needle circle; in other words, east bearings are read on the left half of the circle, where the letter *E* is placed. When the bearing of a line *O P* is northwest, as in Fig. 19 (*a*), or southwest, as in Fig. 19 (*b*), the north end of the needle is to the right of the line of sight; the letter *W* is, therefore, placed on the right side of the line through the south and north points of the circle.



(*a*) (*b*)

Fig. 18

When the letters are marked on the needle circle in this way, the quadrant in which a given line lies is indicated by the letters between which the north end of the needle rests. The angle between the meridian and the line is shown by the number of the graduation opposite the north end of the needle.



(a) (b)

Fig. 19

Thus, in Fig. 18 (*a*), the bearing of the line *OP* is N 70° E because the north end of the needle is opposite the graduation marked 70 between the north and east points. In Fig. 18 (*b*), the bearing of *OP* is S 60° E; the bearing of *OP*, Fig. 19 (*a*), is N 20° W; and that of *OP* in Fig. 19 (*b*) is S 40° W.

Readings can be readily estimated to the nearest 10 minutes. It is a useless refinement to try to make readings closer than this with a compass because of the inaccuracy of the instrument. When the reading is taken, the eye of the observer should be exactly in line with the needle so that the point on the circle opposite the end of the needle may be determined correctly.

29. East and West.—The fact that the letters *E* and *W* are reversed on the needle circle of a compass often causes confusion to the beginner in laying off a line along a given bearing. In this latter case, it is incorrect to reverse east and west. A line whose bearing is northeast should be laid off to the right of the meridian, as at OP_1 in Fig. 3, and not to the left; similarly, a line with a northwest bearing should have the general direction $O\,P_2$.

30. Correcting for Local Attraction.—Special care must be taken to avoid errors that may be caused by local attraction. Its presence may be detected best by comparing the forward and back bearings of the lines. If the needle readings indicate local attraction, the point of set-up at which it exists may be determined in the following manner: Suppose that the forward bearing from O to P, Fig. 20, is N 85° 45′ E and the back bearing from P to O is S 75° 30′ W, local attraction thus being indicated. If OP is a line of a survey, and Q is another point on the survey, the forward and back bearings of PQ are compared. If the bearings of PQ and QP agree, the local attraction must be at O; if the difference between the bearings of PQ and QP is the same as that between the bearings of OP and PO, the local attraction is at P. In rare cases, there is local attraction at two of the points O, P, and Q. Then the survey is continued until the forward and back bearings of some line agree, and the other bearings are corrected from this line as described later in this article.

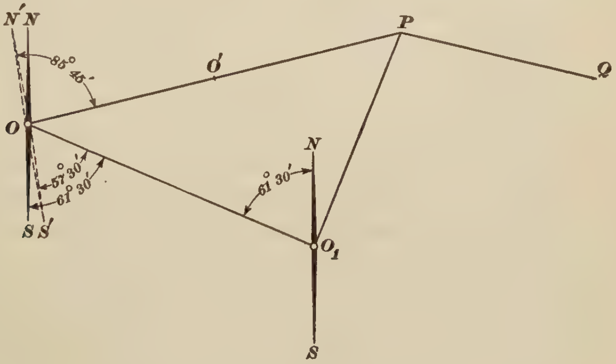


FIG. 20

When OP is not a line of a survey, it is often possible to determine the location of the attraction by setting up at a point, such as O', on OP about midway between O and P;

then the bearing of $O'O$ or $O'P$ is taken. If it agrees with either the bearing of OP or that of PO, it indicates the correct value and shows where the local attraction is. Thus, if the bearing of $O'O$ is S 85° 45′ W, the bearing of OP, previously determined, is correct and the local attraction is at P. If the bearing of $O'O$ is S 75° 30′ W, the bearing of PO is correct and the local attraction is at O. When a slight difference is obtained, say less than 1°, it is customary to take the average of the two values as correct.

If it is not convenient to take an intermediate point on line OP, or if there is reason to suspect that there is local attraction all along OP, forward and back bearings to an outside point, as O_1, must be taken from both O and P. If it is found that there is local attraction at both O and P, the bearing of OP must be corrected by determining the angle by which the needle is deflected by the local attraction. Suppose the bearing of O_1O is N 61° 30′ W and that of OO_1 is, according to the reading of the compass, S 57° 30′ E. Let NS represent the magnetic meridian through O and $N'S'$ the position which the needle at O assumes. If there is no local attraction at O_1, the angle SOO_1 is 61° 30′, and, therefore, the angle by which the needle is deflected is $61° \ 30' - 57° \ 30' = 4°$. As shown in the figure, the deflection is to the west of north. The value of angle NOP is $85° \ 45' - 4° = 81° \ 45'$, and the correct magnetic bearing of OP is N 81° 45′ E.

Attraction that is caused by something in the observer's clothing, as keys or a pocket-knife, can be detected by standing in one position to read the north end of the needle, and then going around to the other side of the compass to read the south end of the needle. If the two readings do not agree, there is an attraction on the observer.

31. To Run Line Having a Given Bearing.—It is frequently required to locate a line making a certain angle with another line whose bearing is known, or to relocate a line of an old survey. In the first case, the bearing of the required line can be calculated by the method explained in Art. **24**; in the second case, the bearing of the line will be given. However, in both problems, a line must be run from a given point in a certain direction.

The bearing of the required line having been determined, the compass is set over the known point on the line and the plate is rotated on the spindle until the north end of the needle indicates that bearing. The line of sight through the slits then has the desired direction, and, with his eye behind the south slit, the surveyor lines in a stake or pole through the north slit. This stake or pole is on the required line.

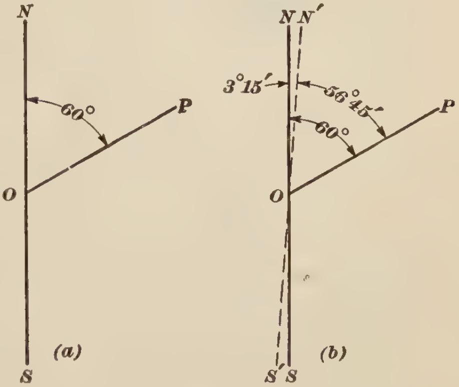


FIG. 21

Suppose, for example, that it is required to run a line OP, Fig. 21 (a), whose bearing is to be N 60° E. The compass is set at O and the plate is rotated until the north end of the needle is opposite the graduation numbered 60 between the north and east points on the needle circle. A pole is then placed at P in line with the sights.

As a check, the compass should be set over the new point and a back bearing taken. If the needle readings indicate local attraction, and it is found to be at the original point of set-up, the line must be relocated as follows: Suppose that a back bearing from P to O, Fig. 21 (a), shows that there is local attraction. Suppose further that the attraction is found to be at O, and it deflects the needle 3° 15′ to the east. Let NS, Fig. 21 (b), be the magnetic meridian at O, and $N'S'$ the position taken by the needle. Then, in order that the correct bearing of OP should be N 60° E, the line OP should be run on a bearing of N 56° 45′ E, as indicated by the needle. If it is desired to lay off a given distance along the line, the stakes or pins at the end of each tape length can be lined in from the compass.

32. Passing Obstacles.—It is often required to determine the bearing of a line between two points, neither of which is visible from the other. Such a case is illustrated in Fig. 22, where the line of sight from A to B is obstructed by a building. With the compass set at A, locate point C at any convenient distance and in any direction from A; record the length and bearing of AC. Then set up the compass at B and run BD with the same bearing and length as AC. Point C should be so selected that C and D will be visible from each other. Set the compass at C and determine the bearing of CD, which is equal to the bearing of AB. It is also permissible to set up at D and take the bearing of DC, which is equal to the back bearing of AB.

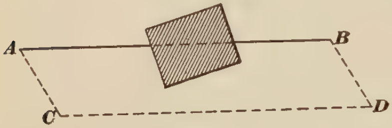


FIG. 22

To find the bearing of a fence or a wall, the compass is set up close to the wall, and a sight is taken on a pole held the same distance from the wall; then, the line of sight is parallel to the wall and has the same bearing.

33. In running a line in a given direction, obstacles are frequently encountered, as in Fig. 23, where K and L are buildings on the line AB. With the compass at A and the line of sight in the proper direction, the point P is set as near to K as possible. Then the compass is moved to P and the line PQ is run in any convenient direction; point Q is marked on the ground and is so selected that QQ' can be easily run parallel to AB. The length and bearing of PQ are recorded. The compass is set at Q, the back bearing of QP is

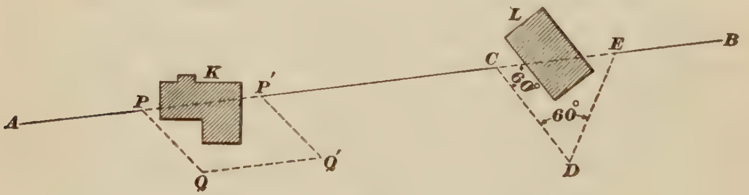


FIG. 23

taken as a check, and the line QQ' is run with a bearing equal to that of AB; the length of QQ' should be such that $Q'P'$ can be run parallel to PQ. If the length of AB is desired, the distance QQ' is measured; otherwise it is not necessary. Then, the compass is set up at Q', a back bearing is taken on Q for a check, and the point P' is so located that the bearing of $Q'P'$ is equal to the back bearing of PQ and its length is equal to that of PQ. The point P' is on line between A and B, and the distance PP' is equal to QQ'. If the compass is set up at P', the line AB can be continued in the proper direction by setting the sights so that the needle indicates the given bearing of AB.

34. The following method of passing the obstacle L, Fig. 23, is often more convenient than that just described. The line AB having been run to C, near L, the line CD, making an angle of 60° with AB, is laid off; the bearing of CD can be calculated by the method explained in Art. **24.** The distance CD is so taken that a line from D, making an angle of 60° with CD, will clear the obstacle; the proper position of D can be judged. The distance CD having been measured, the compass is set up at D, and the point E is so located that the line DE makes an angle of 60° with CD and the distance DE is equal to CD. The point E, thus determined, is on AB, and the distance CE is equal to CD. All bearings should be tested by back bearings.

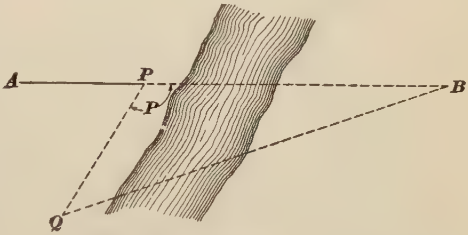


FIG. 24

35. In Fig. 24, the line AB crosses a river; hence, its length cannot be measured directly nor be ascertained by either of the methods just explained. In this case, the bearing of AB is determined and the distance from A to a point P near the bank is measured. Next, any other convenient point Q is selected, and the length and bearing of PQ are recorded;

theoretically PQ may be of any length, but the results will be more accurate if PQ is not less than about one-quarter of PB. Then the compass is set at Q, the bearing of PQ is checked by a back bearing, and the bearing of QB is observed. From the bearings of AB, PQ, and QB, the angles at P, Q, and B can be calculated, as already explained; as a check, their sum should equal 180°. Then in the triangle PQB, the side PQ is known, and the side PB can be calculated by the relation

$$PB = \frac{PQ \sin Q}{\sin B}$$

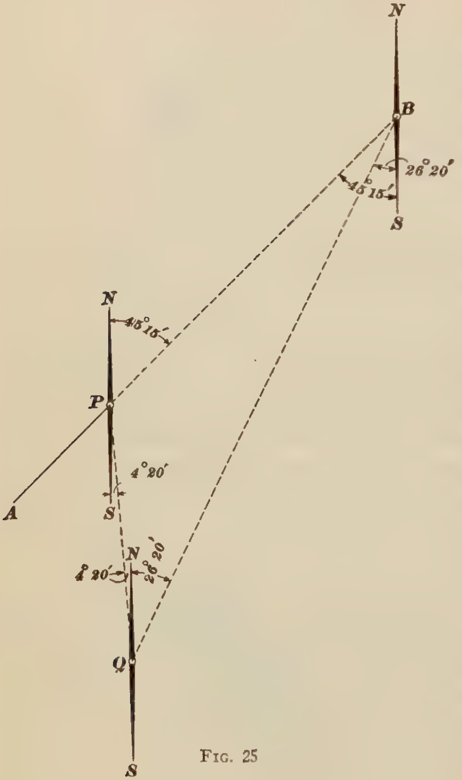


Fig. 25

Example.—Suppose that the length of PQ, Fig. 24, is 100 feet, the bearing of AB is N 45° 15′ E, that of PQ is S 4° 20′ E, and that of QB is N 26° 20′ E. Find the distance from P to B.

Solution.—For convenience in determining the angles between the lines, Fig. 25 is drawn, in which NS represents the meridian. The angle BPQ is $180° - 4° 20' - 45° 15' = 130° 25'$; angle PQB is $4° 20' + 26° 20' = 30° 40'$; and angle PBQ is $45° 15' - 26° 20' = 18° 55'$. As a check, $130° 25' + 30° 40' + 18° 55' = 180°$. Then, $PB = \frac{100 \sin 30° 40'}{\sin 18° 55'} = 157.3$ ft. Ans.

EXAMPLES FOR PRACTICE

1. If, in Fig. 24, the bearing of AB is S 63° 15′ E, that of PQ is S 43° 50′ W, that of QB is N 84° 20′ E, and distance PQ is 150 feet, what is the length of PB? Ans. 181.7 ft.

2. If, in Fig. 24, the bearing of AB is N 45° 15′ W, that of PQ is N 50° 30′ E, that of QB is N 87° 25′ W, and distance PQ is 200 feet, what is the length of PB? Ans. 199.7 ft.

3. The bearing of a line from A to B was measured as S 16° 30′ W. It was found that there was local attraction at both A and B, and, therefore, a forward and a back bearing were taken between A and a point C at which there was no local attraction. If the bearing of AC was S 30° 10′ E and that of CA was N 28° 20′ W, what is the correct bearing of AB? Ans. S 18° 20′ W

NOTE.—In this and similar problems, a sketch is helpful.

MAKING A COMPASS SURVEY

36. Advantage of Compass.—The compass is of great value in a survey where speed and economy are more important than accuracy and where small obstructions such as trees and rocks are frequently encountered along the line. At each obstruction, the compass can be moved to the opposite side and the line continued on the same bearing. Should the instrument as thus located be a foot or two off the correct line, no serious error will result since the new line is run parallel to the true position. It is assumed, of course, that local attraction is absent.

37. Sources of Error.—The chief causes of errors in obtaining the bearings of lines with a compass are local attraction, mistakes in reading the bearing, inaccuracy of the compass, poor adjustment, and variation in declination.

The method of detecting and correcting for local attraction has been explained in a previous article. The adjustment of the instrument has also been described. Errors of observation due to poor sighting or incorrect reading of the needle can be eliminated only by exercising great care and by taking a forward and back bearing for each line. The compass is inaccurate because it is difficult to read the needle closely, and because different compass needles will not take exactly the same position under similar conditions and, therefore, will not indicate the same bearing for a given line. Variation in declination will be treated later.

38. Survey Corps.—The corps for making a compass survey should consist of at least three men, the compassman, and two assistants who act sometimes as flagmen, and sometimes as chainmen. The number of men in the party and the duties of each depend on the purpose of the survey and the character of the country. If the country is thickly wooded, one or more axmen will be required to clear a narrow path in order to afford a fairly long sight for the compassman and to permit straight horizontal chaining. If the survey requires stakes at more or less regular intervals between the points at which the compass is set up, a stakeman is needed to cut, carry, and drive them. Stakes can be made of any well-seasoned wood, but in timbered country, it is often better to make the stakes from saplings as they are needed; the lower end is pointed to permit driving and the upper end is blazed for marking the station number.

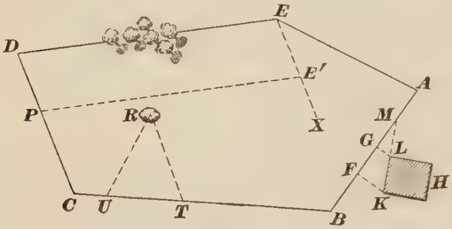


Fig. 26

39. Surveying Established Lines.—If it is desired to survey a series of lines between previously established points, such as the boundaries of the field in Fig. 26, the corners are first found and marked. The measurements sometimes consist only of the lengths and bearings of the boundary lines, but, often, it is also necessary to locate important objects.

A survey line, the length and bearing of which are determined, is commonly called a *course*. A system of connected courses is often called a *traverse*. If the end of the last course is also the beginning of the first course, the system is known as a *closed traverse*. Thus, the traverse shown in Fig. 26 from *A* to *B* to *C* to *D* to *E* and back to *A* is a closed traverse.

In the survey of a traverse, the compass is set up at each corner and the forward and back bearings of each course are taken. While the compass is directed along a side, that side is measured, in order that the compassman can line in the head chainman by looking through the slits. A point of set-up may be either a solidly set stake, a mark in the root of a tree, an X cut in a rock, or any other suitable point. Marks on permanent objects should be fully described so that they can be found and identified at any time. If stakes are placed, they should be referenced by bearings and distances to prominent immovable objects near-by, so that they can be easily relocated in case they are destroyed.

To survey the boundary lines of the field shown in Fig. 26, the compass is set up at any corner, say A, and the bearing of AB is observed. While the sight is directed along AB, its length is also measured. It is customary to take the back bearing of EA, which is the bearing of AE, at this time in order that it will not be necessary to set up again at A at the end of the survey. The compass is next set at B; the bearing of BA, which is the back bearing of AB, is observed; and the bearing and the length of BC are determined. Then the compass is set at C, from which point the bearings of CB and CD and the length of CD are found. The operations are repeated at each corner if possible.

In case one corner is not visible from an adjacent corner, and it is not desirable to cut through the obstruction, the method of passing an obstacle, described in Art. 32, can be employed. Thus, suppose that the line of sight from D to E, Fig. 26, is obstructed by many large trees; then, the length and the bearing of PE', which are equal to those of DE, are determined instead. In this case, the point P is taken on the line DC for convenience, EX is run on the same bearing as DC, and EE' is made equal to DP.

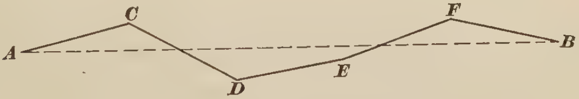


Fig. 27

40. Random Traverse.—It is often required to determine the length and the bearing of a line between two points that are a very great distance apart in wooded and rough country. The best method is to run a random traverse between the two points. For example, suppose that it is desired to find the length and the bearing of the line between A and B in Fig. 27. The compass is set up over one end of the line, say A, and line AC is run on a convenient bearing, as near as possible to that estimated for the line AB; the length and the bearing of AC are recorded. Then the compass is moved to C, a back bearing is taken on A, and the forward bearing and the distance to a point D are measured and recorded. Similarly, lines DE, EF, and FB are run and their lengths and bearings determined. The method of computing the length and the bearing of AB will be explained in another Section.

41. Locating Objects.—Important objects near the line of survey can be located in various ways, depending on circumstances. As explained in *Chain Surveying*, points can be located by distances along the line and perpendicular offsets; thus, the corners K and L of the house H in Fig. 26 can be located by distances BF and FK, and BG and GL, respectively. Another method of locating a point is by taking the length and the bearing of the line from any point on the survey; in Fig. 26, the point L can be located from M by the length and the bearing of ML. Still another method, which is very convenient for locating inaccessible points, is to take the bearings of the lines to the point in question from any two points on the survey. In Fig. 26, the rock R is located by the bearings of UR and TR; if lines are drawn from U and T in the directions indicated by the bearings of UR and TR, R is at their intersection. Bearings to objects should be taken from corners of the field if possible in order to obviate an extra set-up.

FIELD NOTES FOR COMPASS SURVEY

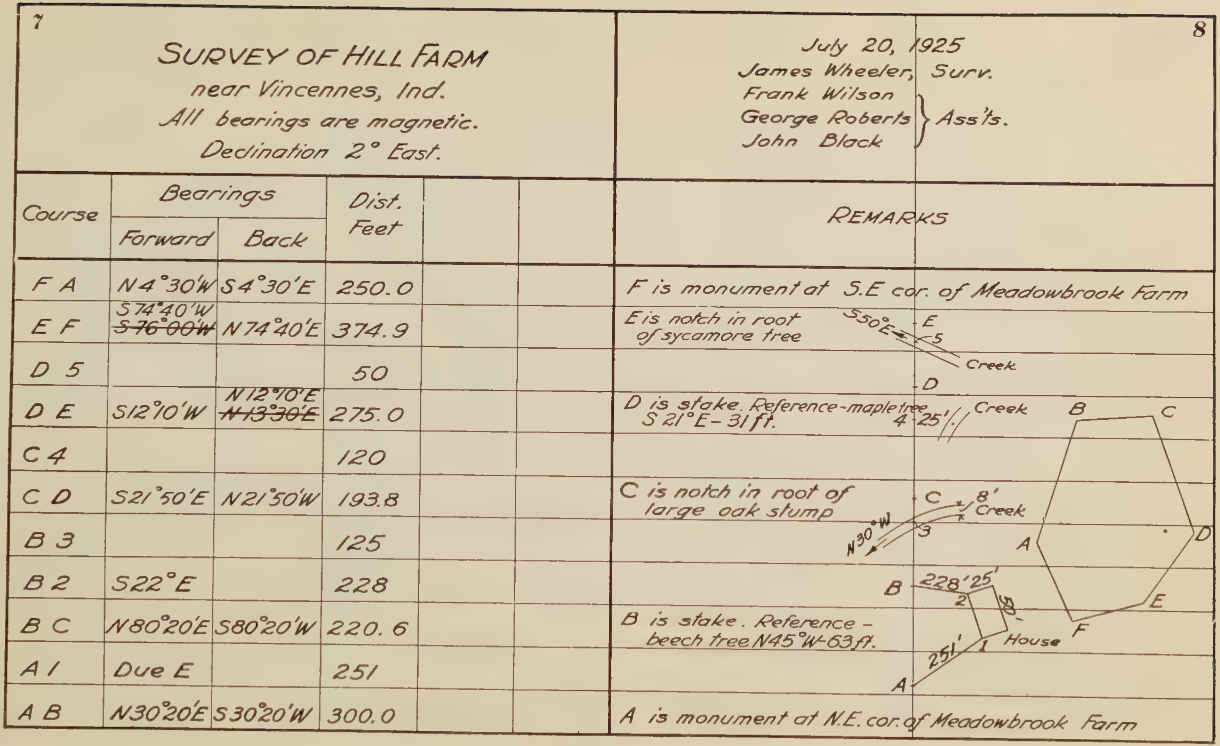
42. General Requirements.—There are many methods of keeping the notes of a compass survey, each surveyor adopting such variations as seem best suited to his particular case. Only one method will be described here as an outline or a guide to illustrate the fundamental principles. The notes should be concise but in sufficient detail to be understood by any one familiar with the principles of surveying. All details intended to be shown on the map should be noted, and nothing should be left to be supplied from the memory of the surveyor. The notebook commonly used, which is known as a *transit book*, has been described in *Chain Surveying*.

The notes should state whether the bearings are magnetic or true. If magnetic bearings are taken, the declination of the needle should be recorded, so that the true bearings can be calculated for future reference. All points of set-up should be fully described. Whenever possible the notes should be illustrated by a sketch. In some cases, all necessary information can be marked directly on the sketch.

The date of the survey and the names of the members of the party should always be given, because, in case of a lawsuit, all data regarding the survey may be required. As explained in *Chain Surveying*, the notes usually read upwards from the bottom of the page.

43. Typical Compass Notes.—In the form of notes shown in Fig. 28, the first column, headed *Course* or *Line*, gives the stations between which the courses are run; thus, BC means the line between B and C, with the set-up at B. In the second and third columns are the forward and back bearings; for the course marked BC, the forward bearing is from B to C, and the back bearing is from C to B when the compass is set up at C. Since the compass was not set up at points *1* and *2*, no back bearings are recorded for *A1* and *B2*. The lengths of the courses are in the fourth column. The remainder of the left-hand page can be used for remarks, but generally these are made with the sketch on the right-hand page.

When the notes are examined, it will be noticed that the forward and back bearings of all the courses except DE and EF agree. This indicates that there was local attraction at D, E, or F. However, since the forward and back bearings of CD agree, there can be no local attraction at D. Similarly, the forward and back bearings of FA show that there is no local attraction at F. Then the correct bearings of DE and EF are, respectively, S 12° 10′ W and S 74° 40′ W. The conditions are shown in Fig. 29; NS represents the magnetic merid-



7

SURVEY OF HILL FARM
near Vincennes, Ind.
All bearings are magnetic.
Declination 2° East.

8

July 20, 1925
James Wheeler, Surv.
Frank Wilson } Ass'ts.
George Roberts } Ass'ts.
John Black } Ass'ts.

Course	Bearings		Dist. Feet			REMARKS
	Forward	Back				
F A	N 4°30'W	S 4°30'E	250.0			F is monument at S.E cor. of Meadowbrook Farm
E F	S 74°40'W ~~S 76°00'W~~	N 74°40'E	374.9			E is notch in root of sycamore tree
D 5			50			
D E	S 12°10'W	N 12°10'E ~~N 13°30'E~~	275.0			D is stake. Reference-maple tree S 21°E – 31 ft.
C 4			120			
C D	S 21°50'E	N 21°50'W	193.8			C is notch in root of large oak stump
B 3			125			
B 2	S 22°E		228			
B C	N 80°20'E	S 80°20'W	220.6			B is stake. Reference – beech tree N45°W–63 ft.
A 1	Due E		251			
A B	N 30°20'E	S 30°20'W	300.0			A is monument at N.E. cor. of Meadowbrook Farm

FIG. 28

ian at E and $N'S'$ is the actual position of the needle. Each of the angles $N'EN$ and $S'ES$ is equal to 1° 20′. The corrected bearings are written above the observed bearings as shown in the notes; the incorrect observed bearing should not be erased. Sometimes, one of the vacant columns is used for the corrected magnetic bearings, and the other column is used for the true bearings, which may be calculated from the magnetic bearings by correcting for the declination by the method which will be explained later. For the courses forming the boundaries of the field, the bearings are read as accurately as possible, usually to the nearest 10 minutes or quarter-degree, but for the lines locating points 1, 2, etc., the bearings are taken only to the nearest degree.

The meaning of the figures in the column of distances is obvious. The length of course AB is 300.0 feet and that of EF is 374.9 feet. The lengths of the boundaries of the field are given to the nearest tenth of a foot; but for the distances to other points, as 1, 2, etc., the nearest foot is accurate enough.

As in the notes for a chain survey, the survey line is usually represented by the red line at the center of the right-hand page, or a line parallel to it, and the notes are supplemented by a sketch. It is sometimes helpful to make a rough diagram of the boundaries of the field, as the polygon $ABCDEF$ in Fig. 28, to show the relation of the various lines. Such a diagram is especially valuable when the main survey lines form a complicated system.

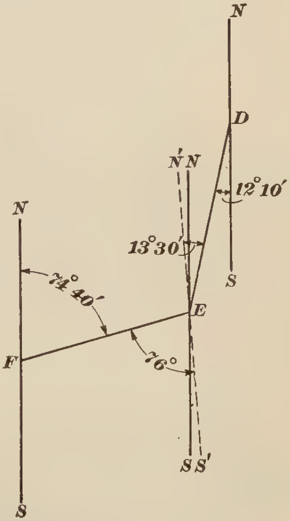


Fig. 29

44. Sketching Method.—In some cases, the sketch is made so complete that additional notes are not necessary. Thus, the sketch for the notes just given may be made as shown in Fig. 30. The title of the survey, the date, and the names of the members of the party should be given as in the written notes. The sketch is not necessarily drawn to scale, but is intended to give only an idea of the shape of the farm and the location of the house and creek. On a map the declination is usually shown by drawing two lines to represent the true and magnetic meridians and marking the angle between them, as shown in Fig. 30. Usually, the true meridian has a full arrowhead at the north end, and the magnetic meridian is indicated by half of an arrowhead at its north end.

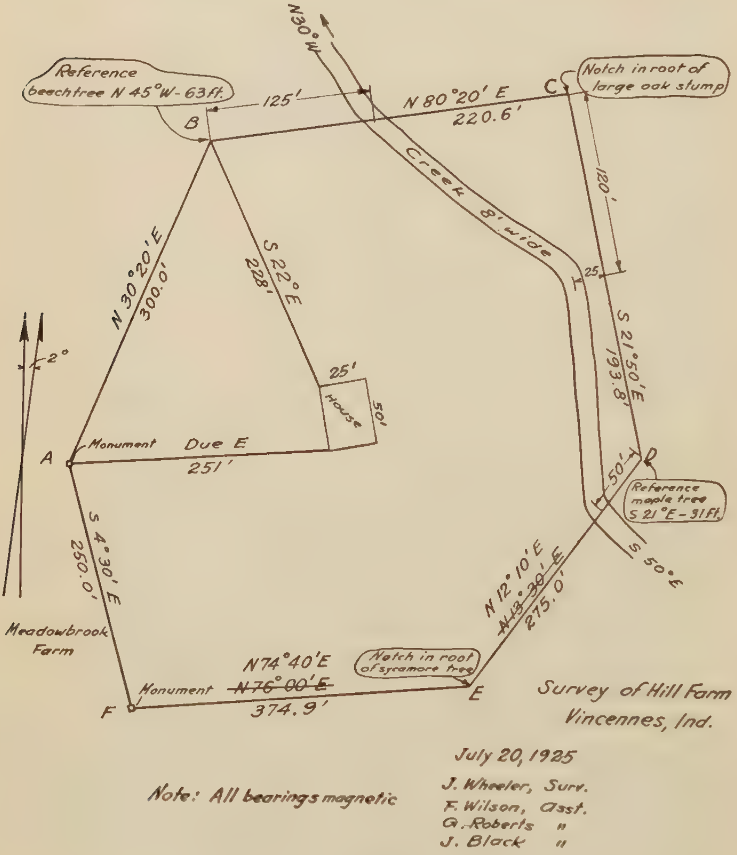


FIG. 30

In writing bearings along lines, it is customary to consider the direction of the line as that in which the bearing is read. For instance, in Fig. 30, the bearing is read from *A* toward *F* and is, therefore, written S 4° 30′ E, although it is observed from *F* to *A* as N 4° 30′ W. If it is desired to write all bearings as they are first observed, that is, as they would be given as forward bearings in written notes, a small arrow is used to indicate reversals. Thus, the bearing of the creek at the upper end is read in a southeasterly direction; since it is written N 30° W, the direction corresponding to the written bearing is indicated by the small arrow. When the back bearing does not agree with the forward bearing, both values are written on the line, as shown for courses *D E* and *E F*. Then the incorrect value is crossed out, but not erased.

Keeping the notes by a sketch alone is often impracticable, because it is necessary to get much information in the small space on an ordinary notebook page; therefore, it is often impossible to make the figures legible and to show clearly to what the dimensions refer. The combination of written notes and sketches is usually best.

DECLINATION

45. Determination of Declination.—In nearly all large cities and in many county seats, the direction of the true meridian has been established by astronomical methods and marked by permanent monuments. The angle between this line and the compass needle gives the declination on the date of observation. For ordinary purposes, the declination at any point in the United States can be found from charts which are published at intervals by the United States Coast and Geodetic Survey, and which can be secured from the U. S. Coast and Geodetic Survey, Washington, D. C.

On these charts are lines, called *isogonic lines*, which connect all points of equal declination; these lines are drawn for each degree of declination. The line of zero declination is called the *agonic line*. The locations of isogonic lines vary continually, but the charts give the amount of this variation in a year so that the approximate declination can be found for any time within several years after the date on the chart. The declinations at points between isogonic lines can be found by interpolation, it being assumed that the declination varies uniformly from line to line.

46. Variation in Declination.—As previously explained, the magnetic meridian at any locality is constantly changing, the total rate of change at any time being due to several independent variations.

First, there is a slow change in the direction of the magnetic meridian, which returns to its original position after several hundred years. This shifting is called the *secular variation*, and, as it is practically periodic, its amount can be determined closely for any date and locality. Values are given on the charts of the United States Coast and Geodetic Survey, which have been previously mentioned.

In addition, there is a change during the day, called the *diurnal variation*. The needle swings back and forth once daily through an arc whose value depends on the locality and on the time of year; in the United States, the total swing varies from about 3 minutes in winter to 12 minutes in summer. The needle reaches the center of its swing at about 11 A. M. and 6 P. M.

Other variations which cannot be predicted are caused by magnetic disturbances in the air. Such disturbances are called *magnetic storms*, and sometimes cause deflections of more than $\frac{1}{2}$ degree. Their presence is indicated by rapid fluctuations of the needle, and observations should not be taken at such a time.

47. Relation Between True and Magnetic Bearings. From the definition of declination, it is evident that the difference between the true and magnetic bearings of a line is equal to the declination of the needle for the locality. Hence, if the declination of the needle is known, the true bearing of a line can be found from its magnetic bearing, and vice versa. Thus, suppose the declination is 2° 18′ east, and the magnetic bearing of a line OX is N 43° E; let it be required to find the true bearing of the line OX. If NS, Fig. 31, is the true meridian and $N'S'$ is the magnetic meridian, angle NON' is 2° 18′ to the right. Since angle $N'OX$ is 43°, angle NOX is $43°+2°\ 18'=45°\ 18'$, and the true bearing of OX is N 45° 18′ E. Suppose it is desired to find the true bearing of a line OA, whose magnetic bearing is S 89° 15′ W, when the declination is 2° 18′ east. In Fig. 31, angle $S'OA$ is 89° 15′ and angle SOA is $89°\ 15'+2°\ 18'=91°\ 33'$. Since this is greater than 90°, the line OA lies in the northwest quadrant with respect to the true meridian, and the angle NOA is $180°-SOA=180°-91°\ 33'=88°\ 27'$. Hence, the true bearing of OA is N 88° 27′ W. It is important to remember that the line is represented but once whereas there are two meridians.

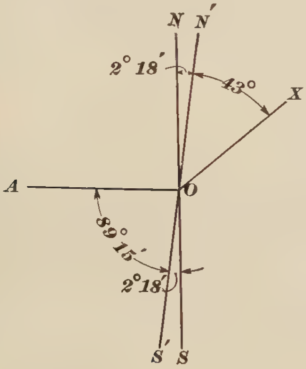


FIG. 31

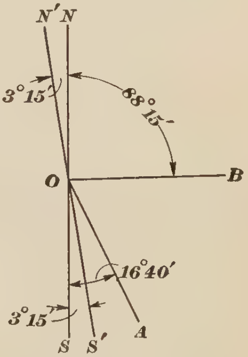


FIG. 32

48. Often the true bearings of lines are given on maps; but to relocate one of these lines with a compass, it is required to find its magnetic bearing. For example, suppose that it is desired to obtain the magnetic bearing of a line OA, Fig. 32, whose true bearing is S 16° 40′ E, when the declination is 3° 15′ west. Here, NS is the true meridian and $N'S'$ is the magnetic meridian. Then angle SOA is 16° 40′, and $S'OA$ is $16°\ 40'-3°\ 15'=13°\ 25'$. The magnetic bearing of OA is, therefore, S 13° 25′ E. Similarly, if the true bearing of OB is N 88° 15′ E, angle $N'OB$ is $88°\ 15'+3°\ 15'=91°\ 30'$. Since this is greater than 90°, the line OB is in the southeast quadrant with respect to the magnetic meridian. Thus, angle $S'OB$ is $180° - 91° 30' = 88° 30'$, and the magnetic bearing of OB is S 88° 30′ E.

49. Declination Arc.—In the foregoing explanations, it was assumed that the line through the zero marks of the needle circle coincided with the line through the sights. For this position of the zero points, the needle readings give magnetic bearings. True bearings may be shown by the needle, however, if the line through the zero points is rotated out of the line of sight by an amount equal to the declination. For the purpose of rotating the needle circle with respect to the line through the sights, the screw w, Fig. 5, is provided. The amount of rotation is measured on a graduated scale x with the aid of a vernier y.

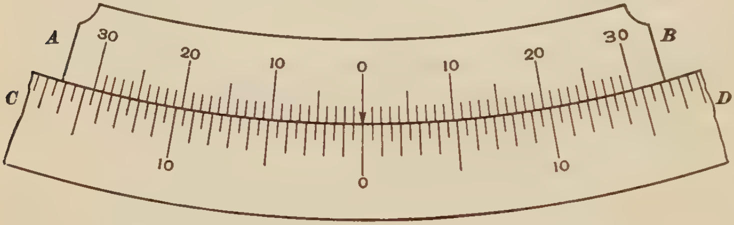
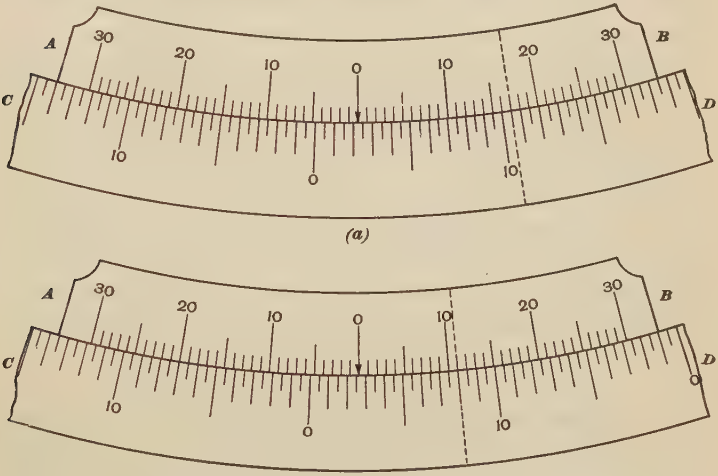


Fig. 33

A *vernier* is an auxiliary scale by means of which a main scale can be read more accurately. The zero of the vernier y is in line with the north and south points of the needle circle, and the vernier rotates with the needle circle. The zero of the scale x is on the line between the slits in the sights, that is, it is on the line of sight, and remains fixed as the vernier rotates.

If the magnetic bearing of a line is wanted, the zero of the vernier y, Fig. 5, should coincide with the zero of the scale x. If the true bearing of a line is required, it can be read directly on the needle circle by setting the zero of the vernier at a point on the scale corresponding to the declination of the needle. Therefore, the combination of the graduated scale x and the vernier y is called the *declination arc* or *declination vernier*.

Both the scale and the vernier are graduated on each side of zero, as shown in Fig. 33, where *A B* is the vernier and *C D* is the scale. If the line of sight is in the true meridian and the declination arc reads zero, the north point of the needle is east of the zero of the needle circle for east declination and west of zero for west declination. If the position of the needle is to indicate true bearings, the needle circle must be rotated to read zero when the line of sight is in the true meridian. To obtain this condition, the needle circle and the vernier, which moves with it, must be rotated in a clockwise direction when the declination is east and in a counter-clockwise direction when the declination is west. Thus, if the declination is east, the zero of the vernier in Fig. 33 is moved toward *C*, or clockwise; and if the declination is west, the vernier is moved toward *D*. In setting the declination, that side of the vernier is used on which the numbers increase in the direction in which the vernier moves. When the vernier is moved toward *C* in Fig. 33, the part of the vernier between 0 and *A* is used; if the vernier moves toward *D*, the part between 0 and *B* is used.



(a)

(b)

Fig. 34

50. The principle on which the construction of a vernier is based is fully explained in another Section. For the purpose of reading and setting the declination arc, the following outline is sufficient. In Fig. 33, each division on the scale *CD* represents half of a degree, or 30 minutes. Therefore, for any position of the vernier, the reading to the next smaller half-degree is given by the number of the scale graduation just preceding the zero of the vernier, that is, between the zero of the scale and the zero of the vernier. To this value is added a number of minutes equal to the number of the vernier graduation mark that coincides with a graduation of the scale (it is not necessary to observe which scale graduation).

Suppose, for example, that the zero of the vernier is set as shown in Fig. 34 (*a*). The scale graduation nearest the zero of the vernier and between the zero of the vernier and the zero of the scale is 2°; the seventeenth graduation of the right part of the vernier coincides with a scale graduation. Hence, the reading is 2° 17′. In Fig. 34 (*b*), the scale graduation preceding the zero of the vernier represents 2° 30′, and the eleventh vernier mark coincides with a scale graduation. The reading, therefore, is $2° \ 30' + 11' = 2° \ 41'$.

To set the vernier at 3° 13′, put the zero of the vernier between the scale graduations representing 3° and 3° 30′, and then bring the thirteenth division of the proper half of the vernier opposite some graduation on the scale. To set the vernier at 4° 38′, place the zero of the vernier between the graduations of 4° 30′ and 5°, and bring the eighth mark of the vernier to coincide with some graduation on the scale.

51. Rerunning Old Surveys.—It is sometimes necessary for a surveyor to retrace the boundaries of a tract of land from bearings and distances given in original land warrants or very old deeds or maps. If the true bearings are given, or if the declination of the needle at the time of the original survey is known, the present magnetic bearings can be readily determined. If the old magnetic bearings are given but the declination is not known, it is sometimes possible to locate one or more of the original lines by means of legally recognized corners, and to measure the true bearings. By comparing the true bearings with the old bearings, the declination can be determined, and the true bearings of the other lines can be readily found from the given original magnetic bearings. The old lines can then be rerun from their true bearings or from their present magnetic bearings. Sometimes, however, there are inaccuracies in the old surveys. In such cases, the old corners or boundaries cannot be altered or moved. Adjustments can be made only by mutual agreement between the interested parties, or by a court decision.

EXAMPLES FOR PRACTICE

1. The declination is 2° 10′ west. Find the true bearings of the lines whose magnetic bearings are (*a*) N 48° 50′ E; (*b*) S 1° W.

$$\text{Ans.}\begin{cases}(a)\ \text{N } 46^\circ\, 40' \text{ E}\\ (b)\ \text{S } 1^\circ\, 10' \text{ E}\end{cases}$$

2. The declination is 4° 15′ east. Find the true bearings of the lines whose magnetic bearings are (*a*) N 87° 10′ E; (*b*) S 3° E.

$$\text{Ans.}\begin{cases}(a)\ \text{S } 88^\circ\, 35' \text{ E}\\ (b)\ \text{S } 1^\circ\, 15' \text{ W}\end{cases}$$

3. The declination is 1° 5′ east. Find the magnetic bearings of the lines whose true bearings are (*a*) S 88° 55′ E; (*b*) S 47° 10′ W.

$$\text{Ans.}\begin{cases}(a)\ \text{Due east}\\ (b)\ \text{S } 46^\circ\, 5' \text{ W}\end{cases}$$

4. The declination is 3° 25′ west. Find the magnetic bearings of the lines whose true bearings are (*a*) S 88° 55′ W; (*b*) S 1° E.

$$\text{Ans.}\begin{cases}(a)\text{N } 87^\circ\, 40' \text{ W}\\ (b)\ \text{S } 2^\circ\, 25' \text{ W}\end{cases}$$

TRANSIT SURVEYING

THE TRANSIT

PRELIMINARIES

DESCRIPTION OF INSTRUMENT

1. Introduction.—The engineer's, or surveyor's, transit is used almost exclusively to measure horizontal and vertical angles in surveying because it combines the features of convenience and a high degree of accuracy. By means of a telescope, the line of sight is well defined and long sights may be taken. Moreover, by the aid of verniers, readings on the graduated scales can be made very accurately. Although the transit is primarily intended for measuring angles without reference to the magnetic needle, most transits have a magnetic needle and a graduated needle circle, and may, therefore, be used as a compass.

While there are many kinds of engineer's transits, all are constructed on the same principles, differing only in minor details. Two forms of transits are shown in Figs. 1 and 2.

2. Telescope.—The telescope *a*, Fig. 1, is similar to that on an engineer's level but is shorter in length; its parts are the objective *b*, the focusing wheel *c*, the cross-wires at *d*, the eyepiece *e*, and the sunshade *f*. The telescope is fixed to the axis *g*, called the *transverse axis*, which rests on the standards *h* and revolves in bearings at the top of the standards. An important feature of the transit shown in Fig. 2 is the rigid construction

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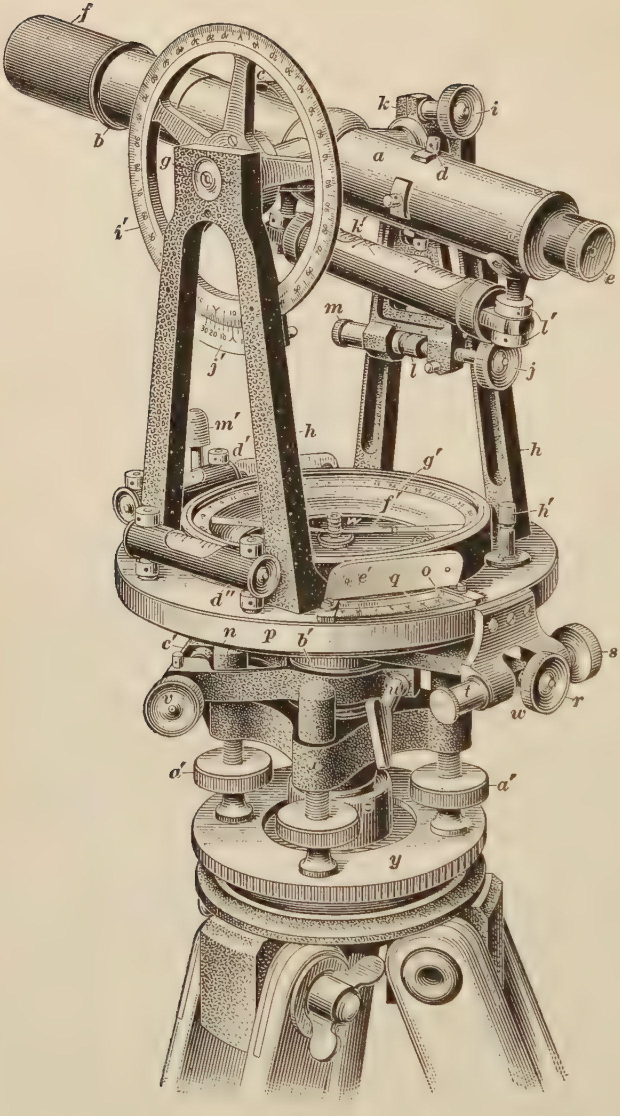


FIG. 1

of the standards, which are cast in one piece. The telescope is held at any desired inclination by means of the clamp screw *i*, Fig. 1, and can be rotated slowly in a vertical plane by means of a tangent screw *j* attached to one standard. The clamp *i* passes through a collar *k* in which the axis *g* revolves when the screw *i* is loose; and a projection *l*, attached to the collar through an arm, fits between the point of the screw *j* and the opposing spring *m*. When the clamp *i* is tightened, the axis *g* is held firmly in the collar *k* and is prevented from moving. If the screw *j* is then turned so that its point pushes against the projection *l*, the axis *g* is caused to rotate in one direction, the

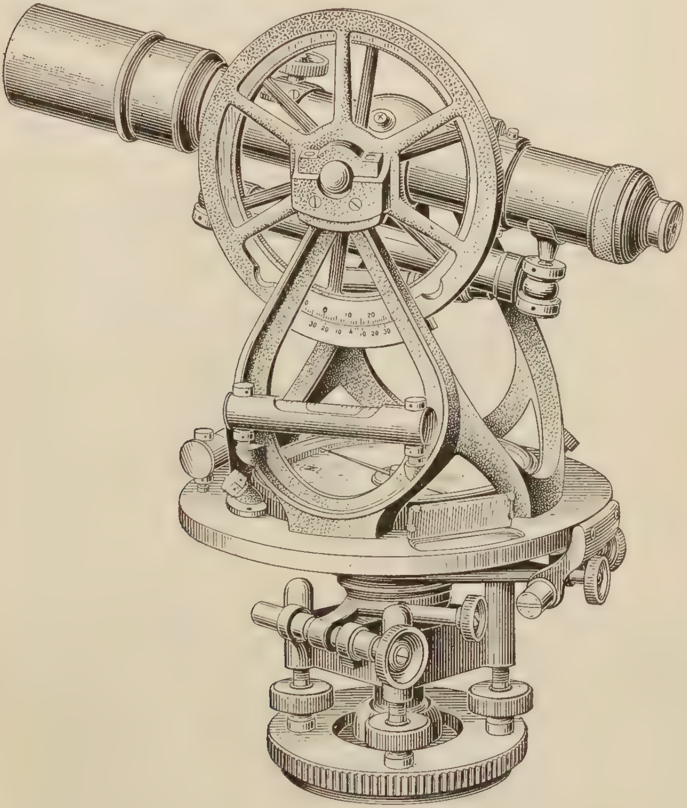


Fig. 2

spring *m* being compressed. When the screw *j* is turned back, the spring forces the projection *l* against the point of the screw and produces rotation of the axis *g* in the opposite direction.

3. Plates and Centers.—In order that horizontal angles may be measured, all transits have two concentric plates, which rotate independently on the same axis, called the *axis of the instrument.* On the lower of these plates *n*, Fig. 1, is a graduated scale, called the *horizontal limb* or the *horizontal circle*, a small part of which is shown at *o*; and on the upper plate *p* are two verniers, one of which is shown at *q*. The upper plate also carries the standards, which are fixed to it.

The upper plate is held in position with respect to the lower plate by tightening the clamp screw *r*, called the *upper clamp;* this operation is known as *clamping the upper plate.* After the upper plate has been clamped, it can be revolved slowly through a small angle by means of the *upper tangent screw s*, which operates against the spring *t*. The lower plate may be secured or clamped against rotation by tightening the *lower clamp u;* this is called *clamping the lower plate.* When the lower plate is clamped, slow motion may be obtained by using the *lower tangent screw v*, which works against a spring. Before either tangent screw is used, the corresponding clamp must be tightened.

The construction of the lower part of a transit is shown in Fig. 3, which represents a cross-section through the axis of the instrument. The conical spindle *n'*, which is connected to the upper plate *p*, Figs. 1 and 3, revolves within a socket attached to the lower plate *n*; the spindle and the socket are held by the nut *o'*, Fig. 3, which screws on the bottom of the spindle. The upper clamp *r*, Figs. 1 and 3, passes through the projection *w* on the upper plate, which fits between the point of the screw *s* and the spring *t*. When the screw *r* is tightened, the collar *p'*, Fig. 3, is pressed against the lower plate and the upper plate is thus secured against rotation on the lower. Slow motion is then obtained by means of the tangent screw *s*.

The socket on the lower plate fits inside of another socket in the *leveling head x*, Figs. 1 and 3. This combination of the spindle and sockets is called the *centers*. The leveling head is connected to the *tripod plate* y by a flexible joint as shown at z, and the inclination of the leveling head and its socket is controlled by the leveling screws a′, which bear on the plate y.

The lower clamp and tangent screw pass through the collar b′, and the projection c′ from the leveling head fits between the point of the screw v and the opposing spring. When the clamp u is tightened, the lower plate cannot turn. By means of the tangent screw v, however, slow motion can be produced.

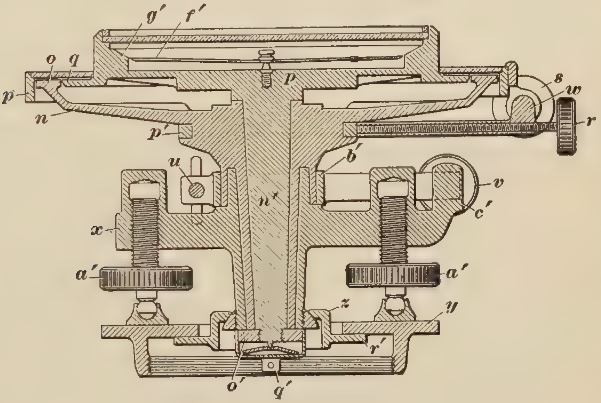


FIG. 3

The spirit levels d′ and d″, Fig. 1, called *plate levels*, are placed at right angles to each other. When each bubble is in the center of its tube at the same time, the plates are horizontal. In Fig. 2, one of the levels is on the standard, but often both are attached to the upper plate, as in Fig. 1.

4. Scales.—Horizontal angles are measured by means of the horizontal limb o, Figs. 1 and 3, marked on the edge of the lower plate, and the verniers q, attached to the upper plate. The upper plate entirely conceals the lower plate except at two small glass-covered openings for the verniers, which are diametrically opposite. The surface of the *reflector* e′, Fig. 1, is specially prepared to reflect light to the vernier and the circle. On the transit shown in Fig. 2, the reflector is hinged so that it can be dropped over the glass when the instrument is not in use, or can be set to any desired inclination.

On the upper plate there is also a magnetic needle *f′*, Figs. 1 and 3, and a needle circle *g′* for use when magnetic bearings are taken. The circle is graduated in the same way as that on a compass, and the north and south points are directly under the line of sight. The screw *h′*, Fig. 1, is for lifting the needle off its pivot.

5. The angle at which the telescope is inclined to the horizontal for any position is measured by means of the graduated scale *i′*, Fig. 1, called the *vertical limb*, and the vernier *j′*. Some transits have an arc, called a *vertical arc*, instead of a full vertical circle as in Fig. 1, while others do not have any vertical limb and do not measure vertical angles. The vertical limb is attached to the transverse axis and revolves with it; the vernier *j′* is screwed to one of the standards. In Fig. 2, the graduated edge of the vertical limb is protected by a guard around it.

A spirit level *k′*, Fig. 1, called a *telescope level*, is attached to the telescope longitudinally whenever there is an arrangement for measuring vertical angles; the telescope level permits the transit to be used also as a leveling instrument. The level tube, which has a graduated scale similar to that on an engineers' level, can be adjusted vertically with respect to the telescope by means of capstan-pattern nuts *l′* at each end; no lateral adjustment is necessary. The rubber stop *m′* prevents the telescope from striking the plate level.

A transit without a vertical limb or a telescope level is called a *plain transit*.

6. Graduations.—The horizontal limb is graduated in various ways, with respect both to the size of the smallest divisions and to the method of numbering. The degrees are marked on all instruments but the number of parts into which each degree is divided and the number of vernier divisions vary. The most common method is to graduate the limb in half degrees and to divide the vernier into 30 parts covering 29 of these half-degree divisions; then, readings can be taken accurately to the nearest minute. On some instruments, the limb is graduated in 20-minute spaces and the vernier is divided into 40 parts; in this case, angles can be measured to 30 seconds. For very precise work, there are instruments whose limbs and verniers are graduated to read to 20 seconds, 10 seconds, and even 5 seconds.

Each subdivision of a degree is marked by a line somewhat shorter than the regular degree-graduations, while each fifth degree is indicated by a longer division line and each tenth degree is marked by a still longer line and also is numbered. Three systems of numbering are in common use, each having its advantage for certain kinds of work. These systems may be described as follows:

1. The *azimuth system*, in which the graduation marks are numbered from 0 continuously around the entire circle to 360.

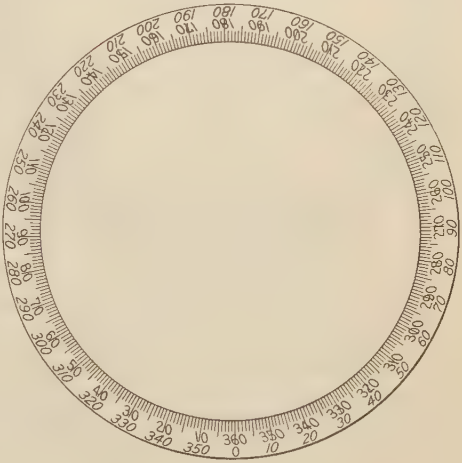


FIG. 4

2. The *transit system*, in which the figures extend from 0 in opposite directions through the adjacent semicircles to 180 at the point diametrically opposite the zero point.

3. The *compass system*, in which the figures extend each way from two 0 points diametrically opposite each other through the adjacent quadrants to the 90° points.

Usually the horizontal limb of a transit has two sets of figures, either of which may be used independently of the other. Ordinarily, the numbers of one set increase from 0 to 360 in a clockwise direction and often the numbers of the other set run from 0 to 360 in the opposite direction, as shown in Fig. 4. However, the other systems are also used, as in Figs. 5 and 6.

On most transits, the direction in which the numbers increase is indicated by the inclination of the figures, as shown in Figs. 4, 5, and 6.

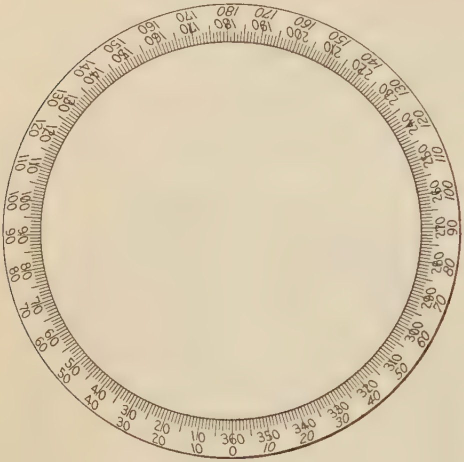


Fig. 5

The vertical limb is usually graduated to degrees and half degrees and, by means of the vernier, readings can be taken to the nearest minute. The vertical circle is always numbered in quadrants with the zero mark opposite the zero of the vernier when the telescope is horizontal. There is only a single row of numbers.

7. Shifting Head.—The position of the instrument over a point on the ground is indicated by a plumb-bob suspended from the lower end of the centers at the central point q', Fig. 3. When the leveling screws a' are tight, the plate r' is held firmly against the plate y; but when the leveling screws are loose, the rest of the instrument drops with respect to the plate y and can be shifted on it. This arrangement, which is called a *shifting head*, is a great advantage in setting up the transit exactly over a point.

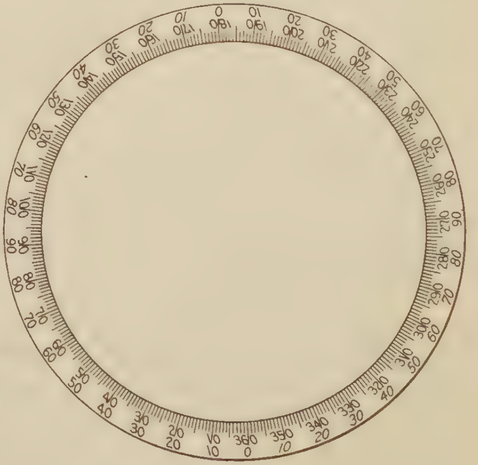
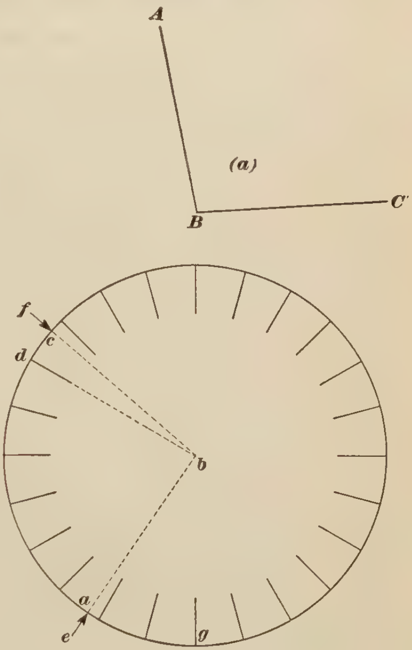


Fig. 6

READING OF VERNIER

8. Measurement of Angles.—In order to measure the angle ABC, Fig. 7 (a), the transit is set at B, the telescope is directed to A, the upper clamp is loosened, and finally the telescope is directed to C. While the line of sight rotates from the direction BA to the direction BC, the horizontal limb of the transit remains stationary and the verniers move along the limb. The amount of the movement in degrees of arc is, therefore, the size of the angle. In order to determine this amount, the location of the zero point of the vernier on the horizontal limb is observed for each position of the telescope.



(b)

Fig. 7

In Fig. 7 (b), let adc represent the horizontal limb; b, the center of the graduated circle; e, the zero point of the vernier when the telescope is directed along BA in (a); f, the zero point of the vernier when the line of sight is directed along BC in (a); and g, the zero point of the horizontal limb. Then, the arc ac in (b), which measures the angle ABC in (a), is found by subtracting the reading of the limb at a from the reading of the limb at c. For convenience the vernier is usually set to read zero when the line of sight is directed to A.

9. Use of Verniers.—One of the features which gives a transit its great accuracy is the fact that fractional parts of the smallest division of the limb can be measured by means of a vernier. Although the limb of a transit is graduated in angular measure and the scale is marked along a curved edge, the principles of the transit vernier are the same as those of the vernier described in connection with leveling rods. For instance, in Fig. 7 (*b*), the arc gdc is considered to be composed of the arc gd, measured on the limb, and the arc dc, measured by the vernier.

Suppose that, in Fig. 8, AB represents a circular scale and CD a vernier that slides along the scale. To measure the arc EF, the zero point of the scale AB is set at E and the vernier is placed so that its zero mark is at F. The arc EF is composed of two parts: the portion EG from the zero of the scale to the scale graduation at G just preceding the zero of the vernier, and the part GF. The zero of the vernier is generally known as the *index* of the vernier, and the reading of the scale opposite the index of the vernier is called the *reading of the vernier*. To determine the reading of a vernier, three steps are performed: (1) the value of the limb graduation preceding the index of the vernier is observed; (2) the value of the fractional part of a division from that graduation to the index of the vernier is found by means of the vernier; and (3) the two values thus obtained are added.

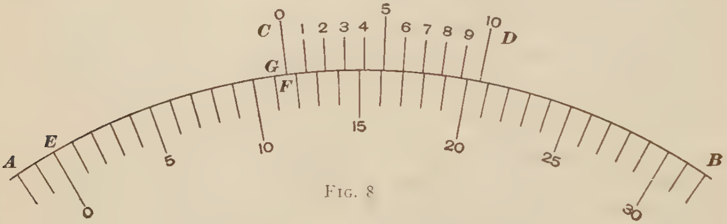


FIG. 8

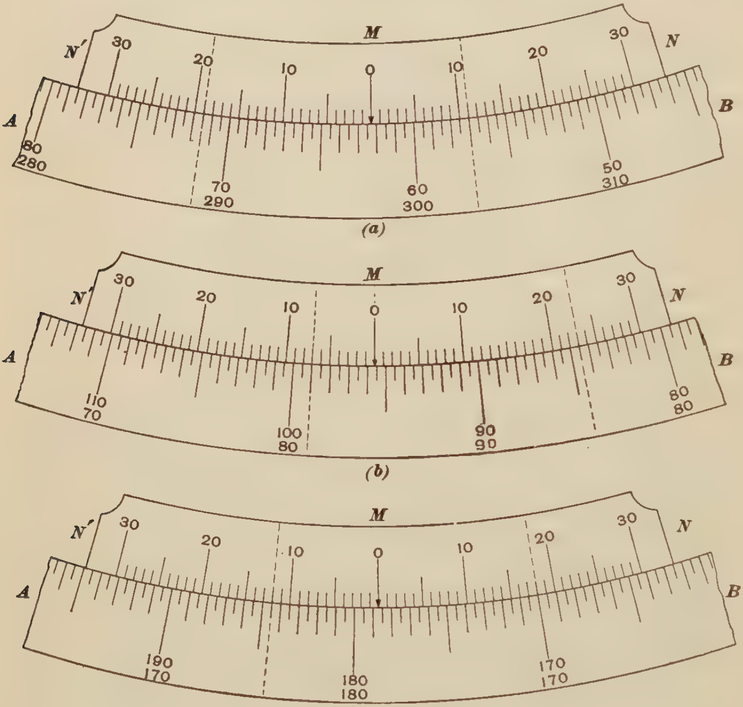
10. Suppose that the limb AB, Fig. 8, is divided in degrees and it is required to measure the arc to tenths of a degree. In this case, the vernier is made with a length equal to 9 limb divisions, and is divided into $9+1$, or 10, equal parts. Then, the total length of the vernier is 9×1, or 9, degrees, and each vernier division is equal to $\frac{1}{10}\times 9=\frac{9}{10}$ degree. Hence, the difference between one limb division and one vernier division is $1-\frac{9}{10}=\frac{1}{10}$ degree. This difference, which is called the *least reading of the vernier*, is always found by dividing the value of a limb division by the number of parts into which the vernier is divided. Thus, in Fig. 8, the least reading of the vernier is equal to $\frac{1}{10}$ degree because each limb division is 1 degree and there are 10 parts in the vernier.

Since the graduation at G is one beyond the mark numbered 10, the arc $E\,G$ represents 11 degrees. The value of the arc $G\,F$ is equal to the product of the least reading of the vernier and the number of the vernier graduation that coincides with a graduation on the limb. Since, in this case, the graduation numbered 6 is opposite a line on the limb, the arc $G\,F$ is equal to $\frac{6}{10}$ degree. This may be proved as follows: The distance from graduation 5 on the vernier to graduation 16 on the limb is equal to the difference between one division on the limb and one on the vernier, or $\frac{1}{10}$ degree; the distance between graduation 4 on the vernier and graduation 15 on the limb is equal to the difference between two limb divisions and two vernier divisions, and so on; finally, the distance from the index of the vernier, which is opposite point F, to the limb graduation preceding it, is equal to the difference between six divisions of the limb and six of the vernier, or $\frac{6}{10}$ degree. Hence, the arc $E\,F$ is $11+\frac{6}{10}$, or $11\frac{6}{10}$, degrees.

11. As in the case of a vernier on a leveling rod, the number of parts into which a transit vernier is divided is one more than the number of limb divisions covered by the vernier. Moreover, the numbers on the vernier increase in the same direction as do those on the limb. Then the value of the part of a division from a limb graduation to the index is found by multiplying the least reading of the vernier by the number of the vernier graduation which coincides with a limb graduation. The number of the limb graduation with which the vernier graduation coincides is not observed.

The numbers of the graduations on the limb of a transit do not increase in the same direction on all parts of the limb, and, therefore, as shown in Fig. 9, the vernier on a transit consists of two similar parts, the numbers of the graduations increasing in both directions from the index; one half of the vernier is used at a time, as will be explained in the following articles.

12. The limb AB in Fig. 9 (*a*), (*b*), and (*c*) is graduated to half degrees; and each part of the vernier NN' has 30



(*a*)

(*b*)

(*c*)

FIG. 9

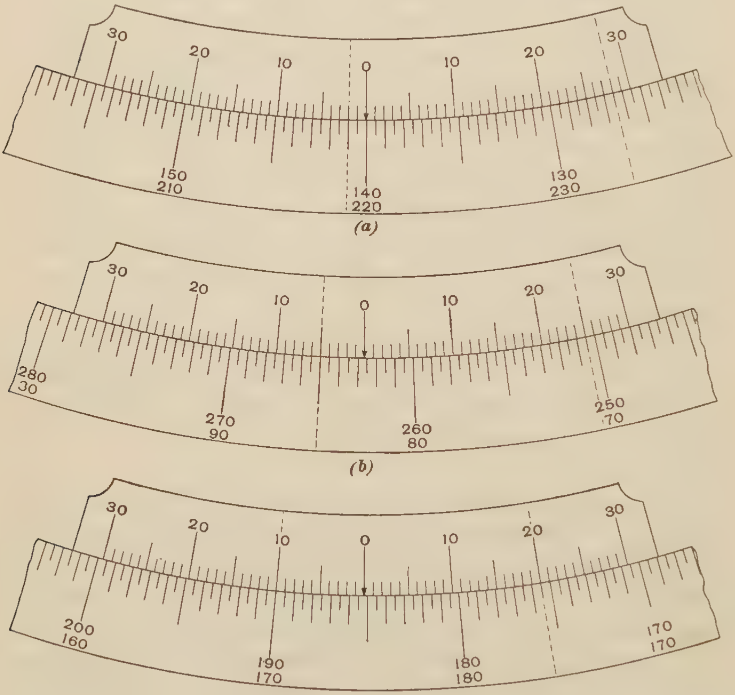
divisions, which cover 29 divisions on the limb. Hence, the least reading of the vernier, which is equal to the value of a limb division divided by the number of parts in the vernier, is $\frac{30'}{30} = 1$ minute. In (*a*) is shown part of a limb on which the graduations are numbered from 0 to 360 in both directions. The figures on the limb and on the vernier of a transit are usually inclined, but they are shown upright here. If the inner scale is read, the numbers increase in a clockwise direction and, therefore, the graduation preceding the index is 62°. As the numbers on the vernier must increase in the same direction as the numbers on the main scale, the vernier from M to N' is used for readings on the inner scale. The nineteenth vernier division coincides with a graduation on the limb, and the least reading of the vernier is 1 minute; hence, the reading of the vernier, which is equal to the product of the least reading and the number of the coinciding vernier graduation, is 1×19, or 19, minutes. The reading of the limb on the inner scale in Fig. 9 (*a*) is, therefore, 62° 19′.

If the outer scale is used, the numbers increase in a counter-clockwise direction and the vernier from M to N is employed. In Fig. 9 (*a*) the graduation preceding the index is 297° 30′ and the vernier reads 11 minutes. Hence, the reading of the limb is 297° 30′+11′, or 297° 41′. It is a common mistake to call the reading 297° 11′; care must, therefore, be taken to include the 30 minutes.

A vernier, such as NN', in which the divisions are numbered in both directions from the center, is known as a *double vernier*. On each transit there are two double verniers, which should be exactly 180° apart on the limb.

13. In Fig. 9 (*b*) is shown part of a limb numbered clockwise from 0 to 360 on the inside, and according to the quadrant system on the outside. The reading for the inner numbers is 95° 30′+7′, or 95° 37′, because the graduation preceding the index is 95° 30′ and the seventh division of the vernier MN' coincides with a graduation on the limb. For the outer set, the numbers between which the index lies increase in a counter-clockwise direction; the reading is 84° 23′, the vernier MN being used. Care must be taken to notice the numbers on each side of the index so that the degrees are not counted in the wrong direction and from the wrong graduation. For instance, the reading for the outer set might be called 95° 37′ by counting clockwise from 90 instead of counter-clockwise from 80.

In Fig. 9 (*c*), AB is part of a limb on which the numbers of the inner set run clockwise from 0 to 360 and those of the outer set run from 0 to 180 in both directions. Since the index lies between 170° and 180° for both sets of numbers, the reading in this case is the same whether the inner or the outer set is used. The value is $178° \ 30' + 12' = 178° \ 42'$. Here, there is a chance of reading in the wrong direction from 180; that is, the reading may be called 181° 18′ by counting counter-clockwise from 180 instead of clockwise from 170.



(a)

(b)

(c)

Fig. 10

EXAMPLE FOR PRACTICE

The verniers shown in Fig. 10 read to minutes; verify the following readings:

	Inner	Outer
(*a*)	140° 2′	219° 58′
(*b*)	262° 35′	82° 35′
(*c*)	185° 10′	174° 50′

AZIMUTHS

14. Forward and Back Azimuths.—Just as a line has a forward and a back bearing, it also has a forward and a back azimuth; that is, the azimuth of a line in one direction is its *forward azimuth* and the azimuth of the same line in the opposite direction is its *back azimuth*. But when the term azimuth is used, the forward azimuth is meant. In Fig. 11, let AB be a line whose azimuth is 115°; NS, the meridian at A; and $N'S'$, parallel to NS, the meridian at B. Then the angle NAB is 115° and the angle $N'BB'$, between the meridian and the prolongation of AB, is also 115°. The azimuth of BA, which is equal to the angle $N'BA$ measured clockwise, is 115°+180°, or 295°. This is also the back azimuth of AB. If the azimuth of BA is given as 295°, then the angle $N'BA$ is 295°, as shown; evidently, the angle NAA' measured clockwise is also equal to 295°. Hence, the azimuth of AB, which is equal to the angle NAB, is 295°−180°, or 115°. This is likewise the back azimuth of BA.

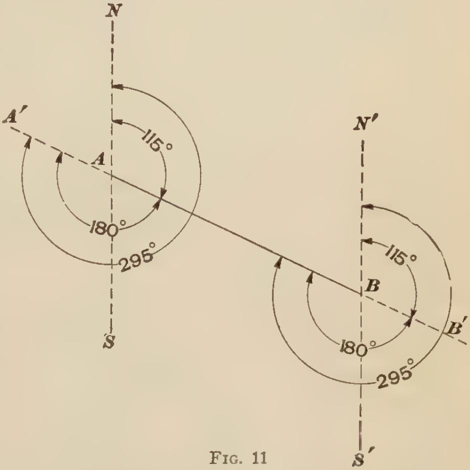


FIG. 11

15. From the preceding explanation, it is seen that the back azimuth of a line can be found from its forward azimuth by one of the following rules:

Rule I.—*If the azimuth of a line is less than 180°, add 180° to find the back azimuth.*

Rule II.—*If the azimuth of a line is greater than 180°, subtract 180° to obtain the back azimuth.*

For example, if the azimuth of a line is 75°, its back azimuth is equal to 75°+180°=255°. If the azimuth of a line is 246°, the back azimuth is 246°−180°=66°.

16. Angles Between Lines.—A line from which an angle is measured in surveying is called a *backsight;* the other side of the angle is a *foresight.* For example, when the angle BAC, Fig. 12 (a), is measured from AB to AC, the backsight is AB and the foresight is AC; likewise, if the angle DAE in (b) is measured from AD, the backsight is AD and the foresight is AE. Sometimes the points to which sights are taken in measuring an angle are also referred to as the backsight and the foresight.

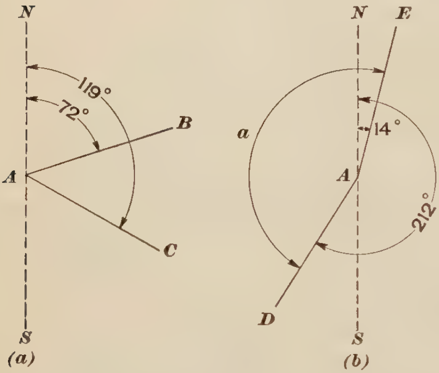


FIG. 12

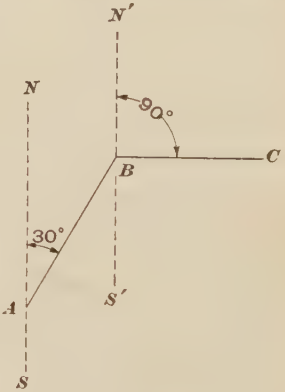


FIG. 13

Suppose that the azimuths of the lines AB and AC in (a) are 72° and 119°, respectively; then, if NS is the meridian through A, the angle NAB is 72° and the angle NAC is 119°. The angle BAC is obviously equal to the difference between NAC and NAB, which is 119°−72°=47°; in other words, the angle between AB and AC is equal to the difference between their azimuths. Now, suppose it is required to find the angle a between the lines AD and AE in (b), the azimuths being, respectively, 212° and 14°. The angle DAN is equal to 360°−212° and the angle NAE is 14°. Hence, the angle DAE, which is equal to the sum of the angles DAN and

NAE, is 360° − 212° + 14° = 360° + 14° − 212° = 162°. Since, in this case, the azimuth of the foresight *AE* is less than that of the backsight *AD*, it is necessary to add 360° to the azimuth of *AE* before the azimuth of *AD* is subtracted from it.

From the preceding explanations, the following rules can be given for finding the angle between two lines when their azimuths are known.

Rule I.—*The angle between two lines, measured clockwise, is equal to the azimuth of the foresight minus the azimuth of the backsight; but in case the azimuth of the foresight is less than that of the backsight, 360° is added to the smaller value before the larger is subtracted from it.*

Rule II.—*The angle between two lines, measured counter-clockwise, is equal to the azimuth of the backsight minus the azimuth of the foresight, 360° being added to the former if necessary before the subtraction is performed.*

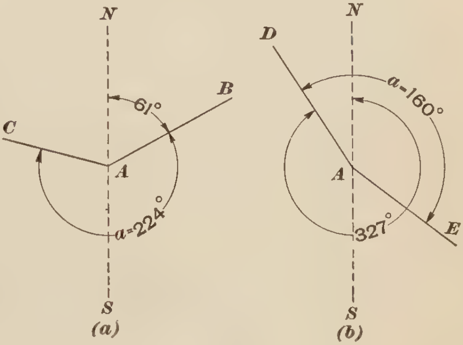


Fig. 14

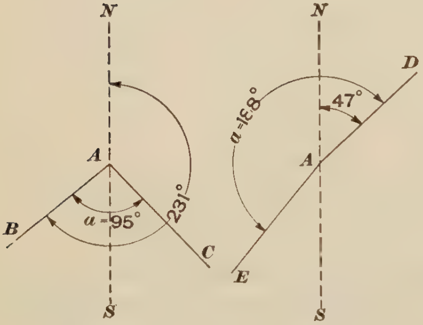
In computing the angle between two lines from their azimuths, both the backsight and the foresight must be taken as starting at the vertex of the angle. For example, let it be required to determine the angle *ABC*, Fig. 13, the azimuths of *AB* and *BC* being 30° and 90°, respectively. The angle *ABC*, counter-clockwise, is equal to the azimuth of *BA* (not *AB*) minus the azimuth of *BC*. The azimuth of *BC* is 90° and that of *BA* is 30° + 180° = 210°; hence, the required angle is 210° − 90° = 120°.

17. Azimuth of Line From Azimuth of Another Line and Angle Between Lines.—Frequently, the angle between a line of known azimuth and a second line is measured, and it is required to compute the azimuth of that second line. For example, suppose that in Fig. 14 (*a*) the azimuth of *AB* is 61° and the angle *a* is 224°; let it be required to find the azimuth of *AC*. Evidently, the angle *NAC* clockwise, which is the azimuth of *AC*, is equal to 61°+224°=285°. Now suppose that the azimuth of *AD* in (*b*) is 327° and the angle *a* is 160°. The angle *NAD* is 360°−327°, and the angle *NAE* is 160° −(360°−327°), or 160°−360°+327°, which may be written as 327°+160°−360°.

Let B=azimuth of given line;
C=angle between that line and another, measured clockwise;
F=azimuth of second line.

Then from the foregoing explanations,

$$F=B+C \qquad (1)$$



(*a*) (*b*)

FIG. 15

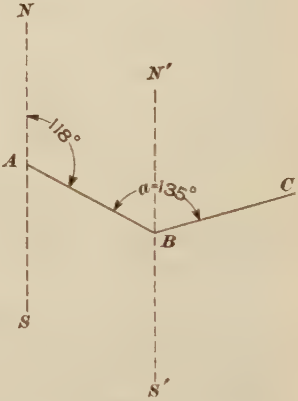


FIG. 16

In case the value of F, obtained by formula **1**, exceeds 360°, it is decreased by 360° in order to make the azimuth less than 360°.

When the angle from a given line to another line is measured counter-clockwise, the formula is

$$F=B-C \qquad (2)$$

in which F and B have the same meanings as in formula **1**, and C is the angle between the lines measured counter-clockwise. In case B is less than C, 360° is added to the value of B before the angle C is subtracted. If, in Fig. 15 (*a*), the azimuth of AB is 231° and the angle a is 95°, then the azimuth of AC may be found by formula **2**; thus,

$$F = B - C = 231° - 95° = 136°$$

Now suppose that in (*b*) the azimuth of AD is 47° and the angle a is 188°. The azimuth of AE is, in this case, $47° + 360° - 188° = 219°$.

When the azimuth of AB, Fig. 16, and the angle a are given, the angle is added to the azimuth of BA (not AB) in order to determine the azimuth of BC. For example, suppose that the azimuth of AB is 118° and the angle a is 135°; then the azimuth of BA is $118° + 180° = 298°$, and the azimuth of BC is $298° + 135° - 360° = 73°$.

18. True and Magnetic Azimuths.—The relations between the true and magnetic azimuths of a line are given by the following formulas:

Let T = true azimuth;
M = magnetic azimuth;
D = magnetic declination.

Then, if the declination is east,

$$T = M + D \qquad (1)$$

and

$$M = T - D \qquad (2)$$

If the declination is west,

$$T = M - D \qquad (3)$$

and

$$M = T + D \qquad (4)$$

EXAMPLE 1.—If the true azimuth of a line is 274° 28′ and the magnetic declination is 2° 40′ west, what is the magnetic azimuth of the line?

SOLUTION.—Since the declination is west, formula **4** applies; hence, the magnetic azimuth of the line is $274° 28' + 2° 40' = 277° 8'$. Ans.

EXAMPLE 2.—The magnetic azimuth of a line is 48° 15′ and the magnetic declination is 3° 15′ east. Find the true back azimuth of the line.

SOLUTION.—Since the declination is east, the true forward azimuth of the line is found by formula **1**; thus,

$$T = M + D = 48° 15' + 3° 15' = 51° 30'$$

The required back azimuth is $51° 30' + 180° = 231° 30'$. Ans.

19. Azimuths and Bearings.—A line in a given direction has but one bearing and one azimuth. The bearing can be found from the azimuth, or vice versa, by one of the following rules:

Rule I.—*If the azimuth is between 0° and 90°, the bearing is northeast and is equal to the azimuth; conversely, if the bearing is northeast, the azimuth is equal to the bearing.*

Rule II.—*If the azimuth is between 90° and 180°, the bearing is southeast and is equal to 180° minus the azimuth; conversely, if the bearing is southeast, the azimuth is equal to 180° minus the bearing.*

Rule III.—*If the azimuth is between 180° and 270°, the bearing is southwest and is equal to the azimuth minus 180°; conversely, if the bearing is southwest, the azimuth is equal to 180° plus the bearing.*

Rule IV.—*If the azimuth is between 270° and 360°, the bearing is northwest and is equal to 360° minus the azimuth; conversely, if the bearing is northwest, the azimuth is equal to 360° minus the bearing.*

Example 1.—Find the bearings corresponding to the following azimuths: (*a*) 62° 10′; (*b*) 111° 31′; (*c*) 200° 14′; and (*d*) 348° 48′.

Solution.—(*a*) By rule I, the bearing is N 62° 10′ E. Ans.
(*b*) By rule II, the bearing is S (180° − 111° 31′) E = S 68° 29′ E. Ans.
(*c*) By rule III, the bearing is S (200° 14′ − 180°) W = S 20° 14′ W. Ans.
(*d*) By rule IV, the bearing is N (360° − 348° 48′) W = N 11° 12′ W. Ans.

Example 2.—Find the azimuths of the lines whose bearings are: (*a*) N 16° 32′ E; (*b*) S 16° 32′ E; (*c*) S 16° 32′ W; (*d*) N 16° 32′ W.

Solution.—(*a*) By rule I, the azimuth is 16° 32′. Ans.
(*b*) By rule II, the azimuth is 180° − 16° 32′ = 163° 28′. Ans.
(*c*) By rule III, the azimuth is 180° + 16° 32′ = 196° 32′. Ans.
(*d*) By rule IV, the azimuth is 360° − 16° 32′ = 343° 28′. Ans.

EXAMPLES FOR PRACTICE

1. Find the true back azimuth of a line whose magnetic forward azimuth is 116° 17′, the magnetic declination being 1° 30′ west.
Ans. 294° 47′

2. If the true azimuth of a line is 249° 21′ and the magnetic declination is 2° 20′ east, what is the magnetic back azimuth of the line? Ans. 67° 1′

3. Find the bearings corresponding to the following **azimuths**: (*a*) 94° 12°; (*b*) 358° 30′; (*c*) 3° 14′; (*d*) 269° 47′.

Ans. $\begin{cases}\text{(a) S 85° 48′ E; (b) N 1° 30′ W;}\\ \text{(c) N 3° 14′ E; (d) S 89° 47′ W}\end{cases}$

4. Find the azimuths of the lines having the following bearings: (*a*) N 88° 16′ W; (*b*) S 5° 7′ E; (*c*) S 5° 7′ W; (*d*) N 88° 16′ E.

Ans. $\begin{cases}\text{(a) 271° 44′; (b) 174° 53′;}\\ \text{(c) 185° 7′; (d) 88° 16′}\end{cases}$

5. If the azimuth of a line *OA* is 130°, that of *OB* is 250°, and that of *OC* is 307°, find the values of the following angles: (*a*) *AOB*, clockwise; (*b*) *AOC*, counter-clockwise; (*c*) *COB*, counter-clockwise, (*d*) *BOA*, clockwise.

Suggestion.—Make a sketch and show the azimuths of the lines as in Fig. 12.

Ans. $\begin{cases}\text{(a) 120°; (b) 183°;}\\ \text{(c) 57°; (d) 240°}\end{cases}$

6. The azimuth of a line *AB* is 110°, and that of *BC* is 175°; find the angle *A B C*, measured clockwise. Make a sketch similar to Fig. 13. Ans. 245°

7. The azimuth of a line *OA* is 130°. (*a*) If the angle *A O B*, clockwise, is 176°, find the azimuth of *OB*. (*b*) If the angle *A O C*, counter-clockwise, is 235°, find the azimuth of *O C*.

Ans. $\begin{cases}\text{(a) 306°}\\ \text{(b) 255°}\end{cases}$

8. If the azimuth of a line *A B* is 45° and the angle *A B C*, clockwise, is 200°, what is the azimuth of *BC*? Ans. 65°

OPERATIONS WITH TRANSIT

GENERAL EXPLANATIONS

20. Transit Points.—A point over which the transit is set up is called a *transit point*. Such a point should be marked as accurately as possible on some firm object. In surface surveys, a firmly embedded rock, or a specially prepared concrete block, called a *monument*, is preferable for locating the most important points; elsewhere, wooden stakes, called *hubs*, are driven flush with the ground. The point is marked on rock by a chiseled cross; in concrete a mark is made while the concrete is soft, or a tack, a nail, or a small pipe is embedded in the soft concrete; the point on a hub is marked by a flat-headed tack, flush with the top of the hub. For identifying points on rock or concrete the necessary facts can be written on the stone with *keel*. To identify a hub and to indicate its location, a projecting stake, which is called a *guard stake* or *witness stake*, is placed near the hub; on it are marked in keel the station number of the hub and any other necessary information, such as the name or the purpose of the survey.

21. Motions of Telescope.—As previously explained, the telescope of a transit has two separate turning motions; namely, rotation about the transverse axis and rotation about the axis of the instrument.

When the telescope is in the position shown in Fig. 1, that is, when the level is under the telescope, it is said to be *normal*. However, it is frequently convenient to turn the telescope on the transverse axis so that it points in the opposite direction and the level is above the telescope. The telescope is then said to be *plunged* or *reversed*. The operation of turning the telescope from its normal to its reversed position, or vice versa, is called *plunging the telescope*. The telescope can be directed toward a point when in either position. For reference, the vernier near the eyepiece when the telescope is normal is marked *A* and the other is marked *B*.

When the instrument is rotated on its vertical axis, it is said to be *rotated in azimuth* because the azimuth of the line of sight changes during the turning. This motion can be effected either by rotating the upper plate alone or by revolving both plates together. In the first case, the lower plate is clamped and the upper plate is unclamped; then the verniers move around the horizontal limb. In the second case, the lower plate is unclamped and the upper plate is clamped; in this motion, the verniers remain fixed with respect to the limb. The operation of turning the instrument through exactly one-half of a revolution, or 180°, is called *reversing in azimuth*. The operations of plunging the telescope and reversing in azimuth both have the effect of pointing the telescope in the opposite direction.

22. Setting Up.—In setting up a transit, it is important to have the center of the instrument, which is the point of intersection of the transverse axis and the vertical axis of the instrument, directly over a given point on the ground; also, the plates and the transverse axis must be horizontal. Much of the work of a surveying party is suspended while the transit is being set up; speed in setting up is, therefore, very desirable.

As previously explained, the location of the center of the transit is indicated by a plumb-bob, suspended from a hook or ring attached to the instrument at q', Fig. 3. The plumb-bob string should be held by a sliding knot in order that the height of the bob can be adjusted; the point of the bob should almost touch the mark over which it is desired to set the transit.

In setting up, the tripod legs are spread, and their points are so placed that the leveling head is approximately horizontal and the telescope is at a convenient height for sighting; the instrument should be within a foot of the desired point but no extra care should be taken to set it very close at once. For convenience in setting up on the side of a hill and over points in rough ground, the legs of the tripod are often made telescopic, in order that their lengths can be adjusted; in setting up on a slope, two legs should be down hill. If the instrument is more than a few inches from the given point, the tripod is lifted without changing the inclinations of the legs and the instrument set as near as possible to the point. By pressing the legs firmly into the ground, the plumb-bob can usually be brought to within a quarter of an inch of the point. If the leveling head is tipped too much, two or, if necessary, all three legs of the tripod are moved to new positions and the plumb-bob is again brought close to the point by pressing the legs into the ground.

When the leveling head is approximately horizontal and the plumb-bob is very near the point, the bubbles of the plate levels are brought almost to the centers of the tubes by means of the leveling screws. If there are four screws, the plates are rotated until one level is parallel to each pair of opposite screws; then each bubble is brought to the center separately by turning the screws of the corresponding pair as explained for the wye level. If there are three screws, the plates are rotated so that one level is parallel to any two screws; both bubbles are then brought to the center at the same time by turning one screw of that pair and the third screw.

After the bubbles are approximately centered, the instrument is loosened on the tripod plate by loosening two adjacent leveling screws, and the plumb-bob is brought exactly over the point on the ground by means of the shifting head. Then the instrument is held securely in position on the tripod plate by tightening the screws that were previously loosened. The bubbles of the plate levels are brought exactly to the centers of the tubes, and the position of the plumb-bob over the point is observed. If the bob has moved off the point in leveling up, it is brought back to the proper position by means of the shifting head, and the instrument leveled again.

23. Review of Functions of Clamps and Tangent Screws. The manner in which the clamps and tangent screws work has already been explained. The subject, however, is here restated in a different form, as a thorough understanding of it is of the greatest importance for an intelligent handling of the transit.

The lower clamp and the lower tangent screw control the motion of the lower plate about the vertical axis. When the lower clamp is set, the lower plate, and, therefore, the instrument as a whole, cannot be revolved in azimuth except very slowly by means of the lower tangent screw. The upper clamp and tangent screw control the sliding motion of the upper plate over the lower. When the upper clamp is set, the upper plate cannot be revolved over the lower except very slowly by means of the upper tangent screw. When the upper clamp is set and the lower loosened, the reading of the instrument is not altered by rotating the telescope. When the lower clamp is set and the upper loosened, the vernier slides along the graduated circle and the reading of the vernier is changed.

24. Directing Telescope to Given Mark.—The telescope, or the line of sight, is said to be directed to a given point when, both plates being clamped, the vertical cross-wire of the transit passes through the point. To apply the method of performing this operation, suppose that, the instrument being set up and leveled at some point, it is desired to direct the line of sight toward a certain mark, as a pole held at one of the stations of a survey.

First one clamp (but not both) must be loosened. If the reading of the vernier is to remain unchanged, the lower clamp should be loosened; otherwise, the upper clamp. The instrument is then revolved in azimuth, one hand being placed near the bottom of each standard. The transitman, looking over the telescope, points it toward the flagpole to be observed, as nearly as he can estimate by the eye. He then looks through the telescope, and, if necessary, turns it to one side or the other, and up or down, until the pole appears in the field of view. Still turning the plate with his hands, he brings the image of the flagpole nearly into coincidence with the vertical cross-wire, as shown in Fig. 17; here *a b* and *c d* are the cross-wires, and *PQ* is the image of the flagpole. He now sets the clamp that has been loosened, and turns the corresponding tangent screw until the intersection of the cross-wires exactly bisects the pole, as shown in Fig. 18. This completes the operation.

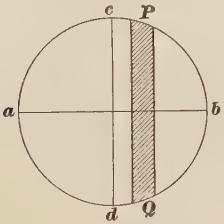


FIG. 17

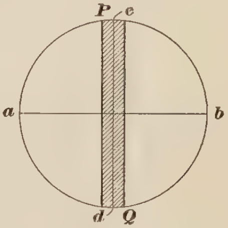


FIG. 18

In transit surveying, the point sighted at should be well defined. The point of a pencil held in the proper position, or the string of a plumb-bob suspended over the point, is preferable for sighting; however, if the sight is very long and such objects cannot be seen distinctly, a range pole can be used. The sight is taken in the same way whether the telescope is normal or reversed. In using a tangent screw, it is important to make sure that the last turn of the screw tightens it against the opposing spring; if the last turn loosens the screw, the spring may not press against the projecting piece between the screw and the spring, and thus the telescope may move after it is set or may not be set properly.

FIELD PROBLEMS

25. Prolonging a Line.—Let AB, Fig. 19, be a line whose position on the ground is fixed by hubs at A and B. The line can be prolonged to C in two ways. In one method, the transit is set up at A, and a sight is taken to B. The point C is located by setting a hub in the line of sight, and marking a point on the hub by means of a tack. If the distance from B to C is to be chained, the chainmen can be lined in by the transitman.

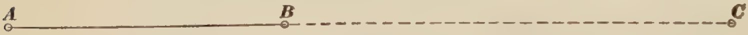


FIG. 19

This method of prolonging a line is often impracticable, since the point C is not always visible from A.

The procedure commonly followed in practice is to set the transit at B, sight to A, and then plunge the telescope so that it will point in the opposite direction, that is, along AB prolonged, or BC. When great accuracy is required, or when there is reason to believe that the transit is not in adjustment, one plate is unclamped, and, with the telescope still reversed, another sight is taken to A. Then the telescope is plunged back to its normal position, and the new direction for AB prolonged is compared with that previously obtained. If the two positions of C do not coincide, the correct point is located midway between them. The process of prolonging a line by taking the average of two sets of sights as just described is called *double centering*.

Just as in the case of turning an angle, the sight along BA is called a backsight and the sight along BC is a foresight. Sometimes, the point A is also referred to as the backsight, and the point C is called the foresight.

26. Measuring Horizontal Angles.—In Fig. 20, let MO and ON be two lines on the ground. To measure the angle

MON, place the transit at *O* and set the index of a vernier, say *A*, to read zero on the horizontal circle by the following method. Unclamp the upper plate and rotate it with respect to the lower plate until the index is within a half degree of the zero of the limb. Then clamp the upper plate and, by means of the upper tangent screw, bring the index exactly opposite the zero of the limb.

With the index clamped at zero and the lower plate unclamped, bring the vertical cross-wire nearly on the point *M* and clamp the lower plate; then bring the intersection of the cross-wires exactly on *M* by means of the lower tangent screw. Next, unclamp the upper plate and bring the vertical wire nearly on the point *N*; clamp the upper plate and bring the intersection of the wires exactly on *N* by means of the upper tangent screw. Then the reading of the horizontal circle at the index of vernier *A* is the value of the angle *MON*. In Fig. 20, the angle is shown as 143° 30′ clockwise, which would be the reading indicated by the inner set of numbers on the limbs shown in Fig. 9. Care must be taken to use the proper tangent screw in each case, as turning the wrong screw is a common source of error.

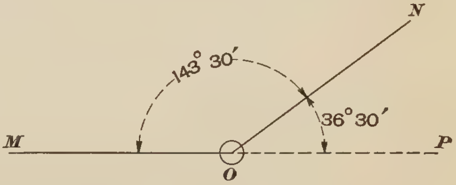


FIG. 20

It is not always necessary to set the index at zero before sighting along the backsight. When the horizontal limb is numbered continuously from 0 to 360 it is sometimes more convenient to read the vernier, wherever it may be, when the instrument is sighted on *M* and both plates are clamped. Then the upper plate is unclamped and the reading for the sight to *N* is taken. The difference between the two readings, taken with the same vernier and on the same set of graduation numbers, is the value of the angle. For instance, the reading for the sight to *M* may have been 140° and that for the sight to *N*, 283° 30′; then the angle *MON* is $283° \, 30' - 140° = 143° \, 30'$. The method explained in the preceding paragraph is usually preferable, however, as there is less chance of confusion and error.

27. Deflections.—The relative directions of two lines may be given by the angle between one line and the prolongation of the other line. Thus, in Fig. 21 (*a*), instead of taking the angle *AOB*, it is often more convenient to consider the angle *A'OB* between *OB* and the prolongation of *AO*. The angle that one line makes with another line produced is called the deflection of the first line from the second. Deflections are measured either clockwise or counter-clockwise from the prolongation of the backsight. Thus, in Fig. 21 (*a*), the deflection of *OB* from *AO* is 20° clockwise, usually designated 20° to the right and written 20° R. In (*b*), the deflection of *OC* from *AO* is 145° to the left, or 145° L. Deflections are always measured in the direction that gives an angle less than 180°.

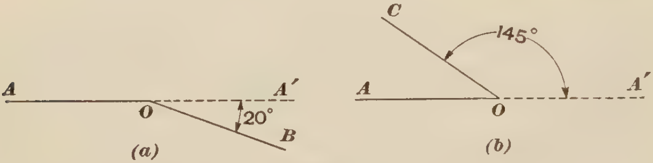


FIG. 21

28. Measuring Deflection Angles.—To measure the deflection of *ON* from *MO* in Fig. 20, the transit is set up at *O*, the index of a vernier, say *A*, is set at zero of the horizontal circle, and, with the telescope reversed, the intersection of the cross-wires is brought exactly on the point *M* by means of the lower clamp and its tangent screw. Then the telescope is plunged back to its normal position, the upper plate is unclamped, and the line of sight is directed to *N* by means of the upper clamp and its tangent screw. The reading of the vernier *A* is the value of the angle *PON*, which is the required deflection angle. In Fig. 20, the angle is given as 36° 30′; since it is turned to the left from *O P*, it would be recorded as 36° 30′ L.

29. Passing Obstacles.—The methods of passing obstacles with a transit are similar to those described for a compass; but, instead of using the bearings of the lines, the angles between the lines are taken. For example, suppose that it is required to determine the length of the line AB, Fig. 22, which crosses a river. First, the line is run to a point P near the bank of the river and the transit is set up at that point. Then any convenient line PQ is laid off, and the angle P and the distance PQ are measured. Finally, the transit is set up at Q and the angle Q is measured. The angle B is equal to $180° - P - Q$; but, if possible, the angle B should be measured in order that the work may be checked by seeing whether the sum of the three angles is 180°. The length of PB is computed by the relation $PB = \frac{PQ \sin Q}{\sin B}$, and the length of AB is found by adding PB and AP.

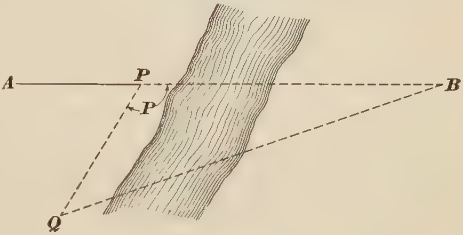


FIG. 22

SURVEYING WITH TRANSIT

RUNNING TRANSIT LINES

INTRODUCTION

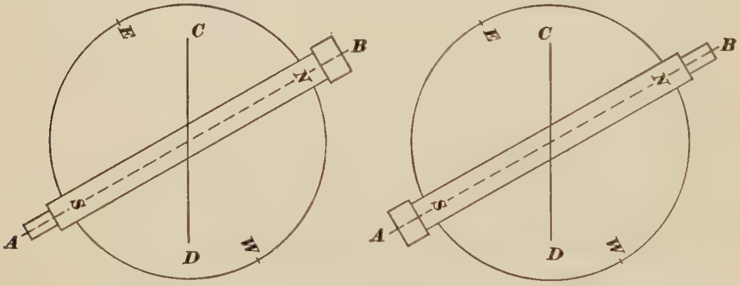
30. Measuring Distances.—Distances in a transit survey should be accurate and, therefore, a steel tape should be used for measurements by chaining. The tape should be held straight, horizontal, and taut; and when the position of a point is to be transferred from the tape to the ground, or vice versa, a plumb-bob should be employed.

As already explained, distances should be measured horizontally, but it is sometimes more convenient to measure a distance parallel to the ground and the angle that the inclined line makes with the horizontal. The required horizontal distance may then be computed by multiplying the inclined distance by the cosine of the vertical angle. Occasionally, the difference in elevation between the ends of the line is determined instead of the angle of inclination. In this case, the horizontal distance is found by the following relation between the sides of a right triangle.

Let a = horizontal distance;
b = inclined measurement;
c = difference in elevation between ends.

Then,
$$a = \sqrt{b^2 - c^2}$$

31. Determining Directions.—There are three common methods of determining the directions of the courses of a traverse by means of a transit: (1) by direct angles; (2) by deflection angles; and (3) by azimuths. Each method has advantages for certain classes of surveys.



(a) (b)

Fig. 23

Most transits have a needle and a needle circle and can, therefore, be used as a compass, the telescope being substituted for the sights. In conjunction with any of the foregoing methods, the bearings of some of the courses in a transit survey are commonly observed on the needle circle as an approximate check to guard against mistakes in reading the limb of the transit. In such cases, the bearings are usually read to the nearest quarter degree.

When a sight is taken with the telescope reversed, the bearing of the line is indicated by the south end of the needle. The following explanation gives the reason for this. Suppose, for example, that the line of sight is directed along the line AB, Fig. 23 (a), with the telescope normal; in this case, the eyepiece is toward A and over the south point S of the needle circle. The needle CD (the north end is at C) indicates that the bearing of AB is northeast, the north end being read when the telescope is normal. Now, suppose that the telescope is plunged to its reversed position. The conditions are then shown in (b); the eyepiece is toward B and the line of sight is directed along BA instead of AB. Since the needle circle has not been rotated, the reading of the needle is not altered. Hence, the north end of the needle still indicates a northeast bearing although the line of sight is now directed southwest. The south end of the needle, on the other hand, properly records a southwest bearing.

METHOD OF DIRECT ANGLES

32. General Description.—In the method of determining the directions of the courses of a transit survey by direct angles, the transit is set up at each corner; and each angle is measured by taking a backsight along one course and a foresight along the next course. The angles may be measured in any order but, to avoid confusion, all angles of the survey should be turned in the same direction, preferably clockwise.

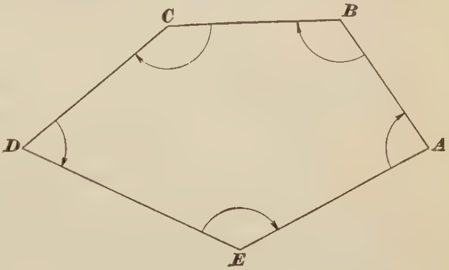


FIG. 24

To survey the boundaries of the field shown in Fig. 24, the transit is set up at any corner, say A, a backsight is taken on E, and the angle EAB is measured clockwise, as indicated by the arrow; the value is recorded in the notes. The magnetic bearing of AB should be taken by reading the needle circle, and the distance AB should be measured. Then the transit is moved to B, and a backsight is taken along BA. The angle

ABC is measured clockwise and the length and the magnetic bearing of BC are determined. The other angles, lengths, and bearings are determined in a similar manner.

The magnetic bearings are used as a check on the angles. Suppose, for instance, that the angle at B, Fig. 24, is 117° 42′, and the bearings of AB and BC are N 37° 30′ W and S 80° 15′ W, respectively. Then the bearing of BC is computed from the observed values of the bearing of AB and the angle at B, as follows: The conditions are shown in Fig. 25; NS represents the meridian through B, the angle SBA is 37° 30′, and the angle ABC is 117° 42′. Then SBC is equal to $ABC - SBA = 117^\circ\ 42' - 37^\circ\ 30' = 80^\circ\ 12'$, and the bearing of BC is S 80° 12′ W. Since this agrees closely with the observed bearing of BC, the angle is taken as correct. In case the observed and calculated values of a bearing do not agree within ½ degree, all readings and computations connected with that bearing should be verified. If no error is found, the difference is probably due to local attraction.

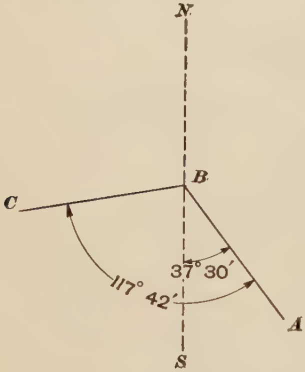
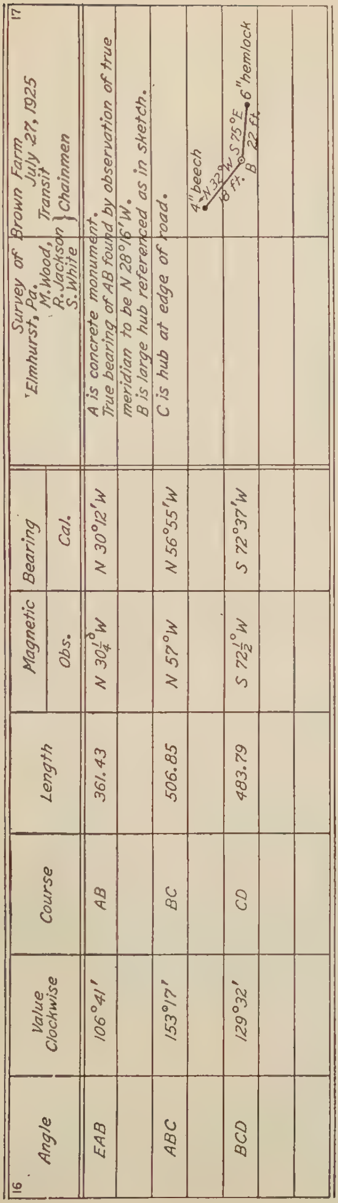


Fig. 25

33. Field Notes.—There are many forms of keeping the field notes for the method of surveying by direct angles, but only one will be shown here to serve as a guide. The title of the survey, the date, the names of the members of the party, and other pertinent information should be given as explained for a compass survey and as shown in Fig. 26. In the first column of the left-hand page of the notebook (a transit book is used) is the angle, the first letter indicating the point on the backsight, the second letter denoting the vertex, and the third letter the point on the foresight; a note to this effect may be inserted at the bottom of the left-hand page to avoid confusion. The second column contains the value of the angle measured clockwise, as indicated by the heading. In the third column is the course, for which the length is given in the fourth column



16						17
Angle	Value Clockwise	Course	Length	Magnetic Bearing		Survey of Brown Farm Elmhurst, Pa. July 27, 1925 M. Wood, Transit R. Jackson, S. White } Chainmen
				Obs.	Cal.	
EAB	106°41'	AB	361.43	N 30¼° W	N 30°12' W	A is concrete monument. True bearing of AB found by observation of true
						meridian to be N 28°16' W. B is large hub referenced as in sketch.
ABC	153°17'	BC	506.85	N 57° W	N 56°55' W	C is hub at edge of road.
						4" beech N 32° W 18 ft.
BCD	129°32'	CD	483.79	S 72½° W	S 72°37' W	B S 75° E 22 ft. 6" hemlock

Fig. 26

and the observed and magnetic bearings in the last two columns of the left-hand page. The notes may be read either from the top of the page downwards or from the bottom of the page upwards.

A complete description of each corner, with references to permanent objects near-by, and any necessary remarks and sketches, should be given on the right-hand page of the notebook. The true bearing of some line of the survey should be determined either from an astronomical observation on the true meridian or from a line of another survey that has been previously established.

34. Measuring Angles by Repetition.—The measurement of direct angles is best adapted to closed traverses where extreme accuracy is required, because the angles can be determined by the following method, called the *method of repetition*, with greater precision than the least reading of the vernier. In this process, the vernier is set at zero and the telescope is directed to the backsight; for instance, if the transit is set up at A in Fig. 24, the telescope is sighted to E. Then the upper plate is unclamped and the telescope is sighted on the foresight, B. The vernier reading is recorded for reference. With the vernier still set in this position, the lower plate is unclamped and the telescope is again directed to E as a backsight. Then, the upper plate is unclamped and another foresight is taken to B. The vernier should now read twice the angle EAB, but it is not necessary to observe the reading. With the vernier clamped at this reading, the lower plate is again unclamped and a third backsight taken on E. Then the upper plate is unclamped and a foresight is taken to B. Now the vernier reads three times the angle $E\,A\,B$. The operation may be repeated as often as desired, the value of the angle being obtained by dividing the final reading by the number of times the angle was turned. For example, suppose the transit can be read to 30 seconds and the first reading is 26° 40′ 30″. Then, after the angle has been turned four times, suppose the vernier reads 106° 41′ 30″. The average value of the angle, which is taken as the correct value, is, therefore, equal to $\frac{1}{4}\times 106° 41' 30'' = 26° 40' 22.5''$. Again, suppose that an angle is found to be about 105° 21′ and that the reading after the angle has been turned six times is 272° 7′. Since the angle is approximately 105° 21′, six times the angle should be about $6\times 105° 21'$, or 632° 6′; hence, the index of the vernier must have turned through more than a complete revolution with respect to the limb, and the final reading may be considered as $360°+272° 7'$, or 632° 7′. The value of the angle, in this case, is taken as $\frac{1}{6}\times 632° 7'$, or 105° 21′ 10″.

By this method a different part of the limb is generally used in each measurement of the angle and the errors due to poor graduation of the limb are practically eliminated. Moreover the unavoidable errors in reading the vernier are not multiplied for each turning.

METHOD OF DEFLECTION ANGLES

35. Outline of Process.—In the method of deflection angles, the directions of the courses are determined by measuring the angle that each line makes with the prolongation of the preceding one. This method is most convenient when all the lines of the traverse have the same general direction, as is usually the case in a survey for a railroad.

To illustrate the process, suppose it is required to run a deflection traverse from the point *F*, Fig. 27, the true bearing of *FG* being known from a previous survey to be S 16° 35′ W. The transit is set up at *F*, and, with the vernier set at zero and the telescope reversed, a backsight is taken to *G*. Since the telescope is reversed, the magnetic bearing of *FG* is observed by reading the south end of the needle. Then the telescope is plunged back to normal, the upper plate is unclamped, and a sight is taken to *A*. The reading of the vernier is recorded as the deflection of *FA*; since *FA* is to the right of *FG* produced, the reading is marked 44° 28′ R. The magnetic bearing of *FA* is also taken and the distance *FA* is measured. The transit is moved to *A*, and the deflection of *AB* from *FA* is measured and recorded as 57° 42′ R; the bearing and length of *AB* are also determined, and the observed bearing is compared with the value calculated from the bearing of *FA* and the deflection of *AB*. The deflections, bearings, and distances of the other courses are obtained in a similar manner.

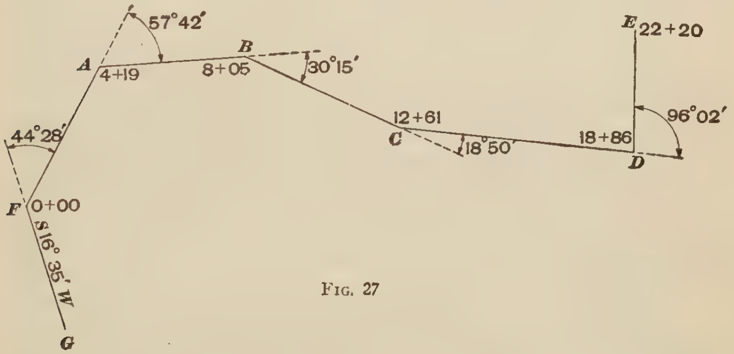


Fig. 27

In a railroad survey, the point *F* would be called Station 0+00 because it is the beginning of the traverse, and the point *A* would be numbered to correspond to the distance *FA*; thus, if *FA* is 419 feet, *A* is Station 4+19. The station numbers of the other points are found in a similar way; the distance from F to B is 419+386=805 feet, and the station number is, therefore, 8+05.

36. Field Notes.—A good method of recording the field notes for a deflection traverse like that shown in Fig. 27 is given in Fig. 28. The general information concerning the purpose of the survey, the members of the party, etc., should be given as in Fig. 26. In the first column of the left-hand page are the station numbers; in the second column, the distances; in the third, the deflections· and in the fourth and fifth, the observed and calculated magnetic bearings. The notes read upwards from the bottom of the page and the values on the line of the notes between any two stations refer to the course in the field joining the same two stations. Thus, 57° 42′ R, between Stations 4+19 and 8+05 in the notes, is the deflection of the course from Station 4+19 to Station 8+05; similarly, 625 in the notes between Stations 12+61 and 18+86 is the distance from Station 12+61 to Station 18+86.

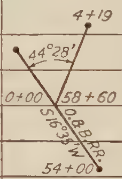
30 Sta.	Dist.	Deflect.	Magnetic Bearing			Remarks	31
			Obs.	Cal.			
22+20						End of Line	
	334	96°02′L	N 32½°E	N 32°33′E			Sta. 0 is at Sta. 58+60
18+86						 4+19 44°28′ 0+00 58+60 S16°35′W O.&B.RR. 54+00	of O. and B.RR. Backsight on
	625	18°50′L	S.51½°E	S 51°25′E			Sta. 54+00 of same line
12+61							True bearing S 16°35′W
	456	30°15′R	S 32½°E	S 32°35′E			and magnetic bearing S15°0′W.
8+05							
	386	57°42′R	S 62¾°E	S 62°50′E			
4+19							
	419	44°28′R	N 59½°E	N 59°28′E			
0+00.							

Fig. 28

The column headed *Remarks* represents the entire right-hand page of the notebook. The reference line from which the survey is started should be fully described, and the relation between the first line of the new survey and the old established line should be given by a sketch as shown. Often the deflections to the right and those to the left are placed in separate columns, headed *right* and *left*, respectively.

METHOD OF AZIMUTHS

37. Orienting.—As previously explained, the azimuth of a line is the angle between the line and a meridian. Although in a survey by azimuths the true meridian is always preferable and sometimes necessary, the magnetic meridian or any other convenient line, called a *reference meridian*, is often used as the direction from which azimuths are measured. The important consideration is that all azimuths of a traverse must be referred to the same meridian, which should be fully described.

When the telescope is directed along a given line and the reading of the vernier of the transit indicates the azimuth of the line of sight, the transit is said to be *oriented*. For example, if the telescope is in the magnetic meridian and the vernier reads zero, the transit is oriented for magnetic azimuths. Likewise, if the true azimuth of a certain line is known to be 30°, and the vernier reads 30° when the telescope is directed along the line, the transit is oriented for true azimuths.

38. Methods of Orienting.—In starting a survey, there must be some line of which the azimuth is either known or assumed and along which the transit can be oriented. There are three methods of orienting, all of which are similar in principle.

One method is based on the fact that the back azimuth of a line is equal to its forward azimuth plus or minus 180°. For example, suppose that the azimuth of OP, Fig. 29, is known to be $64° 21'$, and the transit is set up at P. Then the azimuth of PO, which is the back azimuth of OP, is $64° 21' + 180°$, or $244° 21'$. Hence, if vernier A is set to read $244° 21'$ and the telescope is directed to O, the transit will be oriented along the line PO.

As previously explained, the two verniers on a transit are exactly 180° apart. A second method of orienting utilizes the fact that when vernier *B* reads the azimuth of *OP*, vernier *A* reads that value plus or minus 180°, which is the azimuth of *PO*. Hence, if vernier *B* is set at the azimuth of *OP* and the telescope is directed to *O*, the transit will be oriented for reading vernier *A*.

The third method is based on the principle of prolonging a line by plunging the telescope. If a backsight is taken to *O* with the telescope reversed and then the telescope is plunged back to its normal position, the line of sight will be directed along *PQ*, or *OP* produced, and the azimuth of the line of sight will be equal to the azimuth of *OP*. Hence, in this method of orienting, vernier *A* is set to read the azimuth of *OP*, a backsight is taken to *O* with the telescope reversed, and then the telescope is plunged back to normal. As will be shown in the following article, this third method is the most convenient and is usually preferred.

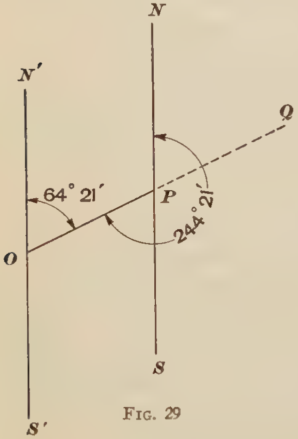


Fig. 29

Often a traverse is started by orienting the transit in the magnetic meridian as follows: Set up over the starting point and loosen the needle. Set the vernier to read zero and then rotate the instrument until the north end of the needle is exactly opposite the north point of the needle circle. Since the vernier reads zero and the telescope is in the magnetic meridian, the transit is oriented.

39. Azimuth Traverse.—Suppose that the traverse shown in Fig. 30 starts at *A*, which is a known point on the line *AF*, whose true bearing was found in a previous survey to be N 42° 36′ W. Since the bearing of *AF* is northwest, its azimuth is 360° − 42° 36′, or 317° 24′. The transit is set up at *A* and is oriented by sighting to *F* with the vernier reading the azimuth of *AF*, or 317° 24′; the magnetic bearing of *AF* is observed and recorded. The upper plate is then loosened and the azimuth of the line of sight for any position of the telescope is given by the reading of the vernier. Thus, if the telescope is directed to *B*, the reading of the vernier, in this case 75° 17′, is recorded as the azimuth of *AB*. The distance from *A* to *B* is measured and recorded, and the bearing of the line is observed.

In using the third method of orienting the transit, the vernier is left set at the azimuth of the preceding course. Thus, when the transit is moved to *B*, the vernier is left at the azimuth of *AB*, and the backsight to *A* is taken with the telescope reversed. Then the telescope is plunged back to normal, the upper plate is unclamped, and the telescope is directed to *C*. The reading of the vernier gives the azimuth of *BC*, in this case 89° 30′; the bearing is also observed as a check, and the distance *BC* is measured. The vernier is left at the azimuth of *BC*; the transit is then set up at *C* and is oriented by backsighting to *B* with the telescope reversed. The operations just described are repeated for each point. In the other methods of orienting, it is necessary to reset the vernier before backsighting, and there is a chance of error in doing so.

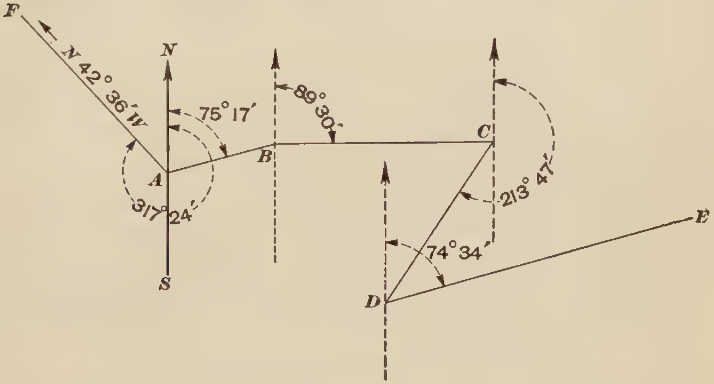


FIG. 30

40. In azimuth surveying, it is convenient to refer the bearings to the same meridian from which the azimuths are measured in order that the bearing and the azimuth of each course may be readily compared in the field. To convert true azimuths to magnetic, or vice versa, the magnetic declination must be known. If it is not given for the locality, it may be found by taking the difference between the true and magnetic bearings of any line. For instance, if the magnetic bearing of AF, Fig. 30, is read on the needle circle as N 40° 15′ W, the angle between the true and magnetic meridians at A is $42° \, 36' - 40° \, 15' = 2° \, 21'$, say 2° 20′. Since the north point of the needle lies to the west of the true meridian, the declination may be taken as 2° 20′ W. If it is suspected that there is local attraction at A, the true and magnetic bearings of some other line must be used.

In most cases where azimuths are used, magnetic azimuths are determined in the field, and the lines are drawn on a map and marked with respect to the magnetic meridian. Then the true meridian is drawn on the map and the angle it makes with the magnetic meridian is indicated; hence, true azimuths can be readily determined. If it is desired to measure true azimuths in the field, true bearings can also be read directly by setting the declination arc that is supplied on most transits.

41. Advantages of Azimuths.—The method of azimuths is best adapted to surveys where many points are to be located from a single set-up as in the case of a topographic survey. One advantage of azimuths is that all angles in the notes are measured in the same direction from the meridian as a backsight. Hence, the chance of an error in recording or interpreting the notes is much less than in recording direct or deflection angles, which must be marked left or right and for which it is necessary to indicate the backsight. Another advantage is that azimuths and bearings can be readily compared in the field; large errors in the transit work can thus be detected at once. In traversing by direct or deflection angles, the work must be delayed while the bearing is calculated from the bearing of the preceding course and the angle, or the calculations must be performed in the office, in which case an error may nullify much field work. A third advantage is that in office calculations, the azimuths or bearings of the courses are usually

62 Course	Length	Magnetic Azimuth	Magnetic Bearing			Survey of Smith-Jones Tract Declination 3°15′ E September 24, 1925 J. Brown, Transit H. Pursel } Chainmen J. Bailey } 63
H-8	192.0	79°0′	N 79°E			Center Rattling Run
H-7	150.86	47°15′	N 47°E			Top of dam
A-H	525.25	41°36′	N $41\frac{1}{2}^\circ$E			H is hub
A-B		8°52′	Checks original azimuth of 8°51′			
G-A	988.99	260°13′	S $80\frac{1}{4}^\circ$W			
G-6	351.33	286°21′	N 74°W			
G-5	298.06	291°14′	N 69°W			
G-4	245.1	316°0′	N 44°W			Center Rattling Run – 8′ wide
G-3	111.4	19°10′	N 19°E			Center Rattling Run – 8′ wide
F-G	547.55	335°55′	N 24°W			G is ☒ on large flat rock
E-F	387.52	228°36′	S $48\frac{1}{2}^\circ$W			F is hub at N. edge of road
D-2	555.1	170°30′				Center Rattling Run – 7′ wide
D-E	628.37	170°31′	S $9\frac{1}{2}^\circ$E			E is hub 30′ N. of road
C-D	594.01	132°45′	S $47\frac{1}{4}^\circ$E			D is hub – Reference 6″ beech – N 78°30′ E, 15.2 ft.
B-1	205.3	70°40′				Center Rattling Run – 7′ wide
B-C	897.46	70°42′	N $70\frac{3}{4}^\circ$E			C is notch and nail in large oak stump B is hub – Reference 8″ oak tree – S 45°50′ W, 22.0 ft.
A-B	659.43	8°51′	N $8\frac{3}{4}^\circ$E			A is stone monument N.E. cor. Hamilton tract. Transit oriented in magnetic meridian. Public road 10′ S.

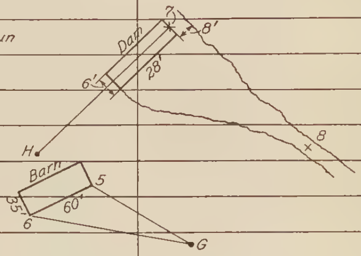
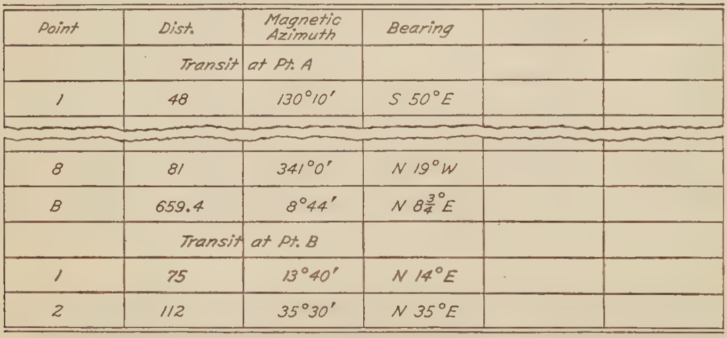


Fig. 31

needed. The fourth important consideration is that plotting with a protractor is much more convenient if azimuths or bearings are used.

42. Field Notes.—When a survey by azimuths does not include many sights from each set-up, the form given in Fig. 31 is convenient for keeping the notes. The general information concerning the purpose of the survey, the members of the party, etc. is given as for the surveys already described. In the first column of the left-hand page are the courses, and in the next three columns are the lengths, magnetic azimuths, and magnetic bearings, respectively. The two remaining columns of the left-hand page and the entire right-hand page are for remarks and sketches.



Point	Dist.	Magnetic Azimuth	Bearing		
	Transit	at Pt. A			
1	48	130°10′	S 50°E		
8	81	341°0′	N 19°W		
B	659.4	8°44′	N 8¾°E		
	Transit	at Pt. B			
1	75	13°40′	N 14°E		
2	112	35°30′	N 35°E		

FIG. 32

The notes read from the bottom of the page upwards. The azimuths of the traverse lines and of important sights are given to the nearest minute; but for locating minor details, as the points on Rattling Run, readings to the nearest 10 minutes are sufficiently close. A complete explanation for the location of point *A* and the method of orienting should be given in the notes as shown. All points should be described fully and accurately. Where possible, the traverse should be closed, as by course *GA*, and, as a check, the azimuth of *AB* should be redetermined by orienting on *G*.

When many points are located from each set-up, the form given in Fig. 32 is commonly used for the left-hand page of the notebook. A sketch, showing the relative locations of the points and what each indicates, should be made on the right-hand page with any other information that cannot be given on the sketch.

SURVEYING BY TRIANGULATION

43. General Principle.—In surveying by triangulation, one line, called a *base line*, is measured, and all other distances are determined by measuring the angles of a triangle and computing the lengths of the sides. The determination of the distance PB in Fig. 22 by measuring the distance PQ and the angles P and Q is an illustration of triangulation. The base line in that case is PQ, the length of which is measured; and, when the angles P and Q are also measured, the distances PB and QB can be calculated by the relations between the sides of a triangle and the sines of the angles. Thus,

$$PB = \frac{PQ \sin Q}{\sin B}$$

and

$$QB = \frac{PQ \sin P}{\sin B}$$

If the distances PB and QB are required with great accuracy, the distance PQ is measured very carefully and the angles are measured by repetition. Moreover, in such a case, the three angles are always measured. A check on the work is thus provided because the sum of the angles should equal 180°.

44. Use of Triangulation.—Triangulation is by far the most accurate and most convenient method of determining distances when the points are very far apart or are separated by rough country. Often, measuring distances in any other way is impracticable because the points may be separated by obstacles which make chaining very difficult; in Fig. 22, the river between P and B is such an obstacle. In surveying by triangulation, it is important that the base line should be as long as possible, and that angles less than 20° should be avoided.

The main use of triangulation, however, is in running a network of lines over a large area in order to locate accurately many points from which other surveys can be started. The most extensive triangulation surveys are being continually made by the United States Coast and Geodetic Survey. In such a survey, the directions of lines are usually referred to the true meridian.

45. Extended Triangulation Survey.—It is not always possible to determine a required distance by means of a single triangle. Then a series of triangles is laid out, in which the base line of each triangle is one of the sides of the preceding triangle. The simplest method of arranging these triangles is shown in Fig. 33; in this case, the process of determining the length and direction of *AH* is as follows: First, the line *B C* in any convenient direction is selected as a base line and its length is carefully measured; if desired, the base line can be run through *A*. Then a number of points, *D*, *E*, *F*, *G*, are selected so as to form a net of triangles, *BCD*, *CDE*, etc., and the three angles of each triangle are measured. Thus, at *C*, the angles *ACB*, *BCD*, and *DCE* are determined; and at *D*, the angles *BDC*, *CDE*, and *EDF* are obtained.

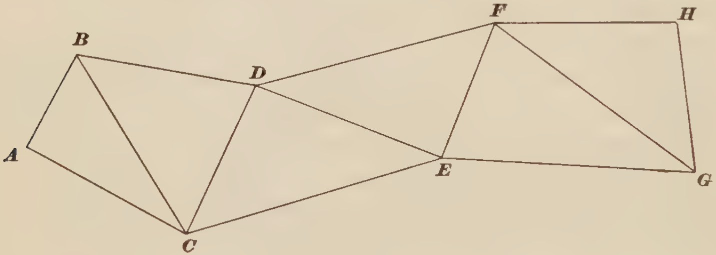


Fig. 33

The azimuth of the base line *B C* is determined or assumed and then the lengths and azimuths of the other lines can be calculated. The length and azimuth of *C D* will be found from the measurements in triangle *BCD*. Then *CD* can be used as a base line for determining *D E* from the measurements in triangle *C DE*. Similarly, *D E* is the base line for triangle *DEF*, *EF* is the base line for triangle *EFG*, and *FG* is the base line for triangle *F G H*. The method of calculating the length and the azimuth of *AH* will be explained in another Section.

It is evident that all the points should be selected before starting the survey in order that all angles having the same point as a vertex can be measured at the same time. The angles may be taken in any order; for example, it may be more convenient to measure the angles at *C*, *E*, and *G* first, and then to proceed to *B*, *D*, and *F*. It is essential that each point not only should be visible from the other points to which sights are taken but also should be accessible for setting up the transit. In the United States Coast and Geodetic Survey, a tower is often built with a platform at the top when no other suitable point is available.

TRIGONOMETRIC LEVELING

46. Measuring Vertical Angles.—As has been explained, trigonometric leveling is the determination of differences in elevation by means of horizontal or inclined distances and vertical angles. For instance, the difference in elevation between the points *A* and *B*, Fig. 34, may be found by measuring the horizontal or inclined distance from *A* to *B* and the angle between a horizontal line and the line through *A* and *B*. This angle may be measured by means of a transit.

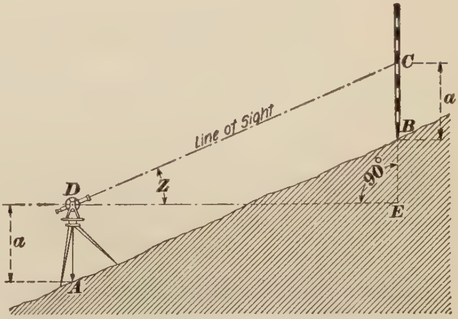


Fig. 34

If a transit is in adjustment, the reading of the vertical limb is zero when the line of sight is horizontal. In such a case the vertical angle between the horizontal and the line of sight for any position of the telescope is given directly by means of the reading of the index of the vernier *j'*, Fig. 1, on the vertical limb *i'*.

To measure the vertical angle between the horizontal and a line between two points on the ground, as *A* and *B*, Fig. 34, the transit is set up over one of the points, say *A*, and the vertical distance a from the ground to the center of the transverse axis is measured by means of a tape or a leveling rod. Then a pole or a rod is held vertically at B, and the point C is marked on it at the height a above the ground. Finally, the horizontal cross-wire of the telescope is set on C, and the required vertical angle Z is read from the vertical limb.

If the objective of the telescope is higher than the eyepiece, the angle is an *angle of elevation* and is marked +, or is given without a sign. If the objective is lower than the eyepiece, the angle is an *angle of depression* and is marked −; the sign − must always be written, as a value without any sign is understood to have the sign +.

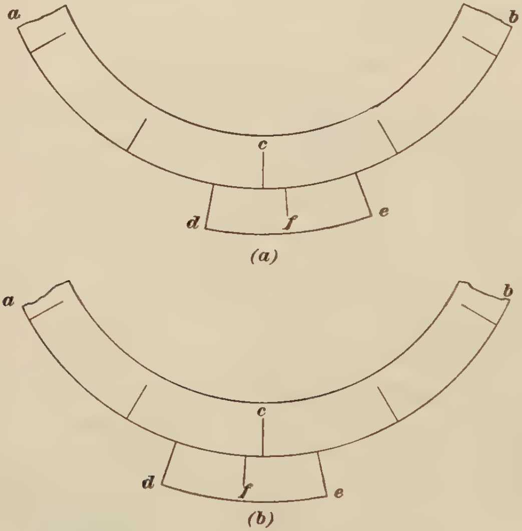


FIG. 35

47. Index Error.—In the method of measuring vertical angles described in the preceding article, it is assumed that the reading of the vertical limb is zero when the line of sight is horizontal. On most transits, the vernier can be shifted slightly on the standard to which it is attached, and the index can thus be set to coincide with the zero of the vertical limb, but it may not be possible, or perhaps not always convenient, to make this adjustment. In case the vernier is not adjusted, its reading when the bubble of the telescope level is centered is called the index error. Suppose, for instance, that the telescope in Fig. 1 is clamped in a horizontal position, as indicated by the fact that the bubble of the telescope level k' is in the center of its tube, but the index of the vernier j' is not opposite the zero of the vertical limb i'. Let $a\ b$, Fig. 35, represent part of the vertical limb, with the zero point at c; and $d\ e$, the vernier, with the index at f. It is assumed that the telescope is normal and the eyepiece is to the right beyond b.

If the zero of the vernier lies between the zero of the limb and the eyepiece of the telescope in its normal position, as in (*a*), the index error is marked +; if the zero of the vernier is between the zero of the limb and the objective of the telescope, as in (*b*), the index error is marked −. When there is an index error, readings of the vertical limb must be corrected by an amount, called the *index correction*, which is equal to the index error. This correction is applied as follows:

Rule I.—*If the error is +, the correction is subtracted from an angle of elevation or added to an angle of depression.*

Rule II.—*If the error is −, the correction is added to an angle of elevation or subtracted from an angle of depression.*

In performing the addition or subtraction, the signs of the angle and of the error are disregarded. For example, if the observed angle is $+10° 45'$ and the error is $+4'$, the corrected value of the angle is, by rule I, $10° 45' - 4' = 10° 41'$. If the angle is $-10° 45'$ and the error is $+4'$, the corrected numerical value is $10° 45' + 4' = 10° 49'$; of course, the negative sign is retained to indicate that the angle is an angle of depression. Again, if the observed angle is $+10° 45'$ and the error is $-4'$, the corrected angle is, according to rule II, $10° 45' + 4' = 10° 49'$. Finally, if the angle is $-10° 45'$ and the error is $-4'$, the corrected numerical value is $10° 45' - 4' = 10° 41'$.

Observations of the index error should be made at intervals during the day, or at least at the beginning and the end of the day's work. It is best to record the actual limb readings and the index error; then the corrected values can be determined in the office at the end of the day.

48. General Formulas.—In Fig. 34, BC is parallel and equal to AD. Hence, $ABCD$ is a parallelogram and DC is parallel and equal to AB. The difference in elevation between A and B is, therefore, represented by the distance CE. From the right triangle CDE,

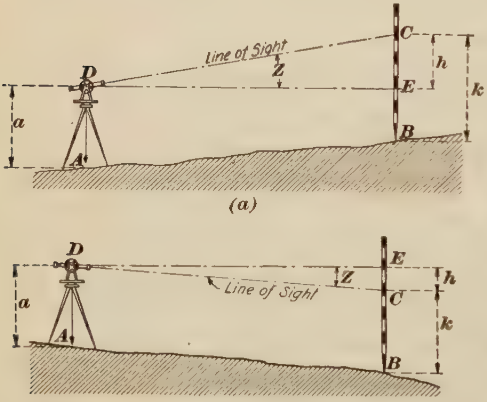
$$CE = DC \sin Z \text{ and } CE = DE \tan Z$$

In general, let

h = difference in elevation between two points;
l = length of inclined line between points;
Z = vertical angle that line makes with horizontal;
s = horizontal distance between points.

Then,
$$h = l \sin Z \qquad (1)$$
$$h = s \tan Z \qquad (2)$$

If the vertical distance is desired with great accuracy, the inclined or the horizontal distance should be measured carefully. Also, the vertical angle should be measured in both directions; for instance, with the transit at A, Fig. 34, a sight is taken to B, and with the transit at B, the reading from B to A is observed. If the two vertical angles differ slightly, the average of the values is taken in calculating the difference in elevation between the points; thus, if the value with the transit at A is $+15° 17'$, and with the transit at B, it is $-15° 19'$, the correct value is taken as $\frac{15° 17' + 15° 19'}{2}$, or $15° 18'$.



(a)

(b)

Fig. 36

Occasionally, it is more convenient to sight on a point which is not at the same height above the ground as the transverse axis of the instrument; thus, in Fig. 36, BC is not equal to AD.

Let h' = difference in elevation between two points;
a = height of transverse axis above ground at first point;
h = distance computed by formula **1** or formula **2**;
k = rod reading at second point.

If the vertical angle is an angle of elevation, as shown in (*a*), the difference in elevation between A and B is found by the formula

$$h' = a + h - k \qquad (3)$$

If the vertical angle is an angle of depression, as in (*b*), the formula is

$$h' = a - h - k \qquad (4)$$

Example 1.—(*a*) If, in Fig. 34, the distance DC is equal to 186.32 feet, the vertical angle from A to B is $+18° 2'$, and the angle from B to A is $-18° 3'$, what is the difference in elevation between A and B? (*b*) If the elevation of A is 110.2 feet, what is the elevation of B?

Solution.—(*a*) The average value of the vertical angle is $\frac{18° 2' + 18° 3'}{2} = 18° 2' 30''$. By formula **1**,

$$h = l \sin Z = 186.32 \times \sin 18° 2' 30'' = 57.7 \text{ ft. Ans.}$$

(*b*) Since B is higher than A, the elevation of B is equal to $110.2 + 57.7 = 167.9$ ft. Ans.

Example 2.—The transit is set up over a point A, Fig. 36 (*b*), with the center of the transverse axis 4.9 feet above the ground; and, with the horizontal cross-wire reading 7.5 feet on a leveling rod held at B, the vertical angle is $-3° 41'$. If the horizontal distance from A to B is 361.74 feet, and the elevation of A is 98.3 feet, find the elevation of B.

Solution.—By formula **2**,

$$h = s \tan Z = 361.74 \tan 3° 41' = 23.3 \text{ ft.}$$

Since the angle Z is negative, C is below E. Then, in formula **4**, $a = 4.9$, $h = 23.3$, and $k = 7.5$; hence,

$$h' = 4.9 - 23.3 - 7.5 = -25.9 \text{ ft.}$$

This means that B is 25.9 ft. below A; the elevation of B is, therefore, equal to $98.3 - 25.9 = 72.4$ ft. Ans.

49. Elevations of Inaccessible Points.—A very useful application of trigonometric leveling is in determining the difference in elevation between two points when one of them is inaccessible. Such a case is shown in Fig. 37, where it is required to find the difference in elevation between A and B, direct leveling and the method explained in Art. 48 being impracticable. If the ground is fairly level for some distance from A, the following procedure is convenient. Set the transit over A, bring the horizontal cross-wire on B, and determine the vertical angle b that the line of sight CB makes with the horizontal line CD; measure also the vertical distance a from the ground to the transverse axis. Next, select a point E on line between A and B, and measure the horizontal distance c. Then, set the transit over E, bring the horizontal wire on B, and determine the vertical angle d between the line of sight FB and the horizontal. Finally, with the transit still at E, find the vertical distance e by setting the line of sight along the horizontal line FG and reading a leveling rod held at A. The difference in elevation between C and F, denoted by f, is equal to $a-e$, and h, the required difference in elevation between A and B, may be computed by the formula

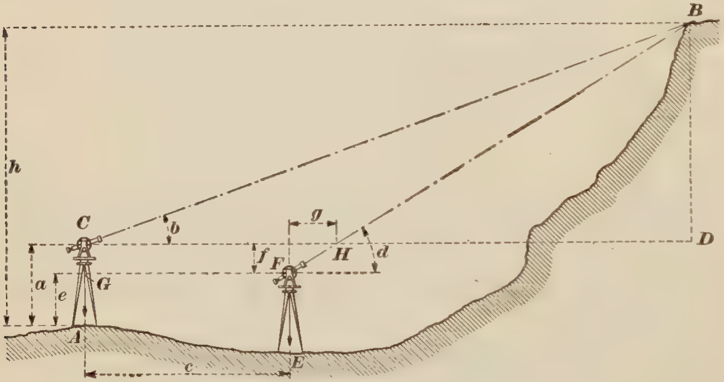


FIG. 37

$$h=a+\frac{c+f\cot d}{\cot b-\cot d} \qquad (1)$$

This relation is derived as follows: The line of sight FB intersects the horizontal line CD at H, the vertical distance between F and H being equal to f, and the horizontal distance g being equal to $f \cot d$. From the figure, $CD = CH + HD$; but $CH = c + g = c + f \cot d$ and $HD = BD \cot d$. Hence,

$$CD = c + f \cot d + BD \cot d$$

Also, in the right triangle BCD,

$$CD = BD \cot b$$

Thus, $$c + f \cot d + BD \cot d = BD \cot b$$

whence, $$BD = \frac{c + f \cot d}{\cot b - \cot d}$$

and $$h = a + BD = a + \frac{c + f \cot d}{\cot b - \cot d}$$

In case the point F is above C, as indicated by the fact that e is greater than a, the value of f is $e - a$ and the formula for the difference in elevation between A and B is

$$h = a + \frac{c - f \cot d}{\cot b - \cot d} \qquad (2)$$

If the horizontal distance CD is required, it may be found by the formula

$$CD = (h - a) \cot b \qquad (3)$$

The point E should be not more than 4 feet vertically below A and not more than 6 feet above A in order that the distance e may be obtained from the set-up at E by reading a rod held on A. In case this is not possible because the ground is too steep, the distance f must be found by direct leveling, a turning point being established between A and E. It is assumed in formulas 1 and 2 that the point B is higher than A; this is almost always the case in practice, because it is hardly possible that B would be visible from both A and E when B is below A.

Example.—In order to determine the elevation of a point B, Fig. 37, on the top of a cliff, a transit having an index error of $+3'$ was set over a point A, the elevation of which was known to be 161.8 feet; the height a of the transverse axis was 5.0 feet, and, when the horizontal wire was brought on B, the vertical limb read $+16° 32'$. A point E was next located at a horizontal distance c from A equal to 200 feet. Then the transit was set over E, the horizontal wire brought on B, and the vertical limb reading observed as $+24° 48'$. Finally, with the telescope horizontal, the reading e on a leveling rod held on A was 7.8 feet. Find the elevation of B.

Solution.—Since the index error was $+3'$, the corrected vertical angle b was $16° 32'-3'$, or $16° 29'$, and the corrected angle d was $24° 48'-3'$, or $24° 45'$. The value of f, which is equal to $e-a$, is $7.8-5.0=2.8$ ft. Then, the difference in elevation between A and B is found by formula **2**. Here,

$$h=5.0+\frac{200-2.8 \cot 24° 45'}{\cot 16° 29'-\cot 24° 45'}$$

$$=5.0+\frac{200-2.8\times 2.16917}{3.37955-2.16917}$$

$$=5.0+\frac{193.93}{1.2104}$$

$$=165.2 \text{ ft.}$$

Hence, the elevation of B is $161.8+165.2=327.0$ ft. Ans.

50. Often, the method of the preceding article cannot be applied because a suitable point cannot be established between the two given points. In such a case, the following method may be used: Let it be required to determine the difference in elevation between A and B, Fig. 38. First, select a point C several hundred feet from A and in such a position that the point B is visible and the distance $A C$ may be readily chained. Then set the transit over A and, with the vernier reading zero,

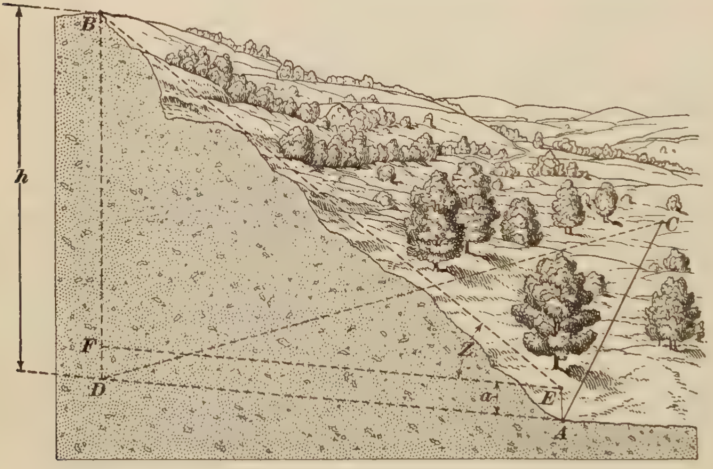


Fig. 38

sight to C. Next, unclamp the upper plate and bring the intersection of the cross-wires on B; for this setting, read the horizontal limb and the vertical limb. Also measure the horizontal distance AC and the vertical distance a from the ground to the transverse axis. Now, set the transit over C and, with the vernier reading zero, sight to A. Finally, loosen the upper clamp, sight to B, and read the horizontal limb. The required difference in elevation is then computed as follows: In the figure, D is a point vertically beneath B and at the same elevation as A. The reading of the horizontal limb when the transit is at A measures the angle between AD and the horizontal projection of AC, and the reading with the transit at C indicates the angle between the horizontal projections of AC and CD. In the triangle ACD, angle ADC is $180° - CAD - ACD$ and

$$AD = \frac{AC \sin ACD}{\sin ADC}$$

The reading of the vertical limb when the transit is at A gives the vertical angle Z between the line of sight EB and the horizontal line EF; from the figure, EF is equal to AD and

$$FB = EF \tan Z = AD \tan Z$$

If Z is an angle of elevation, as in Fig. 38, the difference in elevation between A and B, which is equal to BD and is denoted by h, is $FB + a$ or $AD \tan Z + a$. When the value of AD is substituted, this becomes

$$h = \frac{AC \sin ACD \tan Z}{\sin ADC} + a \qquad (1)$$

If Z is an angle of depression,

$$h = \frac{AC \sin ACD \tan Z}{\sin ADC} - a \qquad (2)$$

If, in formula **2**, h is positive, B is below A; but if h is negative, B is above A.

EXAMPLE.—The horizontal distance AC, Fig. 38, is 500 feet, angle DAC is 89° 15′, angle ACD is 60° 28′, a is 5.0 feet, and the vertical angle Z is $+29° 44'$. Find the difference in elevation between A and B.

SOLUTION.—Angle $A\,D\,C$ is equal to $180° - 89°\ 15' - 60°\ 28' = 30°\ 17'$. Then, by formula **1**,

$$h = \frac{A\,C \sin A\,C\,D \tan Z}{\sin A\,D\,C} + a$$

$$= \frac{500 \sin 60°\ 28' \tan 29°\ 44'}{\sin 30°\ 17'} + 5$$

$$= 492.7 + 5 = 497.7 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The distance between two points M and N, measured parallel to the ground, is 214.69 feet, and the line of sight parallel to the ground makes a vertical angle of $-11°\ 33'$ from M to N and an angle of $+11°\ 35'$ from N to M. If the elevation of M is 81.6 feet, what is the elevation of N?
Ans. 38.6 ft.

2. The horizontal distance from C to D is 811.6 feet; with the transverse axis 5.2 feet above the ground at C and the horizontal cross-wire set at 4.0 feet on a rod held at D, the vertical angle is $+2°\ 56'$. If the elevation of C is 816.5 feet, what is the elevation of D? Ans. 859.3 ft.

3. In Fig. 37, the angle b is $+7°\ 51'$, the height a is 4.8 feet, the distance c is 250 feet, the angle d is $+12°\ 14'$, and the height e is 3.6 feet. If the elevation of the point A is 37.9 feet, what is the elevation of B?
Ans. 139.5 ft.

4. A base line $A\,B$ has a horizontal length of 400 feet. The transit is set up at A with the transverse axis 4.7 feet above the ground; the horizontal angle from $A\,B$ to an inaccessible point C is $87°\ 45'$, and the vertical angle between a horizontal line and the line of sight from A to C is $-35°\ 24'$. The transit is then set up at B and the horizontal angle $A\,B\,C$ is found to be $65°\ 30'$. If the elevation of A is 1,214.7 feet, what is the elevation of C?
Ans. 644.7 ft.

ADJUSTMENTS OF TRANSIT

51. Conditions of Adjustment.—When a transit is in adjustment, the three following conditions are fulfilled:

1. When the bubbles of the plate levels are in the centers of the tubes, the plates are horizontal, the axis of the instrument is vertical, and the transverse axis of the telescope is horizontal.

2. The line of sight is perpendicular to the transverse axis of the telescope, and, therefore, remains in a vertical plane as the telescope is rotated on the transverse axis.

3. When the bubble of the telescope level is in the center of the tube, the line of sight is horizontal and the vertical limb reads zero.

There are five adjustments that must be made in the same order as they are described in the following articles. An open space which is nearly level and which affords an unobstructed sight of about 450 feet should be chosen; and, in setting up, the tripod legs should be planted firmly in solid ground that is not subject to jars from heavy machinery or other causes.

52. First Adjustment.—First, it is necessary to make the axes of the plate levels perpendicular to the vertical axis of the instrument in order that, when the bubbles are centered by the leveling screws, the axis will be vertical and the plates will revolve in a horizontal plane. This adjustment, which is substantially the same as for the compass, is performed as follows: With the upper clamp set and the lower clamp loose, turn the instrument so that each plate level will be parallel to the line determined by a pair of opposite leveling screws; then bring each bubble to the middle of its tube by means of the corresponding pair of leveling screws. Next, revolve the instrument on its vertical axis through 180°. If the levels are in adjustment, the bubbles will remain in the centers of the tubes. If either bubble runs toward one end of the tube, bring it half-way back to the center of the tube by means of the capstan-headed screw at one end of the tube; then bring the bubbles to the centers by means of the leveling screws. Repeat the operation until both bubbles remain in the centers of the tubes in both positions of the instrument.

53. Second Adjustment.—The next operation is to make the line of sight perpendicular to the transverse axis of the telescope. Set up the transit near the center of the open space, as at *A* in Fig. 39. Then with the telescope normal, sight on some well-defined point *B*, a few hundred feet distant, using the point of intersection of the cross-wires. Both plates being clamped, plunge the telescope and set another point a few hundred feet from the instrument; a mark on a wall or fence is very convenient, or a wide stake can be driven on line and the point marked on it. If the line of sight is perpendicular to the transverse axis, this point will be at C in the prolongation of BA.

In order to ascertain whether this is the case, unclamp either plate, rotate the instrument in azimuth with the telescope still reversed, and sight to B again; then plunge the telescope back to normal. If the line of sight strikes the same point as before, no adjustment is necessary. If the line of sight does not pass through the first point, mark a second point on the same object

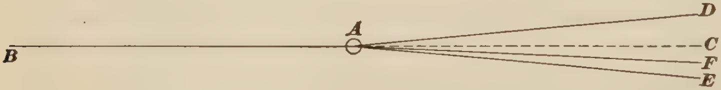


Fig. 39

so that the two points will be very nearly the same distance from the instrument. Suppose D is the point set after the first plunging and E is the second point. Then on the line between D and E, mark point F, making the distance EF from the second point E equal to one-quarter of DE. Move the cross-wires by means of the capstan-headed screws on the sides of the telescope, as described for the wye level, until the line of sight passes through F. For an erecting telescope, loosen the screw toward which the image of the wire should be moved and tighten the opposite screw; for an inverting telescope, loosen the screw away from which the image is to be moved and tighten the other. Thus, for the assumed conditions, loosen the left-hand screw for the erecting telescope or the right-hand screw for the inverting telescope. Repeat the operations until the line of sight after the second plunging passes through the point marked after the first sight.

54. Third Adjustment.—Now the transverse axis of the telescope is made perpendicular to the vertical axis of the instrument in order that, when the instrument is leveled, the transverse axis will be horizontal. This adjustment is made best by sighting, with the telescope normal, to some well-defined point on a high object such as a church spire. In this case, also, the point of intersection of the cross-wires must be used for all sights. Having clamped both plates, depress the objective and set a point on the ground in the line of sight and as far as possible from the instrument. Unclamp one plate, revolve the instrument on its vertical axis, and, with the telescope reversed, sight again to the high point. If the line of sight passes through the first point on the ground, no adjustment is required. If the line of sight does not pass through the first, set a second point near it. Suppose that *A*, Fig. 40, is the high point, *B* is the first point on the ground, and *C* is the second point on the ground. Then mark point *D* half-way between *B* and *C* and bring the line of sight to pass through the point *D* by rotating the telescope on the vertical axis. Next, point the telescope upwards; the line of sight will not pass through *A*, but will strike to one side, say at *E*. Finally, bring the vertical wire on *A* by adjusting the capstan-headed screw underneath one end of the transverse axis. If the line of sight is to move to the right, as from *E* to *A* in Fig. 40, lower the right end of the axis or raise the left end, keeping the telescope in its reversed position; if the line of sight is to move to the left, raise the right end or lower the left end. Repeat the operation until the line of sight in the second position of the telescope passes through the point on the ground determined by the first position. Since the vertical cross-wire is in adjustment, the transverse axis is adjusted in the same manner for both an erecting and an inverting telescope. In case the position of the transverse axis is altered, it is necessary to test again the adjustment of the vertical cross-wire.

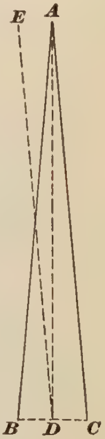


FIG. 40

55. Supplementary Test.—In order to make the cross-wires vertical and horizontal, and thus make it unnecessary to bring the point of intersection of the cross-wires on the point sighted at, the following test is convenient: Level the instrument carefully. Then bring one end of the vertical wire on some well-defined point, and revolve the telescope on its transverse axis so that the point appears to move along the wire. If the point does not remain on the wire throughout the motion, loosen two adjacent screws and rotate the ring holding the wires. Then, repeat the operations. This test should be performed after the transverse axis has been adjusted.

56. Fourth Adjustment.—The line of sight should be horizontal when the bubble of the telescope level is centered, in order that the transit may be accurate for leveling and for the measurement of vertical angles. The adjustment consists of two parts; first the horizontal cross-wire is adjusted, and then the telescope level is tested.

To adjust the horizontal cross-wire, set up at a point *A*, Fig. 41, and drive pegs at points *B* and *C* in a straight line from *A*, and respectively about 15 and 200 feet from *A* in the same direction. Clamp the transverse axis of the telescope so that the line of sight is about parallel to the ground, and read a leveling rod held on the pegs at *B* and *C*. Then plunge the telescope, revolve the instrument on its vertical axis, and clamp the transverse axis so that the horizontal cross-wire cuts the same reading on the rod at *B* as it did before. If the rod reading on the peg at *C* agrees with the reading previously obtained, no adjustment is necessary. If the two readings do not agree, bring the horizontal cross-wire to read the average of the two values by means of the capstan-headed screws on top and bottom of the telescope, keeping the telescope itself clamped in position. The method of moving the cross-wire is the same as described for a level.

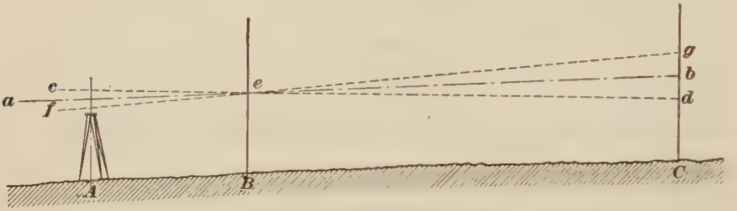


FIG. 41

For the sake of clearness, the conditions are shown greatly exaggerated in Fig. 41, and the telescope of the transit is omitted. Suppose that *a b* represents the theoretical line of sight for an instrument in adjustment; *c d*, the actual line of sight for the first position of the telescope, *e* being the point cut on the rod at *B*; and *f g*, the line of sight for the second position of the telescope, this line being made to pass through *e*. Then the point *b*, midway between *d* and *g*, is marked, and the horizontal wire is set on that point by means of the capstan-headed screws.

The adjustment is tested by taking new readings on the rods at *B* and *C* for the two positions of the telescope. If the readings at *C* do not agree yet, the wire must be moved again. The operations are repeated as often as necessary.

57. The telescope level is adjusted by the peg method described for the dumpy level. All rod readings are taken with the bubble of the telescope level in the center of its tube. Suppose the rod readings on the pegs from the first set-up are r_1 and r_2, and the rod readings from the second set-up are r_3 and r_4. Then if $r_3 - r_4 = r_1 - r_2$, the level is in adjustment. If $r_3 - r_4$ is not equal to $r_1 - r_2$, the corrected value of r_4 is determined as explained in *Leveling*. The telescope is then rotated on its transverse axis so that the horizontal cross-wire reads the corrected value of r_4; the telescope bubble will, therefore, move from the center of its tube. With the telescope clamped in that position, the bubble is brought to the center of the tube by means of the capstan-pattern nuts *l'*, Fig. 1. The operation is repeated until $r_3 - r_4 = r_1 - r_2$.

58. Fifth Adjustment.—The final adjustment is to make the vernier of the vertical limb read zero when the line of sight is horizontal. Set up the instrument and bring the bubble of the telescope level to the center of the tube. Then, if the vernier does not read zero, set it to zero by shifting it on the standards by whatever means are provided for the purpose.

OFFICE WORK IN ANGULAR SURVEYING

LATITUDES AND DEPARTURES

PRELIMINARY EXPLANATIONS

1. Introduction.—Office work in angular surveying consists of the solutions of three main problems: (1) balancing a survey, or correcting the field measurements to distribute unavoidable errors among the courses; (2) plotting the courses of a survey; and (3) computing the area of the land bounded by a closed traverse.

2. Reference Axes.—For the purposes of calculating and plotting, it is convenient to refer the directions of survey courses and the locations of survey points to two lines at right angles to each other; these lines are called reference axes. In Fig. 1, TT' and GG' are reference axes intersecting at the point O. Usually, the axis TT' represents either the true or the magnetic meridian through some point of the survey, and the axis GG' is an east-and-west line. If TT' is not a true or magnetic meridian, it is called an assumed meridian.

3. Latitudes and Departures of Courses.—Ordinarily the relative locations of two points are determined by the length and direction of the straight line between the points. For instance, in Fig. 1, the point Q may be located with respect to P by the length and direction of the line PQ. But the point Q may also be located from the point P by laying off from

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P a given distance PD parallel to TT' and from D a given distance DQ parallel to GG'. Thus, one point may be located from another by constructing a right triangle, the legs of which are parallel to a pair of mutually perpendicular reference axes.

In the case of the courses of a survey, the legs of all the right triangles are drawn parallel to the same pair of reference axes. For example, in Fig. 1, let TT' and GG' represent reference axes at right angles to each other, and PQ and $P'Q'$ two courses of a survey. Then, the lines PD, QE, $P'D'$, and $Q'E'$ are drawn parallel to the axis TT', and the lines QD, PE, $Q'D'$, and $P'E'$ are made parallel to GG'. In the triangle constructed for a given course, the length of the leg parallel to the meridian is called the *latitude* of the course and the length of the leg at right angles to the meridian is the *departure* of the course. In Fig. 1, PD or EQ is the latitude of PQ, and PE or DQ is the departure of PQ. Similarly, $P'D'$ or $E'Q'$ is the latitude of $P'Q'$, and $P'E'$ or $D'Q'$ is the departure of $P'Q'$.

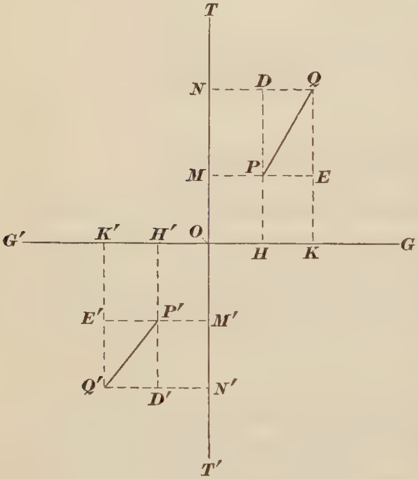

FIG. 1

The latitude and the departure of PQ are also represented by the distances MN and HK. Likewise, $M'N'$ and $H'K'$ are the latitude and the departure of $P'Q'$. In other words, the latitude of a course is the distance measured along the meridian, between lines drawn through the ends of the course at right angles to the meridian; the departure of a course is the distance measured along a line perpendicular to the meridian, between lines drawn through the ends of the course parallel to the meridian.

Latitude is sometimes called *latitude range* or *latitude difference*, and departure is sometimes referred to as *longitude range* or *longitude difference*.

4. Signs of Latitude and Departure.—In order to indicate the direction of a course with respect to a pair of reference axes, positive and negative quantities are used. If the bearing of a course with respect to the reference meridian is northeast or northwest, the latitude of the course is considered positive and is called a *northing*. If the bearing of a course is southeast or southwest, the latitude is negative and is known as a *southing*. For instance, in Fig. 1, the latitude of PQ is $+MN$ or simply MN, and the latitude of $P'Q'$ is $-M'N'$.

If the bearing of a course is northeast or southeast, the departure of the course is assumed to be positive, and is designated as an *easting*. If the bearing of a course is northwest or southwest, its departure is negative and is called a *westing*. Thus, in Fig. 1, the departure of PQ is HK and the departure of $P'Q'$ is $-H'K'$.

COMPUTATIONS INVOLVING LATITUDES AND DEPARTURES

5. General Formulas.—In Fig. 2, let AM represent the direction of a reference meridian; MB, a line at right angles to the meridian; AB, a given course; and G, the angle that the course makes with the meridian. Then, from trigonometry,

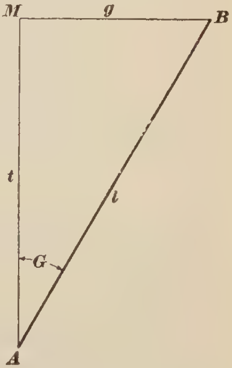

FIG. 2

$$AM = AB \cos G, \text{ and } MB = AB \sin G$$

But AM may be considered as the latitude of the course, and MB, its departure. Thus, the latitude and the departure of a course may be found by the following rules:

Rule I.—*The latitude of a course is equal to the product of the length of the course and the cosine of the angle between the meridian and the course.*

Rule II.—*The departure of a course is found by multiplying the length of the course by the sine of the angle that the course makes with the meridian.*

In general, let

l = length of course;
G = angle between meridian and course;
t = latitude of course;
g = departure of course.

Then,

$$t = l \cos G \qquad (1)$$

$$g = l \sin G \qquad (2)$$

When the latitude and the departure of a course are given, the angle between the meridian and the course, and the length of the course can be computed by the following formulas:

$$\tan G = \frac{g}{t} \qquad (3)$$

$$l = \frac{g}{\sin G} \qquad (4)$$

$$l = \frac{t}{\cos G} \qquad (5)$$

$$l = \sqrt{t^2 + g^2} \qquad (6)$$

These relations hold good for any direction of the course and also apply whether the angle G is given by a bearing or by an azimuth.

6. Given Length and Bearing.—In the case of bearings, the angle between the meridian and a course is always given as less than 90°. Hence, the functions of G are readily found. The numerical values of t and g may then be calculated by formulas **1** and **2**, Art. **5**. They should contain the same number of decimal places as the given value of l. The signs of t and g are determined from the quadrant of the bearing by the following rules:

Rule I.—*If the bearing is northeast, both the latitude and the departure are positive.*

Rule II.—*If the bearing is southeast, the latitude is negative and the departure is positive.*

Rule III.—*If the bearing is southwest, both the latitude and the departure are negative.*

Rule IV.—*If the bearing is northwest, the latitude is positive and the departure is negative.*

It is a common mistake to reverse the latitude and the departure. This will be avoided, or easily detected, if it is kept in mind that for angles less than 45°, the cosine is greater than the sine, and for angles greater than 45°, the sine is greater than the cosine. Therefore, for bearings less than 45°, the latitude of a course is greater than its departure; and for bearings greater than 45°, the departure is greater than the latitude. It is obvious that the latitude of a north-and-south line is equal to the length of the line, and its departure is zero; likewise, the departure of an east-and-west line is equal to the line itself, and the latitude is zero. It will be noticed that the latitude and the departure depend only on the direction of the meridian. They are, therefore, the same for any pair of axes that are parallel and perpendicular to the meridian.

EXAMPLE.—The length of a course is 896.7 feet and its bearing is N 39° 15′ W; find the latitude and the departure.

SOLUTION.—Here $l = 896.7$ and $G = 39° 15'$. Then, by formulas **1** and **2**, Art. **5**, the numerical values of t and g are

$$t = 896.7 \cos 39° 15'$$
$$g = 896.7 \sin 39° 15'$$

If natural functions are used,

$$t = 896.7 \times .77439 = 694.4 \text{ ft.}$$
$$g = 896.7 \times .63271 = 567.4 \text{ ft.}$$

Since the bearing of the course is northwest, rule IV is used for determining the signs of t and g; thus, t is positive and g is negative. The latitude is, therefore, 694.4 ft. and the departure is -567.4 ft. Ans.

Unless the numbers are comparatively easy to multiply, it is preferable to use logarithms. The logarithmic work is conveniently arranged by writing the logarithm of the length with the logarithmic sine of the bearing above it and the logarithmic cosine of the bearing below it; then the addition is performed upwards in one case and downwards in the other. In this problem the work will appear as follows:

$\log g = 2.75385 \qquad g = 567.4$ ft.

$\log \sin 39° \, 15' = 9.80120$
$\log 896.7 = 2.95265$
$\log \cos 39° \, 15' = 9.88896$

$\log t = 2.84161 \qquad t = 694.4$ ft.

The signs of t and g are found by applying rule IV. Thus, $t = 694.4$ ft. and $g = -567.4$ ft. Ans.

7. Given Length and Azimuth.—When the angle between the meridian and a course is given by the azimuth of the course, the angle may be greater than 90°. In this case, the latitude and the departure may be determined in two ways.

In one method, the azimuth is changed to a corresponding bearing, and the latitude and the departure are then computed from the length and the bearing of the course. The various conditions are shown in Fig. 3. Let NS represent the meridian; AB, a given course; Z, its azimuth; Z', its bearing; l, its length; t, its latitude; and g, its departure. In (*a*), the azimuth is less than 90°; thus, the bearing is northeast and the angle is also equal to Z. In (*b*), the azimuth is between 90° and 180°; for this case, the bearing is southeast and the angle Z' is $180° - Z$. In (*c*), the azimuth is between 180° and 270°; the bearing is southwest and the angle Z' is $Z - 180°$. In (*d*), the azimuth is between 270° and 360°; here, the bearing is northwest and the angle Z' is $360° - Z$. In each case the latitude

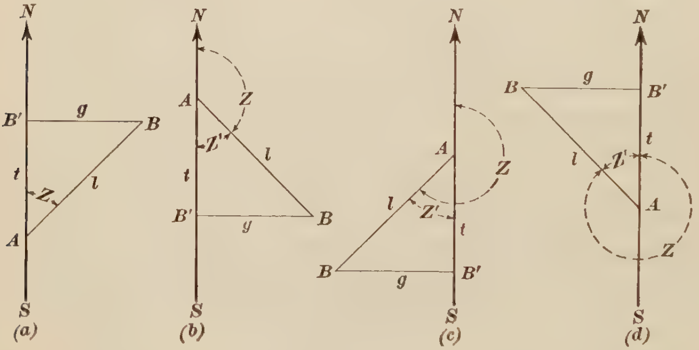

FIG. 3

and the departure with the proper signs may be determined as explained in the preceding article.

In the second method, the latitude and the departure are calculated directly from the length and the azimuth by applying the principles of trigonometry. In Fig. 4, the plane around the point *O* is divided into quadrants by the meridian *N S* and the east-and-west line *E W*. When azimuths are considered, the northeast quadrant *NOE* is the first quadrant; the southeast quadrant *EOS*, the second; the southwest quadrant *SOW*, the third; and the northwest quadrant *WON*, the fourth. Thus, the course *OA*, the azimuth Z_1 of which is less than 90°, is in the first quadrant; the course *OB*, having an azimuth Z_2 between 90° and 180°, is in the second quadrant; the azimuth Z_3 of *OC* is between 180° and 270°, and the course is in the third quadrant; and, since the azimuth Z_4 of *OD* is between 270° and 360°, the course is in the fourth quadrant. Although this method of numbering the quadrants differs from that adopted in trigonometry with regard both to the starting line and to the direction in which the numbers increase, the functions of angles have the same signs in both systems because distances to the right and distances upwards are considered as positive in each case. The values of *t* and *g* for the conditions shown in Fig. 3 will now be determined by trigonometry.

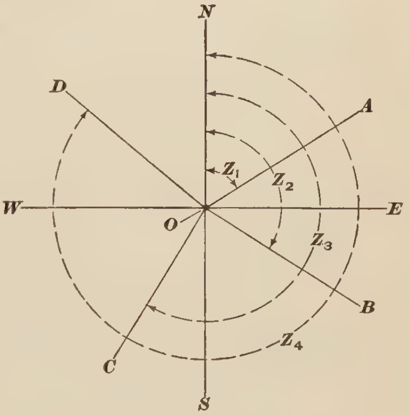

FIG. 4

In (*a*), the course *AB* is in the first quadrant and the angle *Z* is acute; sin *Z* and cos *Z* are both positive, and, therefore, the latitude and the departure of *A B* are also positive. In (*b*), *AB* is in the second quadrant, and the acute angle from which the functions of *Z* are derived is equal to $Z' = 180° - Z$. Then, $\sin Z = \sin (180° - Z') = \sin Z'$, and $\cos Z = \cos (180° - Z')$

$= -\cos Z'$; consequently, the departure is positive and the latitude is negative. In (*c*), *A B* is in the third quadrant and $Z' = Z - 180°$. In this case, $\sin Z = \sin (180° + Z') = -\sin Z'$, and $\cos Z = \cos (180° + Z') = -\cos Z'$; the latitude and the departure are both negative. In (*d*), *A B* is in the fourth quadrant and $Z' = 360° - Z$. Here, $\sin Z = \sin (360° - Z') = -\sin Z'$, and $\cos Z = \cos (360° - Z') = \cos Z'$; therefore, the latitude is positive and the departure is negative.

The calculations in both methods are practically the same but the reasoning is slightly different; it is unimportant which is used. As in the case of bearings, the number of decimal places in the values of t and g should be taken the same as in the given length.

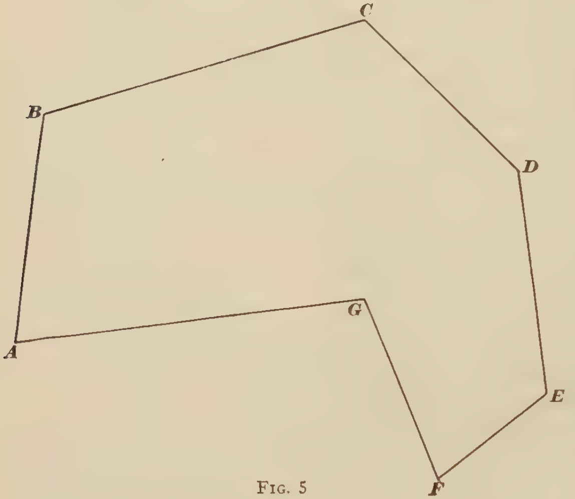

FIG. 5

EXAMPLE.—Find the latitude and the departure of a course having a length of 431.45 feet and an azimuth of 231° 9′.

SOLUTION BY BEARINGS.—Since the azimuth is between 180° and 270°, the bearing is southwest. Hence, the latitude and the departure are both negative. The angle G is equal to $231° 9' - 180° = 51° 9'$. Then, by formulas **1** and **2**, Art. **5**,

$$t = l \cos G = 431.45 \cos 51° 9' = 270.64 \text{ ft}$$
$$g = l \sin G = 431.45 \sin 51° 9' = 336.01 \text{ ft.}$$

Therefore, the latitude is −270.64 ft. and the departure is −336.01 ft. Ans.

It will be noticed that the departure is greater than the latitude because G is greater than 45°.

Solution by Trigonometry.—The course lies in the third quadrant and the acute angle from which the functions of the given angle are derived is equal to $231° 9' - 180° = 51° 9'$. Thus, $\sin 231° 9' = \sin (180° + 51° 9') = -\sin 51° 9'$ and $\cos 231° 9' = \cos (180° + 51° 9') = -\cos 51° 9'$. Then, by formulas **1** and **2**, Art. **5**,

$$t = l \cos 231° 9' = -431.45 \cos 51° 9' = -270.64 \text{ ft. Ans.}$$
$$g = l \sin 231° 9' = -431.45 \sin 51° 9' = -336.01 \text{ ft. Ans.}$$

8. Values for Survey Courses.—The latitudes and the departures of the courses of the transit survey shown in Fig. 5, with other necessary data, are recorded in Table I. In the first three columns are given the courses, the azimuths, and the lengths, which are copied from the field notes. In the fourth and fifth columns are the latitudes of the courses, and in the last two columns are the departures. In order to avoid confusion of signs and also for convenience in balancing the survey, the northings and the southings are placed in separate columns; similarly, the eastings and the westings are separated.

TABLE I

LATITUDES AND DEPARTURES

Course	Azimuth	Length	Latitude		Departure	
			N	S	E	W
AB	8° 51′	659.43	651.59		101.45	
BC	70° 42′	897.46	296.62		847.02	
CD	132° 45′	594.01		403.22	436.20	
DE	170° 31′	628.37		619.79	103.53	
EF	228° 36′	387.52		256.27		290.68
FG	335° 55′	547.55	499.89			223.44
GA	260° 13′	988.99		168.05		974.60

9. Given Latitude and Departure, To Find Length and Direction.—Frequently, the latitude and the departure of a course are known and it is required to determine the length and the bearing or azimuth of the course. If the values of t and g are easy to square or if the direction is not needed, the length may be found conveniently by formula **6**, Art. **5**, which is $l=\sqrt{t^2+g^2}$. In other cases, the angle G is found first by formula **3**, which is $\tan G=\frac{g}{t}$; then l is determined from either formula **4**, which is $l=\frac{g}{\sin G}$, or formula **5**, which is $l=\frac{t}{\cos G}$.

When formula **3** is used, the value of the acute angle G is first computed by temporarily disregarding the signs of t and g, that is, by considering that both are positive. Then the quadrant in which the course lies is determined by considering the given signs of t and g and applying the following rules:

Rule I.—*If t and g are both positive, the bearing is northeast, and the azimuth is equal to G.*

Rule II.—*If g is positive and t is negative, the bearing is southeast, and the azimuth is equal to 180°−G.*

Rule III.—*If t and g are both negative, the bearing is southwest, and the azimuth is equal to 180°+G.*

Rule IV.—*If t is positive and g is negative, the bearing is northwest, and the azimuth is equal to 360°−G.*

In formulas **4**, **5**, and **6**, the signs of g and t are unimportant since the length of the course is always considered positive. When an angle is small, a little difference in the value of the angle has practically no effect on the cosine but changes the sine considerably. Therefore, for values of G less than 20°, formula **5** is more accurate than formula **4**. On the other hand, for angles near 90°, a small difference in the value of the angle affects the cosine much more than the sine. Consequently, when G is greater than 70°, formula **4** is to be used rather than formula **5**. For values of G between 20° and 70°, formula **4** or formula **5** may be employed.

When the given latitude and departure are measured to hundredths of a foot, it is sufficiently accurate to take G

to the nearest minute. If the given distances are to thousandths of a foot, G should be taken to the nearest 10 seconds. In any case, the number of decimal places in the calculated length should be taken the same as in the given latitude and departure.

Example 1.—The latitude and the departure of a course are, respectively, −13.71 chains and 9.38 chains. What are the bearing and the length of the course?

Solution.—To find G, disregard the signs of g and t and use formula **3**, Art. **5**. In this case, g is 9.38 and t is taken as 13.71, the negative sign being neglected temporarily. Then,

$$\tan G = \frac{g}{t} = \frac{9.38}{13.71}, \text{ and } G = 34° \ 23'$$

By rule II, the bearing is southeast, because t is negative and g is positive. Hence, the bearing is S 34° 23′ E. Ans.

Finally, by formula **4**, Art. **5**,

$$l = \frac{g}{\sin G} = \frac{9.38}{\sin 34° \ 23'} = 16.61 \text{ ch. Ans.}$$

The logarithmic work is as follows:

$$\begin{array}{rlrl}
\log \ 9.38 &= 0.97220 & \log 9.38 &= 0.97220 \\
\log 13.71 &= 1.13704 & \log \sin 34° \ 23' &= 9.75184 \\
\hline
\log \tan G &= 9.83516 & \log l &= 1.22036 \\
G &= 34° \ 23' & l &= 16.61 \text{ ch.}
\end{array}$$

First, the difference between the logarithm of 9.38 and the logarithm of 13.71 is the logarithmic tangent of G, from which G is determined. At the same time that G is taken out of the table, its logarithmic sine is obtained. It is subtracted from the logarithm of 9.38 to get the logarithm of l. For the purpose of finding l, G is taken to the nearest minute; however, in a compass survey, where bearings are estimated to the nearest 5 minutes, the bearing of the line would be given as S 34° 25′ E.

Example 2.—The latitude of a course is +125.04 feet and the departure is −216.81 feet; find the azimuth and the length.

Solution.—By formula **3**, Art. **5**,

$$\tan G = \frac{216.81}{125.04}, \text{ and } G = 60° \ 2'$$

Since t is positive and g is negative, rule IV applies; hence, the azimuth is 360° − 60° 2′ = 299° 58′. Ans.

By formula **4**, Art. **5**,

$$l = \frac{g}{\sin G} = \frac{216.81}{\sin 60° \ 2'} = 250.26 \text{ ft. Ans.}$$

The logarithmic work is as follows:

$\log 216.81 = 2.33608$
$\log 125.04 = 2.09705$
$\log \tan G = 0.23903$
$G = 60° \ 2'$

$\log 216.81 = 2.33608$
$\log \sin 60° \ 2' = 9.93768$
$\log l = 2.39840$
$l = 250.26$ ft.

10. Total Departures and Total Latitudes.—In the foregoing articles, the lengths of the courses of a survey and their directions with respect to reference axes are given by means of departures and latitudes. Reference axes are also used for the purpose of locating the points of a survey. In Fig. 6, the point A may be located by the distance CA perpendicular to the meridian TT' and the distance DA perpendicular to the axis GG'. The perpendicular distance from the meridian to a point is called the *total departure* of the point, and the perpendicular distance from the east-and-west axis to the point is its *total latitude*. Thus, the total departure of B is the distance EB perpendicular to TT', and the total latitude of B is the distance FB perpendicular to GG'.

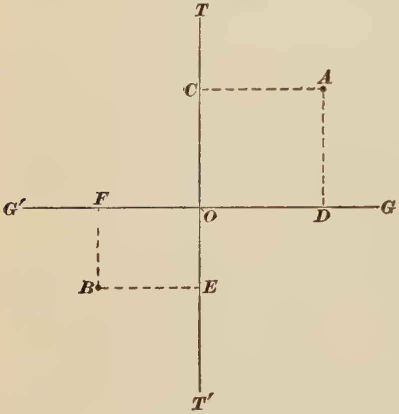

Fig. 6

The total latitude and the total departure of A are also equal to the distances OC and OD, respectively; and the values for B are OE and OF. That is, the total latitude of a point is the distance measured along the meridian from the intersection of the reference axes to the foot of the perpendicular from the point to the meridian. The total departure of a point is the distance measured along the east-and-west axis from the intersection of the axes to the foot of the perpendicular from the point to the axis.

The departure and latitude of a course are distances parallel to the reference axes; and the total departure and total latitude of a point are also distances parallel to the axes. It must be kept in mind, therefore, that the terms departure and latitude refer to a line, whereas total departure and total latitude refer to a point.

The total departure is said to be *east* or *west* according as the point is east or west of the meridian; the total latitude is *north* or *south* according as the point is north or south of the east-and-west axis. North total latitudes and east total departures are considered *positive*, and south total latitudes and west total departures are *negative*. In Fig. 6, the total latitude of A is OC or DA, and that of B is $-OE$ or $-FB$; the total departure of A is OD or CA, and the total departure of B is $-OF$ or $-EB$.

11. Determination of Total Latitudes and Total Departures.—For computing the total latitudes and the total departures of the points of a survey, the reference axes are chosen through some corner of the survey; both values for this point are, therefore, equal to zero. For example, in Fig. 7, the axes GG' and TT' pass through the corner A; the total latitude and the total departure of A are then equal to zero. The total latitude of the point B is EB, which is equal to the latitude of the course AB. The total latitude of the point C is HC, which is equal to $HK+KC$; but since HK is equal to EB or the total latitude of the point B, and KC is the latitude of the course BC, the total latitude of C is equal to the total latitude of B plus the latitude of the course BC. The total latitude of D is $-LD$, which may be considered as equal to $HC-CM$; thus, the total latitude of D is equal to the total latitude of C minus the latitude of CD. The total departure of

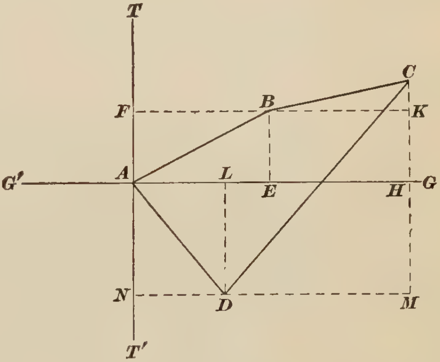

FIG. 7

a point can be found in a similar manner. The computations may, therefore, be made by applying the following rules:

Rule I.—*The total latitude of a point is equal to the total latitude of the preceding point plus or minus the latitude of the course between them. If the latitude of the course is a northing, it is added; and if it is a southing, its numerical value is subtracted.*

Rule II.—*The total departure of a point is equal to the total departure of another point plus or minus the departure of the course between them. If the departure of the course is an easting, it is added; and if it is a westing, it is subtracted.*

The following rules for addition and subtraction will be helpful:

Rule III.—*If both numbers have the same sign, add the numerical values and prefix the common sign.*

Rule IV.—*If the numbers have different signs, subtract the smaller numerical value from the larger and prefix the sign of the larger.*

For example, according to rule III, $+3+2=+5$ and $-4-6=-10$. By rule IV, $+5-3=+2$, $-3+6=+3$, and $+5-6=-1$.

Computations for determining total latitudes and total departures are shown in the following example. In all cases, it is advisable to make a diagram of the conditions.

Example.—The latitudes and the departures of the courses in Fig. 8 are given in the following table. Find the total latitudes and the total departures of B, C, D, and E with respect to axes GG' and TT' through A.

Course	Latitude	Departure
AB	+216	+153
BC	− 97	+271
CD	−244	− 59
DE	−100	−500

Solution.—The total latitude and the total departure of A are evidently equal to zero.

The total departure of B is, by rule II, equal to the total departure of A plus the departure of $A\,B$, or $0+153=153$. Ans.

The total departure of C is equal to the total departure of B plus the departure of $B\,C$, or $153+271=424$. Ans.

The total departure of D is equal to the total departure of C minus the departure of $C\,D$, or $424-59=365$. Ans.

The total departure of E is equal to the total departure of D minus the departure of $D\,E$, or $365-500$. The difference between the numerical values is $500-365=135$; since the larger number has a negative sign, the result is -135. Ans.

This negative sign indicates that E is west of the meridian through A.

The total latitudes are found in a similar manner by applying rule I.

For B, $0+216=216$. Ans.
For C, $216-97=119$. Ans.
For D, $119-244=-125$. Ans.
For E, $-125-100=-225$. Ans.

The negative signs for D and E indicate that these points are south of A.

12. In order to compute the total latitude and the total departure of E, Fig. 8, by the preceding method, it was necessary to determine the values for the points B, C, and D also. Often, the locations of these intermediate points are not needed. In such a case, it is more convenient to apply the following rules:

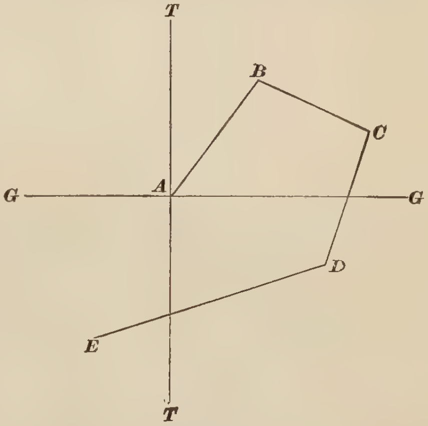

FIG. 8

Rule I.—*To find the total latitude of any point, take the sum of the northings and the sum of the southings of the courses between the starting point and the point in question; find the difference between these sums. If the northings exceed the southings, add this difference to the total latitude of the starting point; if the southings are greater than the northings, subtract the difference from the given total latitude.*

Rule II.—*To find the total departure of a point, take the difference between the sum of the eastings and the sum of the westings. Add the difference to, or subtract it from, the total departure of the starting point according as the eastings or the westings are greater.*

Example.—Using the values given in the example in Art. **11**, find the total latitude and the total departure of E with respect to axes through A.

Solution.—The only northing is 216, and the sum of the southings is $97+244+100=441$. The difference between these sums is $441-216=225$. Since the southings are greater than the northings, the difference is subtracted from the total latitude of A, which is zero. Hence, the total latitude of E is $0-225=-225$. Ans.

The sum of the eastings is $153+271=424$, and the sum of the westings is $59+500=559$. The difference between the sums, which is $559-424=135$, is subtracted because the westings are greater. Hence, the total departure of E is $0-135=-135$. Ans.

13. Latitude and Departure of Line From Total Latitudes and Total Departures of Its Ends.

—In many cases, the total latitudes and the total departures of two points are known, and it is required to find the latitude and the departure of the course between the points. For these calculations, the following rule may be applied:

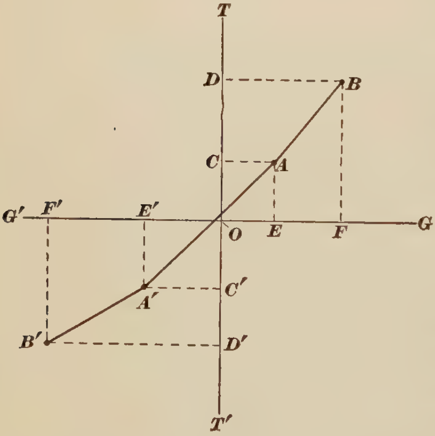

Fig. 9

Rule.—*The latitude, or the departure, of a course is equal to the difference between the total latitudes, or the difference between the total departures of the extremities of the course.*

For example, in Fig. 9, the latitude of the course AB is equal to the total latitude of B minus the total latitude of A; that is, $CD=OD-OC$. Similarly, the latitude of the course $A'B'$ is equal to the total latitude of B' minus the total latitude of A';

or $C'D' = OD' - OC'$. To find the latitude of a course one end of which is north of the reference axis and the other end of which is south of the axis, the negative sign of one of the values must be considered. Thus, to determine the latitude of AA' by the preceding rule, the total latitude of A is taken as $+OC$ and that of A' as $-OC'$. Their difference may then be indicated as $OC - (-OC')$. The rule for subtracting a negative number from a positive one is to change the sign of the subtrahend and add it to the minuend. Hence, $OC - (-OC') = OC + OC'$. Since $OC + OC'$ is equal to CC', which is the latitude of AA', the foregoing rule applies to this condition also.

The departure of a course may be found in a similar manner. For instance, the departure of AB is equal to $OF - OE$; the departure of $A'B'$ is equal to $OF' - OE'$; and the departure of AA' is $OE - (-OE') = OE + OE'$.

When the reference axes pass through one end of a course, the total latitude and the total departure of this end are zero and the latitude and the departure of the course are equal, respectively, to the total latitude and the total departure of the other end of the course.

14. In the preceding article, the numerical values of the latitude and the departure of a course were found, but the direction of the course was not considered. This is determined by inspection of the given values for the ends of the course or by means of a sketch showing the relative positions of the ends.

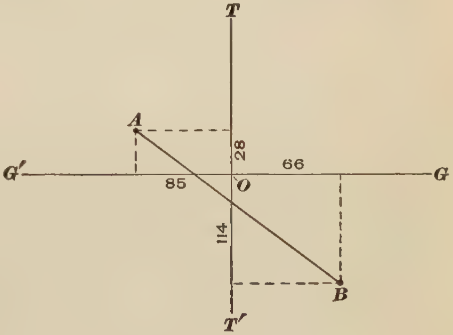

FIG. 10

EXAMPLE 1.—In Fig. 10, the total latitude and the total departure of the point A are, respectively, 28 and -85; and, for the point B, the total latitude is -114 and the total departure 66. Find the latitude and the departure of the course AB.

Solution.—The difference between the total latitudes of *A* and *B* is $28-(-114)=28+114=142$. Since *B* is evidently south of *A*, the latitude of *AB* is a southing and is, therefore, -142. Ans.

The difference between the total departures of *A* and *B* is $66-(-85)=66+85=151$. In this case *B* is east of *A* and the departure of *AB* is an easting. Hence, its value is 151. Ans.

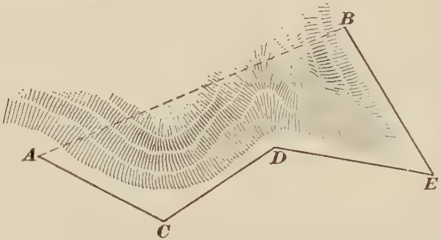

Fig. 11

Example 2.—In Fig. 11, *A* and *B* are two points on opposite sides of a hill, between which it is necessary to construct a tunnel for a railroad. In order to locate a straight line from *A* to *B*, the random survey *ACDEB* was run around the hill. From the field notes given in the following table, compute the length and the azimuth of *AB*.

Course	Length	Azimuth
AC	122.14	121° 45′
CD	118.60	58° 10′
DE	120.48	103° 20′
EB	136.91	334° 3′

Solution.—First, the azimuths are changed to bearings and the latitudes and the departures of the courses are computed. The results are tabulated as follows:

Course	Length	Bearing	Latitude		Departure	
			N	S	E	W
AC	122.14	S 58° 15′ E		64.27	103.86	
CD	118.60	N 58° 10′ E	62.55		100.76	
DE	120.48	S 76° 40′ E		27.79	117.23	
EB	136.91	N 25° 57′ W	123.10			59.91

The reference axes are taken through *A*. The sum of the northings is $62.55+123.10=185.65$; of the southings, $64.27+27.79=92.06$; of the eastings, $103.86+100.76+117.23=321.85$; and of the westings, 59.91. The difference between the northings and the southings is $185.65-92.06=93.59$; the total latitude of B is, therefore, $+93.59$. Similarly, the total departure of B is $321.85-59.91=261.94$. Since the total latitude and the total departure of A are equal to zero, the latitude of AB is equal to the total latitude of B and the departure of AB is the total departure of B. Thus, the latitude t of AB is $+93.59$ ft. and its departure g is $+261.94$ ft. Then, by formula **3**, Art. **5**,

$$\tan G=\frac{g}{t}=\frac{261.94}{93.59}, \text{ whence } G=70° \, 20'$$

Since both g and t are positive, the azimuth of AB is equal to G, or 70° 20′. Ans.

By formula **4**, Art. **5**,

$$l=\frac{g}{\sin G}=\frac{261.94}{\sin 70° \, 20'}=278.16 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. In the second and third columns of the following table are the lengths and the azimuths of the courses in the first column. (*a*) Compute the latitudes and the departures of the courses and compare the results with those given in the fourth and fifth columns. (*b*) Assuming the reference axes to pass through A, determine the total latitude and the total departure of each corner; the values are given in the last columns.

Course	Length	Azimuth	Latitude	Departure	Corner	Total Latitude	Total Departure
AB	638.85	35° 16′	+521.60	+368.86	*B*	+521.60	+ 368.86
BC	943.32	119° 43′	−467.62	+819.26	*C*	+ 53.98	+1,188.12
CD	719.98	171° 58′	−712.92	+100.62	*D*	−658.94	+1,288.74
DE	620.20	260° 38′	−100.94	−611.93	*E*	−759.88	+ 676.81
EF	649.07	352° 56′	+644.14	− 79.85	*F*	−115.74	+ 596.96

2. Determine (*a*) the length and (*b*) the azimuth of the line from A to F in example 1.

Ans. $\begin{cases} (a) \ 608.05 \\ (b) \ 100° \, 58' \end{cases}$

3. The latitude of a course is −317 feet and its departure is 425 feet. Determine (*a*) the length and (*b*) the bearing of the line to the nearest five minutes.

Ans. $\begin{cases} (a) \ 530 \text{ ft.} \\ (b) \ \text{S } 53° \, 15' \text{ E} \end{cases}$

4. Find the latitude t and the departure g of the line from C to E in example 1.

Ans. $\begin{cases} t=-813.86 \\ g=-511.31 \end{cases}$

BALANCING SURVEYS

ERROR OF CLOSURE

15. Total Error of Closure.—If the lengths and the bearings of all the courses of a survey were measured with absolute exactness, and those measurements were plotted with equal accuracy, the courses of a survey that starts and ends at the same point would form a closed polygon. However, no matter how carefully a survey is made, all sources of error cannot be entirely eliminated. Therefore, if the courses are plotted according to the field measurements, the end of the survey never coincides exactly with the starting point. In Fig. 12, $ABCDEA_1$ is the plot of a closed traverse made according to the recorded field notes. The starting point A and the end A_1 represent the same point on the ground but do not coincide on the map because of errors in the work (the distance between A and A_1 is shown exaggerated for clearness). The length of the line joining the beginning of the first course of a closed traverse and the end of the last course determined by the field measurements is called the total error of closure. In Fig. 12, the length of AA_1 is the total error of closure.

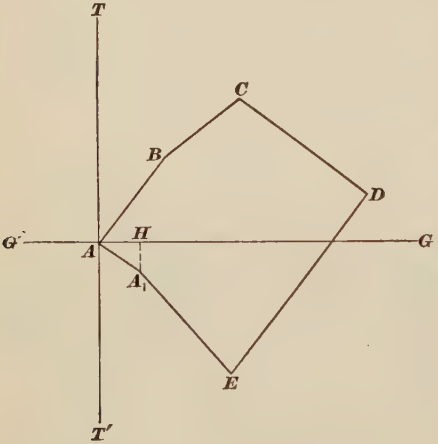

FIG. 12

16. Relative Error of Closure.—It is to be expected that in long surveys the total error of closure will be greater than in short surveys, if the same precautions are taken in both cases. Therefore, the total error of closure does not indicate the relative degrees of accuracy of two surveys of different lengths. As a basis for determining the degree of accuracy of a survey of any length, the error should be reduced to the amount in a unit length. This is found by dividing the total error of closure by the total length of the courses of the survey, and is called the relative error of closure. For example, if the sum of the lengths of the courses is 3,575 feet and the total error of closure is 2.5 feet, the relative error of closure is $\frac{2.5}{3,575}$, or $\frac{1}{1,430}$. The relative error of closure is commonly expressed as a fraction having a numerator equal to unity, or as a ratio such as 1 in 1,430, although it is sometimes given as a decimal, such as .0007. In an ordinary compass survey, the relative error of closure should be less than $\frac{1}{300}$, and in ordinary transit work it should be not greater than $\frac{1}{5,000}$. In some cases, the limit is $\frac{1}{10,000}$, and in very accurate work, the error should be less than $\frac{1}{20,000}$.

17. Formulas for Error of Closure.—In Fig. 12, let the line TT' represent the meridian through A, and GG' the east-and-west reference axis. Draw A_1H parallel to TT'. Then the total latitude of A_1, which is HA_1, is equal to the difference between the sum of the northings and the sum of the southings of the courses. The total departure of A_1, which is AH, is equal to the difference between the sum of the eastings and the sum of the westings.

From the right triangle AHA_1,

$$AA_1 = \sqrt{\overline{HA_1}^2 + \overline{AH}^2}$$

Let E = total error of closure;

S_t = difference between sums of northings and southings;

S_g = difference between sums of eastings and westings.

Then, $E=\sqrt{S_t^2+S_g^2}$ (1)

Also, let e = relative error of closure;

S_l = sum of lengths of courses.

Then, $e=\frac{E}{S_l}$ (2)

In Fig. 12, the reference axes were taken through *A* but formula **1** is true for any positions of the axes.

18. Determination of Error of Closure.—The calculations for finding the error of closure from the notes in Table I will be given for illustration. The sum of the northings is 651.59 + 296.62 + 499.89 = 1,448.10; of the southings, 403.22 + 619.79 + 256.27 + 168.05 = 1,447.33; of the eastings, 101.45 + 847.02 + 436.20 + 103.53 = 1,488.20; and of the westings, 290.68 + 223.44 + 974.60 = 1,488.72. The difference between the northings and the southings is S_t = 1,448.10 − 1,447.33 = 0.77 foot; and the difference between the eastings and the westings is S_g = 1,488.72 − 1,488.20 = 0.52 foot. The total error of closure is, therefore,

$$E=\sqrt{S_t^2+S_g^2}=\sqrt{.77^2+.52^2}=.93 \text{ foot}$$

The sum of the lengths of the courses is S_l = 4,703.33, say 4,703, and the relative error of closure is

$$e=\frac{E}{S_l}=\frac{.93}{4{,}703}=.000198=\frac{1}{5{,}060}$$

19. In computing the error of closure for a survey, only the courses that form boundaries of the field should be considered; lines locating objects, or courses that were taken for reference, should be omitted. On the other hand, the courses considered must form a closed figure in the field.

If the error of closure is very small, say less than 1 in 20,000, it may be disregarded and the measurements assumed to be correct. If the error of closure indicates that the work is sufficiently accurate but not very exact, the field measurements are corrected to eliminate the error and thus balance the survey. If the error of closure is greater than the allowable value for the kind of work, the measurements and the calculations should be checked to locate any mistakes.

EXAMPLE FOR PRACTICE

The field notes and the calculated latitudes and departures for a survey follow. Determine (*a*) the total error of closure and (*b*) the relative error of closure.

Course	Length	Azimuth	Latitude	Departure
AB	638.85	35° 16′	+521.60	+368.86
BC	943.32	119° 43′	−467.62	+819.26
CD	719.98	171° 58′	−712.92	+100.62
DE	620.20	260° 38′	−100.94	−611.93
EF	649.07	352° 56′	+644.14	− 79.85
FA	607.36	280° 57′	+115.37	−596.30

Ans. (*a*) .76
(*b*) .00018

METHODS OF BALANCING

20. Explanation.—In Fig. 12, *A* and A_1 represent the same point on the ground. Hence, in a closed traverse the sum of the northings of the courses should equal the sum of the southings, and the sum of the eastings should equal the sum of the westings. In order to make each of the values, S_l and S_g, in the formula $E=\sqrt{S_l^2+S_g^2}$ equal to zero, and thus reduce the error of closure to zero, it is necessary to revise the notes. The adjustment, which consists of applying corrections to the latitudes and the departures of the courses, is called *balancing the survey*, although it might be called *balancing the notes.*

21. Correcting Measurements.—There are two general methods of correcting the measurements of the courses. One is based on the fact that the chances of error are greater in making a difficult measurement than in making one under more favorable conditions. In the other method, it is assumed that all lines are measured under similar conditions. For ordinary surveys, the second method is usually preferred, as it is considered an unnecessary refinement to apply the first method.

22. Transit and Compass Methods.—In a transit survey, it is commonly assumed that the instrument work and the chaining are done with equal accuracy, but in a compass survey the angular error is necessarily larger than that permissible in transit work. Therefore, the method of balancing for a transit survey differs from that for a compass survey.

23. Angular Error.—In a transit survey, it is customary to test the accuracy of the instrument work before an attempt is made to balance the survey. The error in the measurement of the angles of a closed traverse is determined as follows:

If the method of direct angles is employed, the measured angles are the interior angles of a polygon, and, therefore, their sum should be equal to $180° \times (n-2)$, in which n is the number of courses in the traverse. Thus, the sum of the interior angles of a field having six sides should be $180° \times (6-2) = 720°$. If the sum of the measured angles is 720° 1′, the angular error is 1 minute or 60 seconds.

If the method of deflection angles is used, the difference between the sum of the deflections to the right and the sum of the deflections to the left should be equal to 360°. The amount by which the actual difference varies from 360° is the angular error.

If the method of azimuths is used, the azimuth of one course is found both at the beginning and at the end of the survey; the second value should agree with the first. Thus, in Fig. 7, the azimuth of AD may be determined from A when the survey is started and then the azimuth of DA may be found when the transit is set at D. The back azimuth of DA should be equal to the azimuth of AD; the difference between the two values is the angular error.

The allowable angular error is generally assumed to be proportional to the square root of the number of angles measured.

Let E_a = total allowable angular error;
N = number of angles measured;
e_a = allowable error in one angle.

Then,

$$E_a = e_a \sqrt{N}$$

The value of e_a may be taken as about one-half of the least reading of the transit vernier.

If the actual angular error is not much greater than the allowable error, the transit work may be assumed to be correct. Nevertheless, some of the angles are corrected so that no angular error remains. The angles where errors probably occurred can usually be determined by inspection of the survey. In other cases, the errors should be assumed to be in the angles adjacent to short lines, since any error in sighting or setting up causes a greater angular error when the sides of the angle are short than when they are long.

If the actual error exceeds the total allowable error by more than 1 or 2 minutes, the angles must be measured again to get more accurate results. It is, therefore, advisable to test the accuracy of the transit work while the party is yet in the field.

24. Balancing Transit Surveys.—After the angles of a transit survey have been adjusted, the latitudes and the departures of the courses are corrected. The corrections to be applied are determined by the following formulas:

$$c_t = t \times \frac{S_t}{R_t} \qquad (1)$$

$$c_g = g \times \frac{S_g}{R_g} \qquad (2)$$

in which t = latitude of course;

c_t = correction to latitude;

S_t = difference between sum of northings and sum of southings of courses of survey;

R_t = sum of numerical values of northings and southings;

g = departure of course;

c_g = correction to departure;

S_g = difference between sum of eastings and sum of westings;

R_g = sum of numerical values of eastings and westings.

If the northings exceed the southings, the corrections are to be subtracted from the north latitudes and added to the south latitudes; if the southings are greater, the corrections are added to the north latitudes and subtracted from the south latitudes. Similarly, if the eastings are greater than the westings, the corrections are subtracted from the east departures and added to the west departures; if the westings are greater, the corrections are added to the east departures and subtracted from the west departures.

The lengths and the azimuths of the courses to be used in the description of the survey are calculated from the corrected latitudes and departures. Occasionally, the azimuth of a course is changed again by the changes in the latitude and the departure but, usually, the direction of the line is not affected.

Let g' = corrected departure;
t' = corrected latitude;
G' = corrected bearing;
l' = corrected length.

Then,

$$\tan G' = \frac{g'}{t'} \qquad (3)$$

$$l' = \frac{g'}{\sin G'} \qquad (4)$$

$$l' = \frac{t'}{\cos G'} \qquad (5)$$

$$l' = \sqrt{t'^2 + g'^2} \qquad (6)$$

The azimuth can be found from the bearing.

25. In Table II are shown the calculations for balancing the transit survey for which the notes are given in Table I. It is assumed that the transit work is correct. The azimuths and the lengths of the courses, without parentheses, are copied from the field notes, and the latitudes and the departures, without parentheses, are calculated from them. The difference between the northings and the southings is $S_t = .77$ and their sum is $R_t = 2{,}895.43$. Similarly, $S_g = .52$ and $R_g = 2{,}976.92$. The relative error of closure was found to be $\frac{1}{5{,}060}$; therefore, the work is sufficiently accurate for balancing the survey.

Next, the corrections to the latitudes and departures are computed by means of formulas **1** and **2**, Art. **24.** Since the northings exceed the southings, the corrections will be subtracted from the north latitudes and added to the south latitudes; since the westings are greater than the eastings, the corrections will be subtracted from the west departures and added to the east departures. The corrections are tabulated as follows, $\frac{S_t}{R_t}$ being equal to $\frac{.77}{2{,}895}$, or .00027, and $\frac{S_g}{R_g}$ being equal to $\frac{52}{2{,}977}$, or .00017. Since the method of balancing is approximate, the values of t and g may be taken to the nearest 10 feet. A slide rule will prove convenient for these computations.

COURSE	c_t	c_g
AB	$650 \times .00027 = -.17$	$100 \times .00017 = +.02$
BC	$300 \times .00027 = -.08$	$850 \times .00017 = +.15$
CD	$400 \times .00027 = +.11$	$440 \times .00017 = +.07$
DE	$620 \times .00027 = +.17$	$100 \times .00017 = +.02$
EF	$260 \times .00027 = +.07$	$290 \times .00017 = -.05$
FG	$500 \times .00027 = -.13$	$220 \times .00017 = -.04$
GA	$170 \times .00027 = +.04$	$970 \times .00017 = -.17$

In order to make the error of closure zero, the sum of the numerical values of the corrections to the latitudes must equal S_t and the sum of the corrections to the departures must equal S_g. For this reason, the computed values are adjusted slightly; thus, c_t for *AB* is called .17 although it is a little nearer .18, c_t for *GA* is taken as .04, c_g for *BC* is made .15, and c_g for *GA* is called −.17. The corrected latitudes and departures are written in parentheses above the calculated values. In practice, the calculated values are crossed out, but not erased, and no parentheses are used around the corrected values.

The corrected lengths and azimuths are then determined from the corrected latitudes and departures. For instance, the corrected angle G' for *BC* is found by the relation, $\tan G' = \frac{g'}{t'} = \frac{847.17}{296.54}$, whence $G' = 70° \, 42'$. Since g' and t' are both positive, the azimuth of BC is 70° 42′. The corrected length of BC is

$$l'=\frac{g'}{\sin G'}=\frac{847.17}{\sin 70^\circ\ 42'}=897.62 \text{ feet.}$$

It will be noticed that in some cases the corrected lengths are greater than the corresponding measured distances. Since lengths are likely to be measured too long rather than too short, unless the tape is too long, some engineers prefer to make the adjustments entirely by subtracting from the greater values so that no lengths will be increased.

TABLE II

COMPUTATIONS FOR BALANCING TRANSIT SURVEY

Course	Azimuth	Length	Latitude		Departure	
			N	S	E	W
		(659.27)	(651.42)		(101.47)	
AB	8° 51′	659.43	651.59		101.45	
		(897.62)	(296.54)		(847.17)	
BC	70° 42′	897.46	296.62		847.02	
		(594.11)		(403.33)	(436.27)	
CD	132° 45′	594.01		403.22	436.20	
		(628.55)		(619.96)	(103.55)	
DE	170° 31′	628.37		619.79	103.53	
	(228° 35′)	(387.56)		(256.34)		(290.63)
EF	228° 36′	387.52		256.27		290.68
		(547.41)	(499.76)			(223.40)
FG	335° 55′	547.55	499.89			223.44
		(988.80)		(168.09)		(974.43)
GA	260° 13′	988.99		168.05		974.60
		4,703.33	1,448.10	1,447.33	1,488.20	1,488.72
			1,447.33	1,448.10	1,488.72	1,488.20
			.77	2,895.43	2,976.92	.52

26. Balancing Compass Surveys.—The method of balancing a compass survey differs from that employed for a transit survey only in the formulas for determining the corrections which are applied to the latitudes and the departures. For a compass survey, the following formulas are used:

Let l = length of course;
c_t = correction to latitude;
c_g = correction to departure;
S_l = sum of lengths of courses;
S_t = difference between sum of northings and sum of southings;
S_g = difference between sum of eastings and sum of westings

Then,

$$c_t = l \times \frac{S_t}{S_l} \qquad (1)$$

$$c_g = l \times \frac{S_g}{S_l} \qquad (2)$$

Since the bearings in a compass survey are accurate only to the nearest 5 minutes, they are seldom affected by the adjustment of the latitudes and the departures. The lengths of the courses, however, must usually be changed to agree with the corrected latitudes and departures. The new bearings may be determined by applying formula **3**, Art. **24.** The new lengths may be found by formula **4**, **5**, or **6**, Art. **24**, but they can be determined more easily by applying corrections to the measured lengths. The corrections may be calculated by means of the formula

$$c_l = t \times \frac{S_t}{S_l} + g \times \frac{S_g}{S_l} \qquad (3)$$

in which c_l = correction to length of course;
t = latitude of course;
g = departure of course;

S_l, S_t, and S_g have same meanings as in formulas **1** and **2.**

It will be noticed that the signs of all the terms in the formula are plus, but that is the case only when the corrections are added to both the latitude and the departure of the course in question. The method of applying the formula under other conditions is shown in the following calculations.

27. In the first three columns of Table III are given the courses, the bearings, and the lengths (without parentheses), which are copied from the field notes for a compass survey. In the other columns are the calculated latitudes and departures (also without parentheses). It is required to balance the survey. The corrections to the latitudes and the departures are found by formulas **1** and **2**, Art. **26**. Here, the sum of the northings is 545.1, the sum of the southings is 547.8, and S_t is $547.8-545.1=2.7$. The sum of the eastings is 441.1, the sum of the westings is 439.2, and S_g is $441.1-439.2=1.9$. The sum of the lengths, S_l, is 1,614.3. Then, $\frac{S_t}{S_l}=\frac{2.7}{1,614}=.0017$ and $\frac{S_g}{S_l}=\frac{1.9}{1,614}=.0012$. The computations may be arranged as follows:

COURSE	c_t	c_g
AB	$300\times.0017=.5$	$300\times.0012=-.4$
BC	$220\times.0017=.4$	$220\times.0012=-.3$
CD	$190\times.0017=-.3$	$190\times.0012=-.2$
DE	$270\times.0017=-.5$	$270\times.0012=.3$
EF	$370\times.0017=-.6$	$370\times.0012=.4$
FA	$250\times.0017=.4$	$250\times.0012=.3$

Since the southings exceed the northings, the corrections are added to the north latitudes and subtracted from the south latitudes, as indicated by the signs. Similarly, since the eastings exceed the westings, the corrections are added to the west departures and subtracted from the east departures. Here the corrected values are written in parentheses above the computed values; but in practice the computed values would be crossed out and the corrected values would not be enclosed in parentheses.

The corrections to the original lengths are found by using formula **3**, Art. **26**. For AB, $c_l=260\times.0017-150\times.0012=.3$. Since the correction was added to the latitude of AB, the term $t\times\frac{S_t}{S_l}$ is plus; but since the correction was subtracted from the departure of AB, the term $g\times\frac{S_g}{S_l}$ is minus. The value of c_l is plus and is added to the original length of AB

For BC, $c_l = 37 \times .0017 - 220 \times .0012 = -.2$. The value of c_l is minus and is subtracted from the original length of BC.

For CD, $c_l = -180 \times .0017 - 72 \times .0012 = -.4$. The corrections were subtracted from both the latitude and the departure of CD; hence, both $t \times \frac{S_t}{S_l}$ and $g \times \frac{S_g}{S_l}$ are minus.

For DE, $c_l = -270 \times .0017 + 58 \times .0012 = -.4$.
For EF, $c_l = -100 \times .0017 + 360 \times .0012 = .3$.
For FA, $c_l = 250 \times .0017 + 20 \times .0012 = .4$.

TABLE III

COMPUTATIONS FOR BALANCING COMPASS SURVEY

Course	Bearing	Length	Latitude		Departure	
			N	S	E	W
	(N 30° 15′ E)	(300.3)	(259.4)		(151.1)	
AB	N 30° 20′ E	300.0	258.9		151.5	
	(N 80° 15′ E)	(220.4)	(37.4)		(217.2)	
BC	N 80° 20′ E	220.6	37.0		217.5	
		(193.4)		(179.6)	(71.9)	
CD	S 21° 50′ E	193.8		179.9	72.1	
	(S 12° 15′ W)	(274.6)		(268.3)		(58.3)
DE	S 12° 10′ W	275.0		268.8		58.0
	(S 74° 45′ W)	(375.2)		(98.5)		(362.0)
EF	S 74° 40′ W	374.9		99.1		361.6
		(250.4)	(249.6)			(19.9)
FA	N 4° 30′ W	250.0	249.2			19.6
		1,614.3	545.1	547.8	441.1	439.2
				545.1	439.2	
				2.7	1.9	

Approximate values of t and g are close enough for these calculations. The corrected lengths and bearings are written here in parentheses over the original values in the table.

28. Supplying Omissions.—It is sometimes impossible to determine by direct measurement the lengths and the directions of all the sides of a closed field; moreover, omissions sometimes occur in the notes from accident. In a closed survey in which there are two omissions, the missing parts can be supplied from the other parts by calculation. Suppose, for example, that very thick woods make it impracticable to measure the length and the azimuth of the side CD in the survey for which the notes are given in Table I. Then D would be taken as the starting point of the survey and the notes would be arranged in the following form:

Course	Azimuth	Length	Latitude		Departure	
			N	S	E	W
DE	170° 31′	628.37		619.79	103.53	
EF	228° 36′	387.52		256.27		290.68
FG	335° 55′	547.55	499.89			223.44
GA	260° 13′	988.99		168.05		974.60
AB	8° 51′	659.43	651.59		101.45	
BC	70° 42′	897.46	296.62		847.02	

The length and the azimuth of CD may be found from the total latitude and the total departure of C with respect to axes through D. The sum of the northings is 499.89+651.59+296.62=1,448.10; of the southings, 619.79+256.27+168.05=1,044.11; of the eastings, 103.53+101.45+847.02=1,052.00; and of the westings, 290.68+223.44+974.60=1,488.72. The total latitude of C with respect to D is 1,448.10−1,044.11=403.99, and its total departure is 1,488.72−1,052.00=436.72. Since C is north of D, the latitude of CD is a southing; and since C is west of D, the departure of CD is an easting. Then, by formula **3**, Art. **5**,

$$\tan G = \frac{g}{t} = \frac{436.72}{403.99}$$

Whence

$$G = 47° \ 14'$$

Since g is positive and t is negative, the azimuth of CD is equal to $180° - G = 180° - 47° \ 14' = 132° \ 46'$. The length of CD is, by formula **4**, Art. **5**,

$$l = \frac{g}{\sin G} = \frac{436.72}{\sin 47° \ 14'} = 594.89 \text{ feet}$$

The case shown in Fig. 11 is another in which it is necessary to supply omissions. The line *AB* may be considered as a side of the closed survey *ACDEBA*; and its length and azimuth may be found from the measurements of the other sides as previously shown.

The surveyor should make every measurement that is practicable so as to avoid the necessity of supplying omissions by computation; for, when values are determined in this manner, it must be assumed that the remaining field notes are exactly correct. Consequently, there are no means of balancing the survey, and all errors are thrown in the part or parts supplied.

EXAMPLES FOR PRACTICE

1. In the following table are given the latitudes and the departures of the courses of a closed transit traverse as computed from the field measurements. (*a*) Calculate the total error of closure. (*b*) Verify the corrected latitudes and departures and the corrected azimuths and lengths.

Ans. (*a*) .70

Course	Calculated Latitude	Calculated Departure	Corrected Latitude	Corrected Departure	Corrected Azimuth	Corrected Length
AB	+521.60	+368.86	+521.51	+368.94	35° 17′	638.87
BC	−467.62	+819.26	−467.70	+819.43	119° 43′	943.54
CD	−712.92	+100.62	−713.04	+100.64	171° 58′	720.10
DE	−100.94	−611.93	−100.96	−611.80	260° 38′	620.07
EF	+644.14	− 79.85	+644.03	− 79.84	352° 56′	648.97
FA	+116.18	−597.50	+116.16	−597.37	281° 00′	608.56

2. The following table contains the lengths and the bearings of the courses of a compass survey, the values of the latitudes and the departures as computed from the original notes, and the corrected values of the lati-

Course	Bearing	Length Chains	l	g	Corrected l	Corrected g	Corrected Length
AB	N 41° 30′ E	10.47	+ 7.84	+ 6.94	+ 7.85	+ 6.96	10.49
BC	N 75° 15′ E	11.86	+ 3.02	+11.47	+ 3.03	+11.49	11.89
CD	S 20° 45′ E	11.64	−10.88	+ 4.12	−10.87	+ 4.14	11.64
DE	S 57° 45′ W	15.78	− 8.42	−13.35	− 8.40	−13.32	15.74
EA	N 48° 00′ W	12.52	+ 8.38	− 9.30	+ 8.39	− 9.27	12.51

tudes, the departures, and the lengths. (*a*) Verify the original values of the latitudes and the departures, and the corrected values of the latitudes, the departures, and the lengths. (*b*) Calculate the relative error of closure.

Ans. (*b*) .002

3. From the following notes, determine (*a*) the length and (*b*) the bearing of the line *PQ*.

Ans. { (*a*) 823.0 ft.
(*b*) N 74° 5′ E

Course	Length	Bearing
PA	300.0	N 45° E
AB	200.0	S 70° E
BC	250.0	due east
CQ	163.4	N 60° E

PLOTTING SURVEYS

INTRODUCTION

29. Definition and Methods.—A plot of a survey shows on paper the relative locations of points and objects on the ground. The plot is constructed by drawing lines to represent the courses whose lengths and directions have been determined by measurement and have been recorded in the notes. Although there are many methods of plotting, the points of a survey are commonly located either by latitudes and departures, or by lengths and bearings or azimuths. In a compass survey, the method of lengths and bearings is always used. In a transit survey, the method of latitudes and departures is generally employed for locating important points and the method of lengths and azimuths is used for filling in details.

30. Instruments.—The instruments commonly used in plotting are a drawing board, a sharp hard pencil, a scale, a protractor, two triangles, and a **T** square, all of which have been described in connection with *Geometrical Drawing*. Special forms of the **T** square and the protractor will be described here. A long straightedge, preferably of steel, is sometimes convenient. A *parallel ruler* is often used instead of a **T** square.

31. T Square.—In one style of **T** square, the blade is fixed at right angles to the head, and, therefore, only parallel lines at right angles to the edge of the board can be drawn. For drawing parallel lines not perpendicular to the edge of the board, the type of **T** square shown in Fig. 13 is convenient.

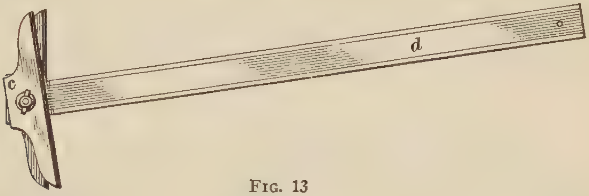

Fig. 13

The head is divided into two separate parts, which are held together by means of a bolt and a nut.

When the nut is loosened, one part can be rotated to the right or to the left with respect to the other section; the two pieces are then held in position by tightening the nut. The upper part of the head, *c*, is rigidly fixed at right angles to the blade *d*, and, when the lower part of the head is kept against the edge of the drawing board, the blade moves parallel to itself as the instrument slides.

32. Parallel Ruler.—The type of parallel ruler generally used is shown in Fig. 14. It consists of a metal straightedge which is mounted on milled rollers of equal diameter attached to a common shaft. If the ruler is carefully rolled over a surface without lifting either roller, the straightedge will move parallel to itself. Hence, if one edge of the ruler is placed along a given line, a parallel line through any point can be obtained by rolling the instrument until an edge passes through the desired point, and then drawing a line along that edge. Owing to the greater cost of parallel rulers and to the fact that

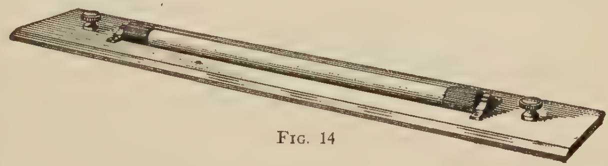

Fig. 14

more care must be exercised to give as good results, parallel rulers are not used so much as **T** squares.

33. Protractors.—The simple semicircular protractor has been described in a preceding Section. The type shown in Fig. 15 is an ordinary metal protractor with a rotating arm, or blade, *a*, which extends from the center of the graduated circle. In order that the center may be set accurately at the desired point on the paper, it is marked by cross lines on a transparent celluloid disc in the joint *b* by which the blade is attached. For laying off angles in transit work, the arm of the protractor is supplied with a vernier *c*, by means of which settings can be made to the nearest minute. In Fig. 15 the half-degree graduations are omitted to make the illustration clearer.

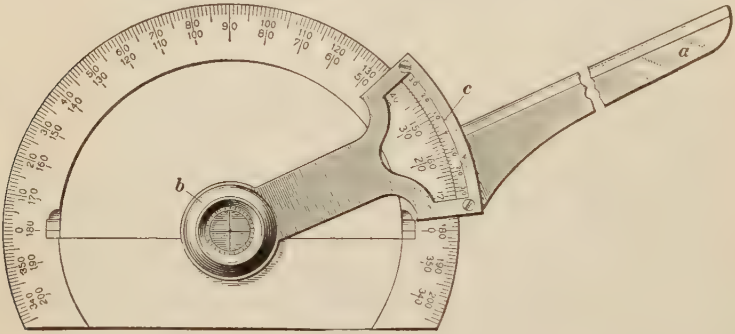

Fig. 15

In using protractors such as those already described, it is necessary to move the protractor for every new point where angles are measured; thus, much time is required and there is likely to be some error in placing the protractor in the proper position at each point. The form of protractor often used in plotting lines by bearings or azimuths is the paper protractor, which can be tacked to the drawing board and kept in one position for plotting many lines. The *paper protractor* is a full circle printed from accurately engraved plates on a sheet of drawing paper, tracing paper, or bristol board. The circle is from 8 to 14 inches in diameter and is graduated to half or quarter degrees; the numbers of the graduations are not printed on the sheets but are written on by the draftsman in the manner most convenient for his purpose. For bearings, the graduations are numbered in quadrants from 0° to 90° in each direction, similar to those on the needle circle of a compass, so that bearings can be readily found. For azimuths the numbers increase from 0 to 360 clockwise, as on the horizontal limb of a transit. The center of the graduated circle is marked by a cross.

PLOTTING BY LENGTHS AND BEARINGS

34. Preliminaries to Plotting.—Before plotting is started, it is necessary to select the scale, the direction of the meridian, and the location of the first point of the survey, so that the plot will not run off the paper. It is a slight advantage in plotting with a **T** square to have the meridian parallel to the edge of the paper, but there is no objection to having the meridian in any other direction. However, north should always be toward the top of the paper. The best scale and the location of the first point can be determined from the latitudes and departures, or by careful examination of the field notes and a rough sketch of the survey.

In ordinary work the scale used is so small that the corrections to the measurements that are required to balance the survey do not affect the plotting. Hence, the plot can be drawn from the measurements in the notes, and it is not necessary to balance the survey before plotting. A considerable error of closure in the plot indicates a mistake either in the field measurements or in the plotting, and should be investigated.

35. Plotting Bearings With Paper Protractor.—When a paper protractor is used, a long line representing the direction of the meridian is drawn in a convenient position on the paper, and at any place along this line the protractor is tacked to the board so that the zero points coincide with the line. In Fig. 16, the meridian is indicated by the line with a half arrowhead. The starting point *A* of the survey is marked by a pinhole with a small circle around it. For making this hole, a needle inserted in a small piece of wood as a handle is very convenient. If a parallel ruler is used, one edge of it is set on the protractor to read the bearing of the first course. In Fig. 16, the bearing of *AB* is N 40° E and the ruler is, therefore, placed in the position *aa*. Then, it is rolled to the position *a′a′*, where the edge passes through *A*, and an indefinite line is drawn. On this line, the length of the course *AB* is laid off to the proper scale, and the point *B* is located and marked with a pinhole and a circle. The parallel ruler does not have to extend across both parts of the graduated circle, but may be set to pass through the center mark and the proper graduation in one quadrant. Next the ruler is set on the protractor in the position *bb* to read the bearing of *BC*, which is N 70° E, and is rolled to the position *b′b′* so that the edge passes through *B*. An indefinite line is drawn along the edge and, on this line, the length of *BC* is laid off to scale to locate *C*. Point *D* is located in a similar manner. The ruler is first placed in the position *c c* to read a bearing of S 30° E, and is then rolled to the position *c′c′* to pass through *C*; the distance *CD* is laid off in the direction of the ruler.

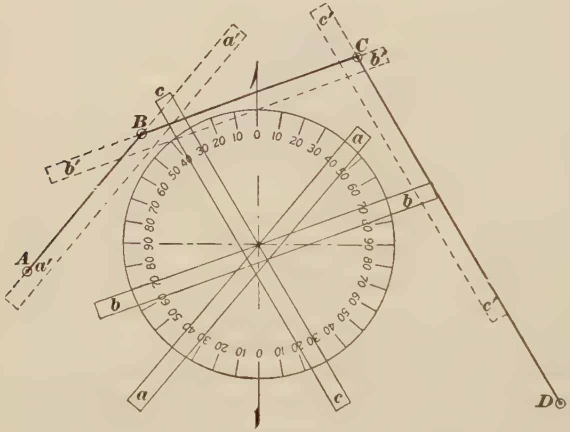

FIG. 16

In case it becomes impossible to transfer a direction from the protractor to the required position of the line by means of the parallel ruler, the protractor can be shifted to any other position on the paper. When the protractor is to be moved, a line is drawn parallel to the meridian, and the zero points of the protractor are set on this line.

If no parallel ruler is at hand, the directions of the lines can be transferred from the protractor to the required points by means of an adjustable **T** square, or better, by means of two triangles used for drawing parallel lines as explained elsewhere. If the triangles have long edges, as 12 or 14 inches, the direction of a line can be transferred with great rapidity after a little practice; of course, smaller triangles can also be used but they require more shifting than the larger ones. Azimuths can be laid off with a protractor in the same way as bearings

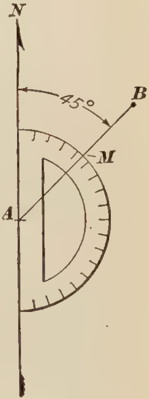

FIG. 17

36. Plotting With Other Protractors. When an angle is to be laid off with a metal or celluloid protractor, the center of the protractor must be set over the vertex of the angle. Therefore, the protractor must be placed at each point over which the transit was set in the field.

The direction of the meridian is marked by a long line at some convenient position on the paper and the starting point of the survey is located. If a semicircular protractor without a rotating arm is used, a meridian is drawn through the starting point and the protractor is set with its center at that point and its zero graduations on the meridian. If the bearing of the line is northeast or southeast, the protractor is placed to the right of the meridian, and if the bearing is northwest or southwest, the protractor is placed to the left of the meridian. In Fig. 17, *A* is the starting point and *N S* is the meridian. Then a point is marked at the edge of the protractor opposite the graduation corresponding to the bearing of the first course. In Fig. 17, the bearing of *A B* is taken as N 45° E and the point *M* is marked at the edge of the protractor opposite the

45° graduation to the east from the north. The protractor is moved out of the way and an indefinite line is drawn through A and M. The length of AB is laid off to scale on this line and the point B of the survey is located and marked. A meridian is then drawn through the point B, and the operations are repeated to locate the next point of the survey.

If a semicircular protractor with an arm is used, the bearings can be laid off in less time and with less trouble. There is no necessity of drawing the meridian through each station since the straight part of the protractor can be placed against the edge of the **T** square, parallel ruler, or triangle, which is parallel to the meridian. By shifting the **T** square and by moving the protractor along the edge of the **T** square, the center of the protractor can be brought over the desired point. Then the blade of the protractor is set at the required bearing, and an indefinite line is drawn along the edge of the blade. The length of the course is laid off on this line to locate the next point of the survey. When only one bearing is laid off from a point, it is probably better to set the blade of the protractor at the desired bearing first, and then to shift the protractor to the proper position by bringing any part of the blade to the station from which the bearing is measured.

PLOTTING BY LATITUDES AND DEPARTURES

37. Introduction.—The first step in plotting by latitudes and departures is to select the scale of the map, the direction of the meridian, and the location of the starting point on the paper. Then the reference axes, to which the total latitudes and the total departures are referred, are drawn parallel and perpendicular to the meridian. In order to facilitate plotting, the paper is divided into a number of squares by drawing lines parallel to the axes and exactly 10 inches apart in both directions. These lines should be drawn very carefully, and the lengths of the diagonals of each square should be measured as a a check to see that they are 14.14 inches long. Then, points may be located by measuring from the nearest line rather than from a reference axis. For instance, if the scale of the map is

1 inch to 100 feet, the distance between the lines represents 10×100, or 1,000, feet. A point whose total departure is 2,300 feet may, therefore, be located by measuring 300 feet from the 2,000-foot line instead of 2,300 feet from the meridian.

38. Locating Points.—The values in Table IV are the total latitudes and the total departures of the points of the traverse for which the notes are given in Table I. The reference axes are assumed through *A*, and the corrected latitudes and departures taken from Table II are used.

The plot of the points by total latitudes and total departures is shown in Fig. 18. Station *B* is found by measuring 651.42 feet from *A* along the meridian to *B′* and then 101.47 feet from *B′* at right angles to the meridian. Point *C* is located by measuring 947.96 feet from *A* to *C′* and then 948.64 feet from *C′* at right angles to the meridian or parallel to the east-and-west axis. Point *C′* should not be located by measuring 296.54 feet from *B′*, because any mistake in *B′* would also be carried to *C′*. All distances should be measured from one of the carefully drawn lines that divide the paper into 10-inch squares. In locating a point, it is always convenient to measure the longer distance along one of these lines and the shorter distance from that line. Thus, point *E* can be located best by measuring along the east-and-west axis 488.46 feet from the auxiliary line 1,000 feet east of the meridian to *E″*, and then from *E″*

TABLE IV

TOTAL LATITUDES AND DEPARTURES

Station	Total Latitude	Total Departure
A	0	0
B	+651.42	+ 101.47
C	+947.96	+ 948.64
D	+544.63	+1,384.91
E	− 75.33	+1,488.46
F	−331.67	+1,197.83
G	+168.09	+ 974.43

75.33 feet at right angles to the east-and-west axis. The work can be checked by measuring the lengths of the courses.

After the important points have been located by latitudes and departures, the details can be filled in by means of a protractor. The complete plot of the survey from the notes in Fig. 19 is shown in Fig. 20. The directions of lines on a map are generally given by bearings, even though azimuths were measured in the field. The numbers of the points locating objects would not be shown on the finished map but are given here for reference. In some surveys, the lengths and the bearings of the courses would not be shown either. When they are written on the map, however, the corrected values, and not the measured ones, should be used. Thus, the corrected values taken from Table II are used in Fig. 20.

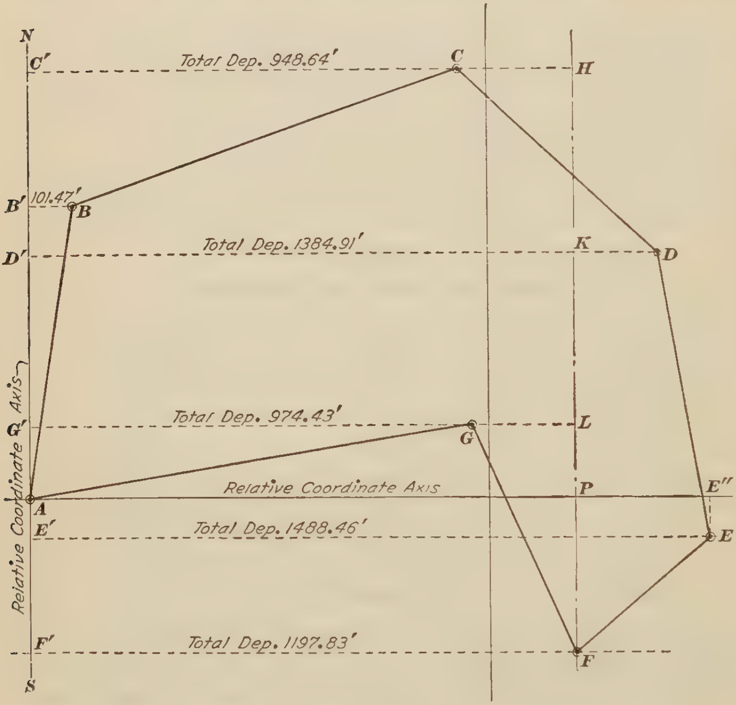

FIG. 18

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Survey of Smith-Jones Tract
Declination 3°15′ E   September 24, 1925
J. Brown, Transit
H. Pursel } Chainmen
J. Bailey

Course	Length	Magnetic Azimuth	Magnetic Bearing			
H-8	192.0	79°0′	N 79°E			Center Rattling Run
H-7	150.86	47°15′	N 47°E			Top of dam
A-H	525.25	41°36′	N $41\frac{1}{2}^\circ$E			H is hub
A-B		8°52′	Checks original azimuth of 8°51′			
G-A	988.99	260°13′	S $80\frac{1}{4}^\circ$W			
G-6	351.33	286°21′	N 74°W			
G-5	298.06	291°14′	N 69°W			
G-4	245.1	316°0′	N 44°W			Center Rattling Run - 8′ wide
G-3	111.4	19°10′	N 19°E			Center Rattling Run - 8′ wide
F-G	547.55	335°55′	N 24°W			G is ☒ on large flat rock
E-F	387.52	228°36′	S $48\frac{1}{2}^\circ$W			F is hub at N. edge of road
D-2	555.1	170°30′				Center Rattling Run - 7′ wide
D-E	628.37	170°31′	S $9\frac{1}{2}^\circ$E			E is hub 30′ N. of road
C-D	594.01	132°45′	S $47\frac{1}{4}^\circ$E			D is hub - Reference 6″ beech - N 78°30′ E, 15.2 ft.
B-1	205.3	70°40′				Center Rattling Run - 7′ wide
B-C	897.46	70°42′	N $70\frac{3}{4}^\circ$E			C is notch and nail in large oak stump B is hub - Reference 8″ oak tree - S 45°50′ W, 22.0 ft.
A-B	659.43	8°51′	N $8\frac{3}{4}^\circ$E			A is stone monument N.E. cor. Hamilton tract. Transit oriented in magnetic meridian. Public road 10′ S.

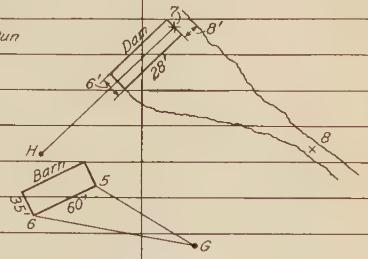

Fig. 19

CALCULATING AREAS

DOUBLE MERIDIAN DISTANCES

39. Explanation.—The most common method of calculating the area of a field bounded by straight lines is known as the double-meridian-distance method. The *double meridian distance* of a course, abbreviated *D. M. D.*, is equal to the sum of the total departures of the extremities of the course. For example, in Fig. 18, the double meridian distance of *CD* is equal to $C'C + D'D$, and that of AB is $B'B + 0$, or $B'B$. Just

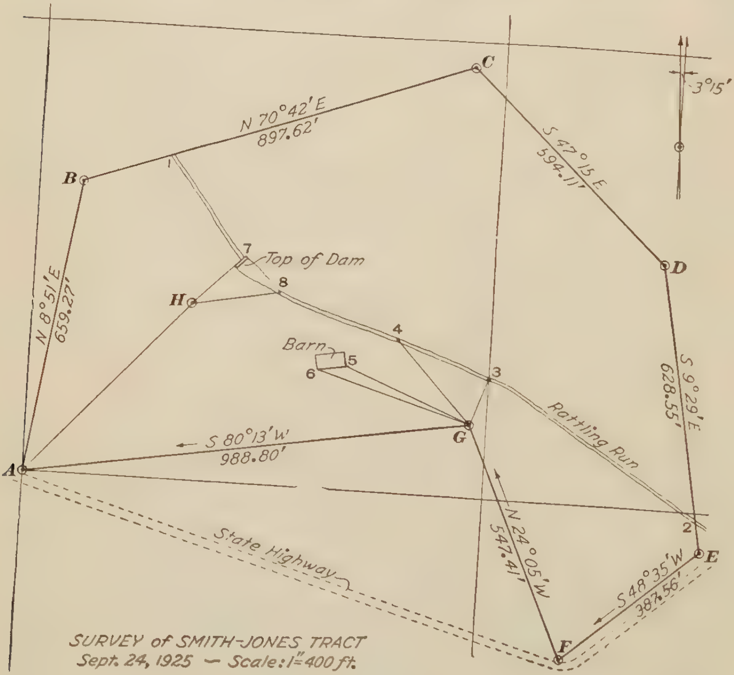

FIG. 20

as the total departures of the points of a traverse can be measured from any axis, the double meridian distances of the courses can be referred to any convenient meridian. If, in Fig. 18, the reference meridian is taken through F, the total departure of C is $-CH$ and the total departure of D is $+KD$. Hence, the double meridian distance of CD is equal to $KD - CH$. The numerical value of the double meridian distance of GA is equal to $GL+AP$; but, since the total departures of G and A are both negative, the double meridian distance of GA must be taken as $-(GL+AP)$. In order to make the calculations of double meridian distances easier and to avoid negative values, the reference meridian is usually taken through the most westerly corner of the field.

If the field has been plotted, the most westerly corner is readily selected. Sometimes, however, a study of the departures of the courses is sufficient to determine the proper corner. Occasionally, it may be necessary to assume a point as the most westerly and to calculate the total departures of the other corners from it. If all the values come out positive, the assumption is correct; otherwise, the corner having the greatest west total departure is the one to be selected. In Fig. 18, A is the most westerly corner.

40. Calculation of Double Meridian Distances.—The double meridian distances of the courses of a traverse may be calculated by determining the total departures of the corners and taking the sum of the total departures of the extremities of each course as explained in the preceding article. The survey should be balanced before the area is computed, and the corrected departures should be used in calculating the double meridian distances. If, in Fig. 18, the reference meridian is taken through A, the double meridian distance of BC, which is equal to the departure of B plus the departure of C, equals $101.47+948.64=1{,}050.11$.

The calculations may be made more easily by means of the following rule:

Rule.—*The double meridian distance of any course is equal to the sum of the double meridian distance of the preceding course,* *the departure of the preceding course, and the departure of the course itself. East departures are added and west departures are subtracted.*

In this method the courses must be taken in order, starting from the reference meridian. The double meridian distance of

Calculations for Determining Double Meridian Distances

First Method		Second Method	
Total Dep. *A*	= 0		
Total Dep. *B*	= 101.47		
D. M. D. *AB*	= 101.47	D. M. D. *AB*	= 101.47 = Dep. *AB*
Total Dep. *B*	= 101.47	Dep. *AB*	= 101.47
Total Dep. *C*	= 948.64	Dep. *BC*	= 847.17
D. M. D. *BC*	= 1,050.11	D. M. D. *BC*	= 1,050.11
Total Dep. *C*	= 948.64	Dep. *BC*	= 847.17
Total Dep. *D*	= 1,384.91	Dep. *CD*	= 436.27
D. M. D. *CD*	= 2,333.55	D. M. D. *CD*	= 2,333.55
Total Dep. *D*	= 1,384.91	Dep. *CD*	= 436.27
Total Dep. *E*	= 1,488.46	Dep. *DE*	= 103.55
D. M. D. *DE*	= 2,873.37	D. M. D. *DE*	= 2,873.37
		Dep. *DE*	= 103.55
Total Dep. *E*	= 1,488.46	Sum	= 2,976.92
Total Dep. *F*	= 1,197.83	Dep. *EF*	= −290.63
D. M. D. *EF*	= 2,686.29	D. M. D. *EF*	= 2,686.29
		Dep. *EF*	= −290.63
Total Dep. *F*	= 1,197.83	Diff.	= 2,395.66
Total Dep. *G*	= 974.43	Dep. *FG*	= −223.40
D. M. D. *FG*	= 2,172.26	D. M. D. *FG*	= 2,172.26
		Dep. *FG*	= −223.40
Total Dep. *G*	= 974.43	Diff.	= 1,948.86
Total Dep. *A*	= 0	Dep. *GA*	= −974.43
D. M. D. *GA*	= 974.43	D. M. D. *GA*	= 974.43 = −Dep. *GA*

the first course of the traverse is equal to the departure of the course, and, as a check on the work, the double meridian distance of the last course should be equal to the departure of the course with its sign changed. The corrected departures for the survey plotted in Fig. 18 are given in Table II. The double meridian distance of the course AB is equal to its departure, or 101.47. The double meridian distance of BC is equal to the double meridian distance of AB, which is 101.47, plus the departure of AB, which is 101.47, plus the departure of BC, which is 847.17; the result is $101.47+101.47+847.17 = 1{,}050.11$, as obtained by taking the sum of the total departures of B and C.

The calculations for determining the double meridian distances of all the courses by both methods are conveniently arranged in the accompanying tabulation. Since the departures of EF, FG, and GA are westings, they are subtracted in applying the rule. The computed value of the double meridian distance of the last course GA is 974.43, and the departure of the course is -974.43. The computations are, therefore, correct because these values are numerically equal and have different signs. Since the meridian is taken through A, which is the most westerly corner, all the double meridian distances are positive in this case. However, if the meridian had been taken through any other corner, some of the values would have been negative.

COMPUTATION OF AREA

41. The area of any polygon can be computed by dividing the given figure into triangles, rectangles, and trapezoids, finding the area of each portion, and taking the sum of these partial areas. Sometimes, it is more convenient to compute the area of a figure that encloses the given polygon and then deduct from it the areas not included in the given polygon.

For instance, in Fig. 18, the area of the field $ABCDEFGA$ may be taken as equal to the area $C'CDEFF'C'$ minus the area $C'CBAC'$ minus the area $AGFF'A$. But area $C'CDEFF'C'$ is equal to $C'CDD'+D'DEE'+E'EFF'$; area $C'CBAC'$ is equal to $C'CBB'+B'BA$; and area $AGFF'A$ is equal to

$G'GFF'-G'GA$. Hence, area $ABCDEFGA$ is equal to $C'CDD' + D'DEE' + E'EFF' - C'CBB' - B'BA - G'GFF' + G'GA$.

The area of a triangle is equal to one-half the product of the base and the altitude, and the area of a trapezoid is equal to one-half the product of the altitude and the sum of the bases: thus, area $B'BA$ is equal to $\frac{1}{2}\ AB' \times B'B$, and area $C'CBB'$ is equal to $\frac{1}{2}\ B'C' \times (B'B + C'C)$. But AB' and BB' are, respectively, the latitude and double meridian distance of AB; $B'C'$ is the latitude of BC, and $B'B + C'C$ is its double meridian distance. Hence, area $B'BA$ is equal to one-half the product of the latitude and the double meridian distance of AB, and area $B'BCC'$ is equal to one-half the product of the latitude and the double meridian distance of BC. Similarly, the other areas are equal to one-half the products of the latitudes and the double meridian distances of the other courses. If the latitude and the double meridian distance are multiplied, the product is the *double area*, since the area itself is equal to one-half the product. It is convenient to consider double areas and then, after they have been combined, to divide the result by 2.

In the preceding expression for area $ABCDEFGA$, it will be noticed that areas $C'CDD'$, $D'DEE'$, $E'EFF'$, and $G'GA$ are to be added, and the courses CD, DE, EF, and GA, which form the inclined sides of the respective areas, all have south latitudes. On the other hand, areas $C'CBB'$, $B'BA$, and $G'GFF'$ are to be subtracted and the courses BC, AB, and FG have north latitudes. Therefore, if all the double meridian distances are positive, the area may be computed by the following rule:

Rule.—*Multiply the double meridian distance of each course by the latitude of the course. Take the sum of the products for the courses having north latitudes and the sum of the products for the courses having south latitudes. Subtract the smaller result from the larger and divide the remainder by 2. The area is taken as positive in every case.*

The calculations for the area of the field in Fig. 18 are shown in Table V. North latitudes and east departures are indicated +, and south latitudes and west departures are indicated −.

TABLE V

CALCULATION OF AREA

Course	Latitude	Departure	D. M. D.	Double Area	
				North	South
AB	+651.42	+101.47	101.47	66,100	
BC	+296.54	+847.17	1,050.11	311,400	
CD	−403.33	+436.27	2,333.55		941,191
DE	−619.96	+103.55	2,873.37		1,781,374
EF	−256.34	−290.63	2,686.29		688,604
FG	+499.76	−223.40	2,172.26	1,085,608	
GA	−168.09	−974.43	974.43		163,792
				1,463,108	3,574,961
					1,463,108
					2)2,111,853
					1,055,926 sq. ft.
					=24.241 acres

EXAMPLES FOR PRACTICE

1. Verify the double meridian distances of the courses for the following notes:

Course	Latitude	Departure	D. M. D.
AB	+521.55	+368.79	368.79
BC	−467.51	+819.20	1,556.78
CD	−712.99	+100.57	2,476.55
DE	+112.01	−596.25	1,980.87
EF	− 97.30	−612.49	772.13
FA	+644.24	− 79.82	79.82

2. Calculate the area of the field in example 1.

Ans. 24.140 acres

CIRCULAR CURVES

THEORY OF CIRCULAR CURVES

DEFINITIONS AND FUNDAMENTAL PRINCIPLES

23.[*] **Tangents.**—The line of a railroad consists of a series of straight lines connected by curves. Each two adjacent lines are united by a curve having the radius best adapted to the conditions of the surface. The straight lines are called **tangents,** because they are tangent to the curves that unite them. In order to determine the curve by which two tangents are to be united, the angle between them must be known.

24. Intersection of Tangents.—Let *A B* and *C D*, Fig. 17, be two intersecting tangents. It is desired to determine their point of intersection and the angle that they make with each other. A picket is first set up at *B* and another at *A*, or at some other point on

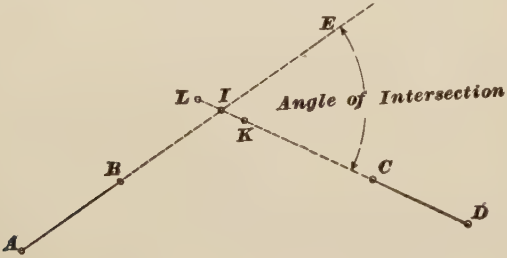

FIG. 17

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[*] On account of the omission, after this section was printed, of some matter that was transposed to another section, the article and figure numbers begin here with 23 and 17, respectively, instead of 1.

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the line *A B*. The transit is set up at *C*, the telescope is backsighted to *D*, then reversed forward and sighted at a flag held in the line given by the transit and also about in range with the pickets at *A* and *B*, as nearly as the flagman can judge. With a little practice he can determine approximately the intersection *I* of the two lines. Two temporary hubs are then driven at *K* and *L* in the line *C D*, one on either side of the estimated position of the point of intersection *I*. These hubs are centered by driving a tack half its length in the top of each, exactly on the line *C D*, and a cord is stretched between the tacks. The instrument is then set up at *B*, the telescope is backsighted to *A*, reversed forward, and a flag lined in at *I* where the line of sight intersects the cord, which will be the intersection of the line *A B* prolonged with *C D* prolonged. A permanent hub is now driven at *I* and centered by driving a tack at the exact point where the prolongation of *A B*, as given by the instrument, crosses the cord connecting the tacks in the hubs at *K* and *L*. Having thus located the point *I*, the instrument should be set over it, and the angle *E I C* should be measured.

The point *I* is the intersection of the tangents *A B* and *C D*. Such a point is called a **point of intersection;** it is commonly designated by the letters P. I., and the guard stake indicating its position is marked P. I. The external angle *E I C*, formed by the intersecting tangents, is called the **intersection angle** or the **angle of intersection.**

25. Curves.—Railroad curves are usually circular and are divided into three general classes, namely: simple, compound, and reverse curves.

A **simple curve** is a curve having but one radius, as the curve *A B*, Fig. 18, whose radius is *A C*.

A **compound curve** is a continuous curve composed of two or more arcs of different radii, as the curve *C D E F*, Fig. 19, which is composed of the arcs *C D*, *D E*, and *E F*, whose respective radii are *G C*, *H D*, and *K E*. In the general class of compound curves may be included what are known as **easement curves, transition curves,** and **spiral curves,** now used very generally on the more important railroads. These do not belong properly to the subject of Surveying.

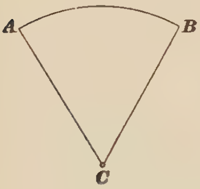


FIG. 18

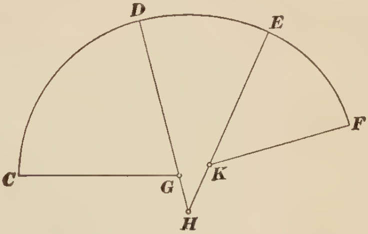


FIG. 19

A **reverse curve** is a continuous curve composed of the arcs of two circles of the same or different radii, the centers of which lie on opposite sides of the curve, as in Fig. 20. The two arcs composing the curve meet at a common point *M*, called the **point of reversal,** at which point they are tangent to a common line perpendicular to the line joining their centers. Reverse curves are becoming less common on railroads of standard gauge.

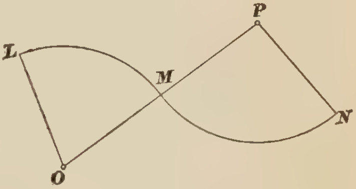


FIG. 20

26. Geometry of Circular Curve.—In studying the subject of curves, a thorough knowledge of the principles of geometry relating to the circle is very essential. The elementary parts of a circular curve and the related geometric values required in laying it out are illustrated in Fig. 21. *A B* and *C D* represent two tangents that are united by the curve *B G H K C*. The prolongations of these tangents intersect at *E* and form the angle of intersection *F E C*. The following principles of geometry are of especial importance as relating to curves, and are restated here for convenience of reference, the form of statement being modified to suit present requirements by substituting the subtending chord for the arc:

1. A tangent to a circle is perpendicular to the radius at its tangent point. Thus, $A\ E$, Fig 21, is perpendicular to $B\ O$ at its tangent point B, and $C\ D$ is perpendicular to $C\ O$ at C.

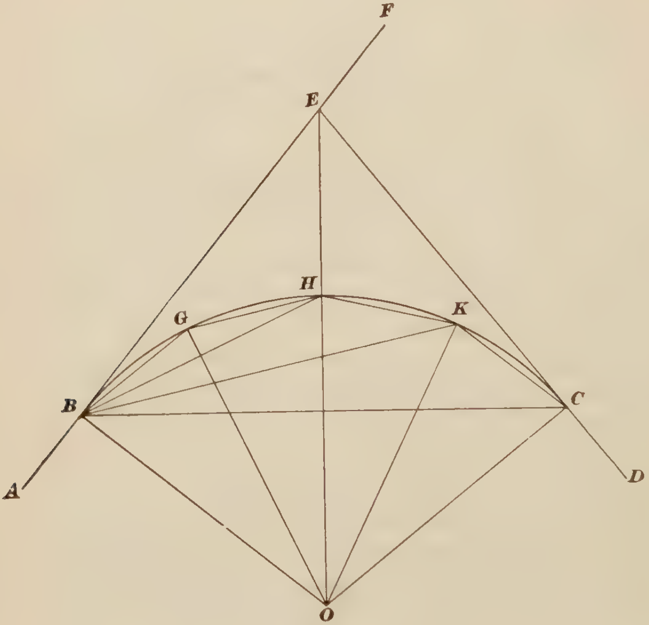


FIG 21

2. Two tangents to a circle from any point without the circle are equal in length, and make equal angles with the chord joining their points of tangency. Thus $B\ E$ and $C\ E$ are equal, and the angles $E\ B\ C$ and $E\ C\ B$ are equal.

3. An angle not exceeding 90° formed by a chord and the tangent at one of its extremities is equal to one-half the central angle subtended by the chord. Thus, the angle $E\ B\ C = E\ C\ B =$ one-half $B\ O\ C$.

4. An angle not exceeding 90° having its vertex in the circumference of a circle and subtended by a chord of the

circle, is equal to one-half the central angle subtended by the chord. Thus, the angle *G B H*, whose vertex *B* is in the circumference, is subtended by the chord *G H* and is equal to one-half the central angle *G O H*, subtended by the same chord *G H*.

5. Equal chords of a circle subtend equal angles at its center and also in its circumference, if the angles lie in corresponding segments of the circle. Thus, if *B G*, *G H*, *H K*, and *K C* are equal, $B O G = G O H$, $G B H = H B K$, etc.

6. The angle of intersection between any two tangents of a circle is equal to the central angle subtended by the chord joining the two points of tangency. Thus, the angle $C E F = B O C$.

7. A radius that bisects any chord of a circle is perpendicular to the chord.

27. The Angular Unit.—The rate of divergence of the two lines forming an angle, from their common or angular point, determines the size of the angle. As we know from geometry, the unit of angular measurement is the degree, equal to $\frac{1}{360}$ part of a circle. In the calculations relating to curves, it is convenient to remember that two lines forming an angle of 1° with each other will, at a distance of 100 feet from the angular point, diverge by 1.745 feet, very closely.

In Fig. 22, the lines *A B* and *A C*, meeting at the point *A*, are supposed to form an angle of 1°, and the angle *B A C* is measured by the arc *B C*, described with the radius *A B*, which is 100 feet in length. The chord *B C*, which is the straight line joining the extremities of that arc, is equal to twice the sine of one-half the angle *B A C*, multiplied by 100. The natural sine of 30 minutes, expressed to seven decimal

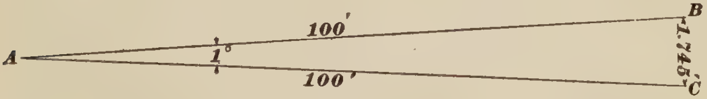


FIG. 22

places, is .0087265,* and 2 × .0087265 × 100 = 1.7453. Hence, the length of the chord *B C*, expressed to three decimal places, which is close enough for all practical purposes, is 1.745 feet.

28. Degree of Curvature.—The sharpness of a curve, or the rapidity with which it turns, may be designated by its radius; that is, by the radius of the circular arc composing it. In this country, however, this property of a curve is usually designated by the number of degrees contained in the central angle of the curve subtended by a chord of 100 feet. This is called the **degree of curvature,** or **degree of curve.** Thus, if the chord *B G*, Fig. 21, is 100 feet long, and the angle *B O G* is 1°, the curve is called a **one-degree curve**; but if with the same length of chord the angle *B O G* is 4°, the curve is called a **four-degree curve**; while if the angle *B O G* is 10°, the curve is called a **ten-degree curve**; etc. These are commonly written 1° curve, 4° curve, 10° curve, etc. That is to say, a chord of 100 feet on the curve, or, at the circumference of the circle of which the curve is an arc, will subtend an angle of 1° at the center of a 1° curve, an angle of 4° at the center of a 4° curve, or an angle of 10° at the center of a 10° curve, etc., formed between two radii.

It should be noticed, however, that the term *degree of curvature*, as used in this sense, is not a perfectly general term, since it applies only to curves of comparatively long radii. Evidently it can be applied only to curves having diameters not less than the designated length of chord (100 feet), and, consequently, the extreme limit of its application is reached in the case of a circle whose diameter is 100 feet. The degree of curvature of an arc of such a circle would be 180°. But, although this is the theoretical limit of application of the expression *degree of curvature*, the practical limit is reached in the designation of curves of much larger radii, such as will contain several chords of 100 feet each in a semi-circumference. In sharper curves, or curves of smaller radii, it is customary to designate the rate of curvature by the length of the radius.

---

* The value of the sine is here stated to seven decimal places, because the value .00873, as given in a table of five decimal places, would give for the divergence of the lines in 100 feet the value 2 × .00873 × 100 = 1.746, which is not as close as the value 1.745.

29. Deflection Angle.—The angle formed between any chord of a curve and a tangent to the curve at one extremity of the chord is the **deflection angle** for the chord; that is, it is the angle by which the chord is deflected from the tangent. According to the principle stated in Art. **26,** 3, this angle is equal to one-half the central angle subtended by the chord. Thus, $E B G$, Fig. 21, is the deflection angle by which the chord $B G$ is deflected from the tangent $B E$. According to the principle just referred to, it is equal to one-half the central angle $B O G$. Since it is customary to designate a curve by its degree of curvature, which is equal to the central angle subtended by a chord of 100 feet, it follows from what has just been stated *that the deflection angle for a chord of 100 feet is equal to one-half the degree of curvature.*

But, according to the principle stated in Art. **26,** 4, the angle $G B H$, which the chord $B H$ makes with the preceding chord $B G$, is equal to one-half the central angle $G O H$, subtended by the chord $G H$, so that if the chord $G H$ is equal to the chord $B G$, the central angle $G O H$ will be equal to $B O G$ and the angle $G B H$ equal to $E B G$. The same is true of the angles $H B K$ and $K B C$. Hence, it also follows from this principle that *when a series of chords is deflected from a common point on a curve to consecutive stations 100 feet apart along such curve, the common difference between the consecutive deflection angles is always equal to one-half the degree of curvature.*

But while the name deflection angle is applied to the angle between any chord of a curve and the tangent at one extremity of the chord, it is customary to restrict its application to an angle of this character that is subtended by a chord of 100 feet, and the term deflection angle is commonly understood to mean the deflection angle subtended by such a chord. When used in this sense it may be designated as the **regular deflection angle.** Hence, in any curve, *the regular deflection angle is equal to one-half the degree of curvature.*

30. Relation Between Deflection Angle and Radius. In Fig. 23, let $O L$ be perpendicular to the chord $B G$ at its center. In the right triangle $B O L$, we have

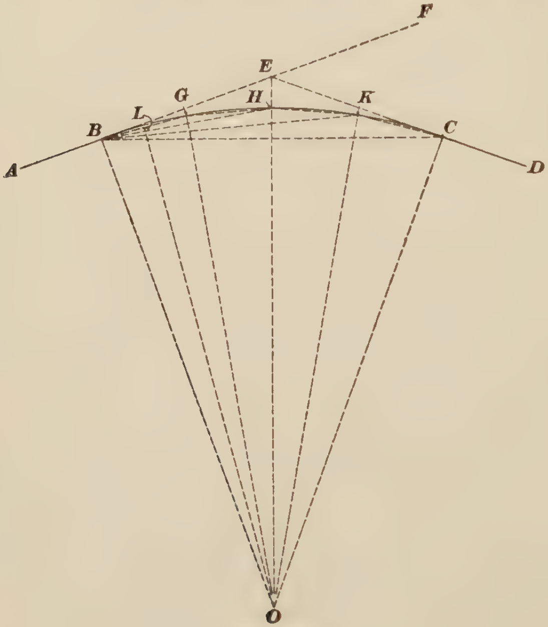


FIG. 23

$$O B = \frac{B L}{\sin B O L};$$

but $O B$ is the radius of the curve, $B L$ is one-half the chord $B G$, and the angle $B O L$, being equal to one-half $B O G$, is equal to the deflection angle $E B G$. If we now denote the radius $O B$ by R, the chord $B G$ by c, and the deflection angle $E B G = B O L$ by D, the equation written above becomes

$$R = \frac{\frac{1}{2} c}{\sin D} \qquad (9)$$

This formula is perfectly general and applies to any length of chord. If we let D_{100} denote the deflection angle for a chord of 100 feet and substitute 100 for c, we shall have

$$R = \frac{50}{\sin D_{100}} \qquad \textbf{(10)}$$

31. Approximate Value of Radius.—When the degree of curve is not too great, its radius can be determined approximately and closely enough for most practical purposes by dividing the radius of a 1° curve by the degree of the given curve. The deflection angle of a 1° curve is $\frac{1}{2}°$ or 30 minutes. By using seven-place logarithmic tables and substituting the logarithmic sine of 30 minutes in formula **10,** we have

$$\begin{array}{ll} \log 50 & = 1.6989700 \\ \log \sin 0° \, 30' & = \bar{3}.9408419 \\ \hline \log R & = 3.7581281 \end{array}$$

Hence, $R = 5{,}729.65$ feet.

This is the correct value of the radius of a 1° curve to two decimal places, but in order to obtain it seven-place logarithmic tables must be used; it will not be given exactly by five-place tables. In practice, the radius of a 1° curve is commonly taken at 5,730 feet.

If we now refer to a table of natural sines, we shall see that for small angles the sines are very nearly proportional to the angles, so that the sine of 1°, the deflection angle of a 2° curve, is approximately and very closely equal to twice the sine of 0° 30′, the deflection angle of a 1° curve, etc. It is therefore evident that the above formula will give values of the radius R that are very nearly inversely proportional to the deflection angles, and consequently, to the degrees of curve.

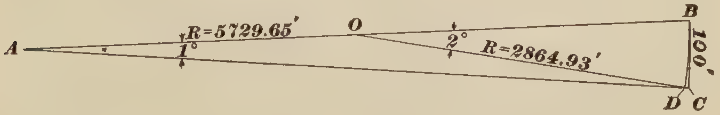


FIG. 24

In order to illustrate this, let $A\,B$ and $A\,C$, Fig. 24, be radii 5,729.65 feet in length, forming an angle of 1° at the center A; then the arc BC subtended by these radii will be 100 feet in length. The curve BC is called a 1° curve. If, from the point O as a center, with a radius OB equal to 2,864.93 feet, we describe an arc BD 100 feet in length, the radii OB and OD will form an angle of 2° at the center O, and the curve BD is called a 2° curve. But, the radius of the 1° curve, divided by 2, or $5{,}729.65 \div 2 = 2{,}864.83$, is very nearly equal to 2,864.93, the radius of the 2° curve. And likewise, the radius of the 1° curve divided by 3, or $5{,}729.65 \div 3 = 1{,}909.88$, is very nearly equal to the true radius of a 3° curve, which is 1,910.08 feet, etc. Hence, if we assume the radius of a 1° curve to be 5,730 feet, and denote the degree of curve by D_c, the approximate value, in feet, of the radius of a curve of greater degree can be found by the formula

$$R = \frac{5{,}730}{D_c} \qquad \textbf{(11)}$$

The values of the radii given in the Table of Radii and Deflections were calculated by formula **10,** seven-place logarithmic tables being used; values quite close to these, however, can be obtained by means of five-place tables.

The results obtained by formula **11,** however, are sufficiently accurate for most practical purposes, for curves of from 1° to 10°. But for sharp curves, that is, for those exceeding about 10°, the radii should be found by means of formula **10,** especially if they are to be used as a basis for further calculation. The error increases as the degree of curve increases, as is shown by the following examples:

Example 1.—What is the radius of a 4° curve?

Solution.—Applying formula **10,** knowing that the deflection angle D_{100} of a 4° curve is 2°, we have

$$R = \frac{50}{.0349} = 1{,}432.66 \text{ ft.}$$

Applying formula **11,** we have

$$R = \frac{5{,}730}{4} = 1{,}432.5 \text{ ft.}$$

In this case the error is only .16 foot, and may be ignored in practical work.

Example 2.—What is the radius of a 30° curve?

Solution.—Applying formula **10,** we have

$$R = \frac{50}{.25882} = 193.18 \text{ ft.}$$

Applying formula **11,** we have

$$R = \frac{5{,}730}{30} = 191 \text{ ft.}$$

In this case the error is 2.18 feet.

EXAMPLES FOR PRACTICE

Note.—In the following exercises, compute the radius, (*a*) by formula **10** and (*b*) by formula **11,** and compare the results.

1. What is the radius of a 5° curve? Ans. (*a*) 1,146.3 ft. (*b*) 1,146.00 ft.
2. What is the radius of a 9° curve? Ans. (*a*) 637.27 ft. (*b*) 636.67 ft.
3. What is the radius of a 15° curve? Ans. (*a*) 383.05 ft. (*b*) 382.00 ft.
4. What is the radius of a 20° curve? Ans. (*a*) 287.94 ft. (*b*) 286.50 ft.

OTHER VALUES RELATING TO CURVES

32. Subchords.—On curves of short radii, that is, curves of about 20° and upwards, center stakes are usually driven at intervals of 25 feet, nominally, that is, at intervals of one-fourth the length of the arc subtended by a chord of 100 feet. Chords shorter than 100 feet are commonly called **subchords.** For curves of very long radii, the chord and arc may be assumed to be of the same length. But as the degree of curvature increases, the difference in length between the arc and chord also increases, and for curves above 20° the excess in the length of the arc over that of the chord becomes very considerable. If in Fig. 23, the chord *B C* is 100 feet long, the arc *B G H K C* must have a length greater than 100 feet; and if the arcs *B G*, *G H*, *H K*, and *K C* are each equal to one-quarter the arc *B H C*, then the equal chords *B G*, *G H*, *H K*, and *K C* subtending these equal arcs must each have a length greater than one-quarter BC, or, since $BC = 100$ feet, each of these chords must have a length greater than 25 feet. The actual length of the subchord can be calculated by means of formula **9,** which, when transposed to express the value of the chord, gives

$$c = 2R \sin D \qquad \textbf{(12)}$$

This formula applies to any length of chord when D is the deflection angle for the chord considered. But if the chords BG, GH, HK, and KC each subtend one-fourth of the arc BHC, they are equal to each other, and according to the principles stated in Art. **26,** 3 and 5, the angles EBG, GBH, HBK, and KBC are also equal to each other, and consequently, are each equal to one-fourth the deflection angle EBC. Hence, the length of one of the chords BG, GH, HK, or KC, subtending one-fourth of the arc BHC, is given by merely substituting in the above formula the sine of one-fourth the deflection angle EBC. We thus have the following important principle: *The deflection angle for any fractional part of an arc is equal to the corresponding fractional part of the deflection angle for the whole arc.*

In applying this principle for the purpose of computing the deflection angles for subchords, it is customary to consider the deflection angle to be proportional to the length of the chord. Though this practice is not strictly correct, the resulting error is very slight in curves of large radius.

EXAMPLE 1.—Suppose the curve BHC, Fig. 23, to be a 20° curve, and the chord BC subtending it to be 100 feet in length. What is the length of the subchord BG subtending one-fourth the arc BHC?

SOLUTION.—Since the degree of curve is the central angle subtended by a chord of 100 feet, and the chord BC is 100 feet in length, the central angle BOC is 20°, and as the arc BG is one-fourth of the arc BHC, the central angle subtended by the chord BG is equal to $\frac{1}{4} \times 20° = 5°$. The deflection angle for the chord BG, being equal to one-half the central angle subtended by the same chord, is equal to $\frac{1}{2} \times 5° \ 00' = 2° \ 30'$. Or, also, since the deflection angle for the chord BC is $\frac{1}{2} \times 20° = 10°$, according to the principle just stated, the deflection angle for the chord BG is equal to $\frac{1}{4} \times 10° = 2° \ 30'$. Sin $2° \ 30' = .04362$.

By formula **10** the radius of a 20° curve is equal to $\frac{50}{\sin 10^\circ} = \frac{50}{17365}$ = 287.94 feet. Hence, by substituting these values in formula **12,** we get for the length of the subchord BG the value $c = 2 \times 287.94 \times .04362 = 25.12$ ft. Ans.

Consequently, in measuring each of the subchords BG, GH, HK, and KC, 25.12 feet should be used instead of 25 feet.

EXAMPLE 2.—What is the deflection angle for a chord of 15 feet in a 3° curve?

SOLUTION.—The deflection angle for a chord of 100 feet in a 3° curve is equal to one-half the degree of curve, or $1^\circ\ 30' = 90'$. According to the principle stated above, therefore, the deflection angle for a chord of 15 feet in a 3° curve is equal to

$$\frac{15}{100} \times 90' = 13.5' \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The degree of a curve is 5° 30′; what is the deflection angle for a chord of 16.2 feet? Ans. 0° 26.7′

2. The degree of a curve is 7° 15′; what is the deflection angle for a chord of 38.4 feet? Ans. 1° 23½′

3. What is the length of the chord subtending one-fifth of the arc subtended by a chord of 100 feet in a 16° curve? Ans. 20.061 ft.

4. In a 10° curve, what is the length of the chord subtending .12 of the length of the arc subtended by 100 feet? Ans. 12.013 ft.

33. Required Degree of Curve.—In proceeding to unite two tangents by a curve when the angle of intersection between the tangents has been measured, the first matter to be decided is the degree of the curve that is to unite them. This will depend on the character of the work and the topographical conditions of the surface. In railroad work the requirements of the anticipated traffic naturally impose a limit on the sharpness of curve or degree of curvature allowed. On the other hand, while it is always desirable to connect the tangents by as light and easy a curve as possible, the conditions of the surface usually limit this, especially in hilly country. No satisfactory rule can be given, but in general, the curvature should be as easy, that is, the degree of curve should be as small, as the conditions will permit.

The smaller the degree of curve, the greater will be the length of curve necessary to effect a given amount of curvature, and, consequently, the degree of curve is usually determined by the length of the line or form and extent of the space within the limits of which the required amount of curvature must be effected, this being governed by the topographical features of the surface and other controlling conditions. In level country, the degree of curvature will be determined by the angle of intersection and the tangent distances. (See Art. **34.**) When the angle of intersection and the length of the radius of the curve have been determined, the tangent distances can be calculated by means of formula **13,** or when the angle of intersection and the tangent distances have been determined, the radius of the curve can be calculated. For the same angle of intersection, the length of the curve will vary inversely as the degree of curvature. It is customary to fix a limiting or maximum degree of curvature, according to the requirements of the anticipated traffic, which must not be exceeded.

34. Tangent Distances.—When the degree of curve has been decided, the next step in order is the location of the points on the tangents where the curve begins and ends. The point where the curve begins is called the **point of curve,** and is designated by the letters P. C.; the point where the curve terminates is called the **point of tangency,** and is designated by the letters P. T. According to the principle stated in Art. **26,** 2, these two points are equally distant from the point of intersection of the tangents. The distance of the P. C. and P. T. from the P. I. is called the **tangent distance.** The chord connecting the P. C. and P. T. of a curve is commonly called its **long chord.**

In Fig. 23, let AB and CD be tangents intersecting at the point E. From the principle stated in Art. **26,** 6, we know that $BOC = FEC$; hence, the angle $BOE = \frac{1}{2} FEC$. From the right triangle EBO, we have

$$\frac{BE}{BO} = \tan BOE = \tan \tfrac{1}{2} FEC$$

If we now let I denote the angle of intersection FEC, and T the tangent distance BE, and, remembering that BO is the radius R, substitute these values in the foregoing expression, we shall have, by clearing of fractions,

$$T = R \tan \tfrac{1}{2} I \qquad (\mathbf{13})$$

From the point of intersection, the tangent distance, as determined by formula **13,** is measured back on both tangents, thus determining the tangent points, as the points B and C, Fig. 23. Plugs are driven at both points and centered as to line and measurement, and guard stakes are driven to indicate their positions. If the numbering of the station runs from B toward C, the stake at B will be marked P. C., and the stake at C marked P. T., each stake being marked also with the number of the station and the plus.

EXAMPLE.—Suppose that the intersection angle FEC, Fig. 23, is equal to 40° 00′ and it is decided to unite the tangents AB and CD by a 10° curve. What is the tangent distance?

SOLUTION.—By applying formula **10,** we find that for a 10° curve the radius R is equal to 573.7 feet. One-half the intersection angle is 20° 00′, and the natural tangent of 20° 00′ = .36397. Hence, by applying formula **13** we find the tangent distance to be

$$T = 573.7 \times .36397 = 208.81 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The point of intersection of two tangents is at Station 20 + 37 and the angle of intersection is 16° 13′. If the two tangents are united by a 3° curve, what is (*a*) the tangent distance, and (*b*) the station of the P. C.?

$$\text{Ans.}\begin{cases}(a) & 272.13 \text{ ft.}\\ (b) & \text{Sta. } 17 + 64.87\end{cases}$$

2. The point of intersection of two tangents is at Station 5 + 84.32 and the angle of intersection is 59° 20′. If the two tangents are united by an 8° 30′ curve, what is (*a*) the tangent distance, and (*b*) the station of the P. C.?

$$\text{Ans.}\begin{cases}(a) & 384.32 \text{ ft.}\\ (b) & \text{Sta. } 2\end{cases}$$

3. Suppose that the point of intersection of two tangents is at Station 40 + 25, and the angle of intersection is 21° 35′. Assuming the two tangents to be united by a 4° 15′ curve, determine (*a*) the tangent distance, and (*b*) the station of the P. C.

$$\text{Ans.}\begin{cases}(a) & 257.03 \text{ ft.}\\ (b) & \text{Sta. } 37 + 67.97\end{cases}$$

FIELD WORK

35. To Lay Out a Curve With a Transit.—The tangent points for a curve having been set as described in the preceding article, the curve can easily be run in between them. Continuing the example of the preceding article, suppose the point *B*, Fig. 23, which is the P. C. of a 10° curve to the right, happens to be at Station 58 of the tangent *A B*, and that it is desired to run in the curve between *B* and *C*, setting a stake at each full station, that is, at the end of each chord of 100 feet. The transit is set up at *B*, the P. C., and, with the vernier set at zero, the telescope is sighted to the point of intersection *E*, or it may be backsighted to *A*, if a more convenient or better sight can be obtained. Since the central angle *B O G*, measured by a chord of 100 feet, is 10°, the deflection angle *E B G* subtended by the same chord will be one-half *B O G*, or 5°. An angle of 5° 00′ is turned to the right on the vernier, giving the line *B G*, a distance of 100 feet is measured from *B* along the line so given, and at the extremity of this measurement the flag is lined in by the instrument, giving the point *G*, or Station 59, at which a stake marked 59 is driven. An additional 5° 00′ is then turned off, making 10° 00′ from the tangent, and at the end of another measurement of 100 feet the flag is lined in and a stake marked 60 is set for the point *H*. The operation is continued by turning in succession angles of 5° 00′ each, and measuring for each angle a chord of 100 feet, until a total angle of 20° 00′, or one-half of the intersection angle, is reached. If the work has been performed correctly, the last deflection will bring the head chainman to the point of tangent *C*. Then, moving the transit to *C*, and backsighting to *B*, the angle *B C E*, turned from the chord *B C* to the tangent *C E*, should also be 20° 00′, = *E B C*.

The P. C. rarely occurs at a full station, however. When it comes at a substation, the chord between it and the next full station will be a subchord. Had the P. C. of the curve come at the substation, say, 57 + 32, the length of the subchord, or distance to the next full station, would be 100 − 32 = 68 feet, and the deflection angle for the subchord of 68 feet would be found as follows: The deflection angle for a full station, that is, for 100 feet, is 5° 00′ = 300′; for 1 foot it is $\frac{300}{100}'$ = 3′, and for 68 feet it is 68 × 3 = 204′ = 3° 24′. This angle is turned off from zero and a stake is set on the line given by the transit at a distance of 68 feet from the P. C., which is at Station 58. The remainder of the curve is then run in as already explained, except that the last chord preceding the P.T. will be a subchord also.

36. Length of Curve.—The length of a curve uniting two tangents, as usually understood, is not the actual length of the arc, but it is the length between the tangent points *measured in chords of 100 feet*, which length is always somewhat less than the length of the arc. Since the degree of curve is equal to the central angle subtended by a chord of 100 feet, the number of such chords in the curve will be equal to the quotient obtained by dividing the total angle at the center by the degree of curve. Since also the total angle at the center is equal to the angle of intersection between the tangents, it follows that when the angle of intersection has been measured and the degree of curve decided on, the length of the curve, expressed in stations of 100 feet, can be found by dividing the angle of intersection by the degree of curve. The following rule is a statement of this principle:

Rule.—*Express the angle of intersection in degrees and decimals by reducing the minutes, if any, to decimals of a a degree, and divide by the degree of curve; the quotient will be the length of the curve, in chords of 100 feet and decimals thereof. This quotient, when multiplied by 100, will be the length of the curve in feet, as measured in chords of 100 feet.*

The P. C. and P. T. having been set and the station of the P. C. determined by subtracting the tangent distance from the station of the P. I., the station of the P. T. is found by adding the calculated length of the curve to the station of the P. C.

EXAMPLE.—Suppose that the angle of intersection is 32° 42′, that the tangents are to be united by a 6° curve, and that the station of the P. C. is 57 + 32. What is (*a*) the length of the curve and (*b*) the station of the P. T.?

SOLUTION.—(*a*) The angle of intersection 32° 42′ reduced to the decimal form is equal to 32.7°. As each central angle of 6° will subtend a chord of 100 feet, there will be as many such chords in the curve as 6 is contained times in 32.7, which is 5.45; that is, there will be in the curve five chords of 100 feet each, plus one chord of 45 feet, or in all a length of 500 + 45 = 545 feet, which is the required length of the curve. Ans.

(*b*) The length of the curve 5 + 45, added to the station of the P. C., 57 + 32, gives 62 + 77 as the station of the P. T. Ans.

Having set all the full stations on the curve, the last chord measurement is in this case 77 feet, while the total deflection angle from the tangent is 16° 21′, or half of the intersection angle 32° 42′.

Another method of calculating the length of the curve is as follows: The sum of all the deflection angles is equal to one-half the intersection angle. The intersection angle being 32° 42′, one-half of which, or the total deflection for the P. T., is equal to 16° 21′, which, reduced to minutes, equals 981′. The deflection for 100 feet is $\frac{6}{2}^\circ = 3^\circ = 180'$, and the deflection for 1 foot is $\frac{180'}{100} = 1.8'$; then, 981′, the total deflection, divided by 1.8′, gives 545 feet as the required length of the curve. Ans.

EXAMPLES FOR PRACTICE

1. Suppose that 16° 13′ is the angle of intersection between two tangents that are united by a 3° curve, and that the P. C. is at Station 17 + 64.87. What is (*a*) the length of the curve and (*b*) the station of the P. T.?

Ans. $\begin{cases} (a) & 540.56 \text{ ft.} \\ (b) & \text{Sta. } 23 + 5.43 \end{cases}$

2. Suppose that 59° 20′ is the angle of intersection between two tangents that are united by an 8° 30 curve, and that the P. C. is at Station 2. What is (*a*) the length of the curve and (*b*) the station of the P. T.?

Ans. $\begin{cases} (a) & 698.04 \text{ ft.} \\ (b) & \text{Sta. } 8 + 98.04 \end{cases}$

3. Suppose that 21° 35′ is the angle of intersection between two tangents that are united by a 4° 15′ curve, and that the P. C. is at Station 37 + 67.97. What is (*a*) the length of the curve and (*b*) the station of the P. T.?

Ans. (*a*) 507.84 ft.
(*b*) Sta. 42 + 75.81

4. Suppose that 20° is the angle of intersection between two tangents that are united by a 4° curve, and that the P. C. is at Station 7 + 47.37 What is (*a*) the length of the curve and (*b*) the station of the P. T.?

Ans. (*a*) 500 ft.
(*b*) Sta. 12 + 47.37

INTERMEDIATE INSTRUMENT POINTS ON CURVE

37. Method by Zero Tangent.—It happens frequently that there are obstructions in the line of curve so that the entire curve cannot be run in from the P. C. on account of obstructions to the view. This is usually the case in either hilly or timbered country. In such cases the curve is run in as far as the stations are visible from the P. C., and at the last station that can be seen a hub is set and centered,

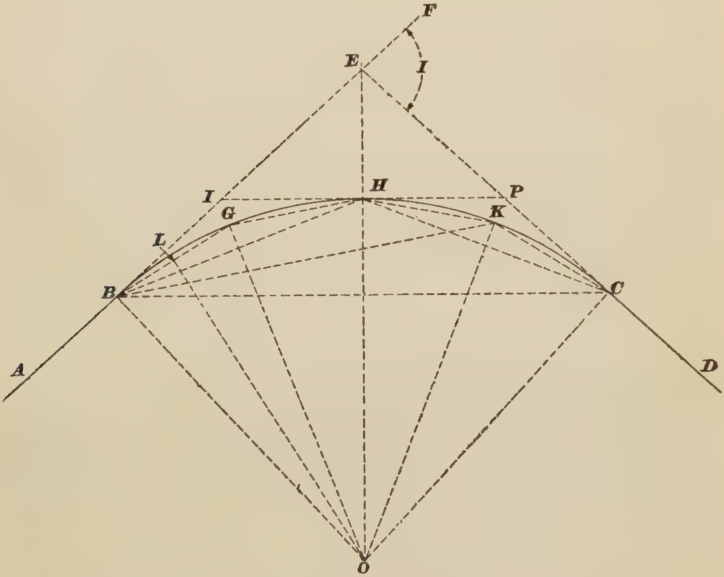


FIG. 25

and the transit moved forward and set up at this intermediate transit point. For example, in Fig. 25, suppose that the station at *H*, 200 feet from the P. C., which is at *B*, is the last point on the curve that can be set from the P. C. A plug is driven at *H* and centered carefully by a tack driven at the point. The transit is now moved forward and set up at *H*. Since the deflection angle *E B H* is 10° to the right, an angle of 10° is turned to the left from zero and the vernier clamped. The instrument is then sighted to a flag at *B*, the lower clamp set, and by means of the lower tangent screw the cross-hair is made to exactly bisect the flag. The vernier clamp is then loosened, the vernier set at zero, and the telescope plunged. The line of sight will then be on the tangent *I P*, and the deflection angles to *K* and *C* can be turned off from this tangent and the stations at *K* and *C* located in the same manner that the stations at *G* and *H* were located from *B*. For according to the principle stated in Art. **26,** 2, the angle at *I H B* between the tangent *I H* and the chord *B H* is equal to the angle *E B H* between the tangent *E B* and the same chord.

This method of setting the vernier for the backsight when the instrument is moved forward to a new instrument point on the curve is favored by many engineers. It is sometimes called the **method by zero tangent.** The essential principle of the method is that *the vernier always reads zero when the instrument is sighted on the tangent to the curve at the point where the instrument is set, and the deflection angles are made to read from the tangent to the curve at this point in the same manner as though this point were the P. C. of the curve.*

38. Method by Continuous Vernier.—A method of turning off the deflection angles that is commonly employed in railroad practice is what is known as the **method by continuous vernier.** In this method, when the curve has been run in as far as expedient from the P. C. and the instrument is moved forward and set up over another station on the curve, the vernier is set at zero before taking the backsight on the P. C., where the instrument was previously set up. The backsight is then taken, the centers clamped, the telescope reversed, the plates unclamped, and the regular deflection angle for the next station is turned off the same as though the instrument were at the P. C. The regular deflection angles for the stations following the instrument point are turned off in order, in the same manner, that is, from the zero at the P. C., the same as though the instrument were at the P. C. This will be understood more clearly from an example.

Suppose that the Station *C*, Fig. 25, instead of being the P. T. of the curve is merely a point on the curve 400 feet from the P. C., which is at *B*, and that it is the last point on the curve that can be set with the instrument at *B*. Having located the point *C*, and the transit having been moved forward and set up over this point, the vernier is set at zero and a backsight taken on the P. C. at *B*. According to the principle stated in Art. **26,** 2, the angle *B C E* is equal to the deflection angle *E B C* for the Station *C*; hence, the vernier being set at zero when the backsight is taken to *B*, if the deflection angle for Station *C*, which in this case is 20°, is turned off to the right, the telescope will be in line with the tangent *E D* at the point *C*. The following stations along the curve can therefore be located from this tangent by turning off the deflection angles in consecutive order the same as though the instrument were at the P. C. at *B*, since the deflection angle for each station is greater than that for the preceding station by the deflection angle for a 100-foot chord. For a 10° curve, the deflection angle for a chord of 100 feet is 5°, and in this case, therefore, each station following *C* can be located by deflecting angles of 5°, 10°, 15°, etc. in consecutive order from the tangent *E D*. But since, after backsighting to the P. C., a deflection of 20°, corresponding to the regular deflection angle for Station *C*, was turned off in order to bring the telescope in line with the tangent *E D*, it is evident that adding 5°, 10°, 15°, etc., consecutively, to the deflection angle for Station *C* will give 25°, 30°, 35°, etc. as the deflection angles for the stations following *C* in their respective order. Hence, these deflection angles may be used in locating the respective stations when the instrument is at *C*, the same as though it were at *B*, if the vernier is set at zero when the backsight is taken to the P. C. at *B*.

If it is necessary to move the instrument again and set it up at some following station on the curve, from which it is impossible to observe the P. C., the vernier may be set at the deflection angle for the last instrument point *C*, in this case 20°, and the backsight taken on this point instead of setting the vernier at zero, as when sighting to the P. C. The stations following the new instrument point can then be located by their regular deflection angles, the same as if they were located from the P. C. In like manner, with the instrument set at any station of the curve, and the backsight taken on any station previously located, if previous to taking the backsight the vernier is set at the deflection angle for the station sighted to, any station visible along the curve can then be located by its regular deflection angle, the same as though the instrument were set up at the P. C.

This method possesses the advantage of permitting the deflection angle for each 100-foot station along the curve to be calculated in regular order by adding one-half the degree of curvature to the deflection angle of the preceding station. Any portion of the curve can be run in from the notes without further calculation. The length of curve between any two stations can also be calculated from the difference between their deflection angles, by dividing twice this difference by the degree of curvature.

DEFLECTION OFFSETS

39. Tangent and Chord Deflections.—Let *A B*, Fig. 26, be a tangent joining the curve *B C E H* at *B*. If the tangent *A B* is prolonged to *D*, the perpendicular distance *D C* from the tangent to the curve is called a **tangential**

distance, a **tangent deflection offset,** or simply a **tangent deflection.** If the chord BC is prolonged to the point G, the distance GE is called a **deflection distance,** a **chord deflection offset,** or simply a **chord deflection.**

If the values of the radius OB and the chord BC are known, the value of the tangent deflection $DC = FE$ can be found in the following manner: Let OM be drawn to the middle point M of the chord BC; according to the principle stated in Art. **26,** 7, OM is perpendicular to BC, and since also the tangent BD is perpendicular to the radius OB, the right triangles OMB and BDC are similar, and we can write, $OB : BM = BC : DC$. But OB is the radius, BC is a chord, and DC is the tangent deflection. If we denote these values by R, c, and f, respectively, then, since $BM = \frac{1}{2}c$, this proportion can be written in the form $R : \frac{1}{2}c = c : f$, from which

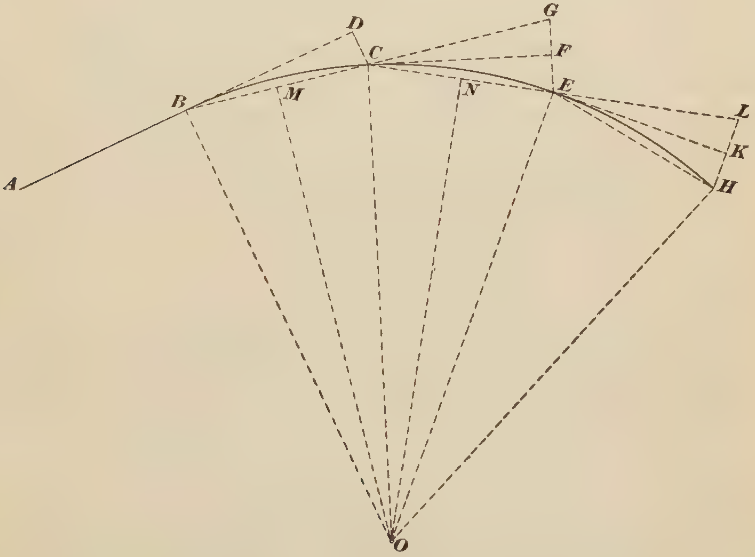


FIG. 26

$$f = \frac{c^2}{2R} \qquad \textbf{(14)}$$

This formula will give the tangent deflection for any length of chord irrespective of any other condition.

If the chord BC is prolonged to G, and if the distance $CG = BC = CE$, the chord deflection GE can be found in the following manner: Since BG and CE are perpendicular respectively to OM and ON, the angle $GCE = MON$, and since $BC = CE$, angle $BOC = COE$, and angle MOC, being $\frac{1}{2}\,BOC$, is equal to angle CON, which is $\frac{1}{2}\,COE$. Hence, angle $COE = MON = GCE$. The triangles OCE and CGE are therefore similar, since both are isosceles, and the angle GCE = angle COE. Hence, we have the proportion $OC : CE = CE : GE$. Denoting the chord deflection GE by d, and substituting R, c, and d for OC, CE, and GE, the above proportion may be written in the form $R : c = c : d$, from which

$$d = \frac{c^2}{R} \qquad \textbf{(15)}$$

From the principles stated in Art. **26,** 3 and 5, we know that, since the chords BC and CE are equal, they form equal angles with a tangent to the curve at C, and since CF is such a tangent and CG is the prolongation of the chord BC, the triangles CFG and CFE are equal; consequently, $GF = FE$ and the chord deflection GE is double the tangent deflection $FE = DC$, as is also shown by the two preceding formulas. It should be well understood, however, that this is the case only when the chords BC and CE are equal.

40. Tangent and Chord Deflections for Subchords. As a basis of calculation, it is convenient to remember that, for a chord of 100 feet preceded by a chord of the same length, the chord deflection for a 1° curve is 1.745 feet. For in Fig. 26, since $BC = CE$ and triangle CGE is similar to triangle OCE, if the angle COE is 1° and the chord CE is 100 feet, then from Art. **27,** we know that the chord deflection GE is 1.745 feet. Assuming the radius of a 2° curve to be equal to one-half that of a 1° curve, etc., formula **15** shows that the chord deflection for a 2° curve is double the deflection for a 1° curve, or 3.49 feet, and so on. The tangent deflection, being one-half the chord deflection, will be .873 foot for a 1° curve, 1.745 feet for a 2° curve, etc.

In calculating tangent and chord deflections, distances measured either on chords or tangents are expressed as decimal parts of a station length of 100 feet, which is taken as the unit. Thus, the tangent deflection for 75 feet is expressed as the tangent deflection for .75 of a station. This method of expression, however, is confined entirely to the calculation; the deflection is spoken of as the deflection for 75 feet.

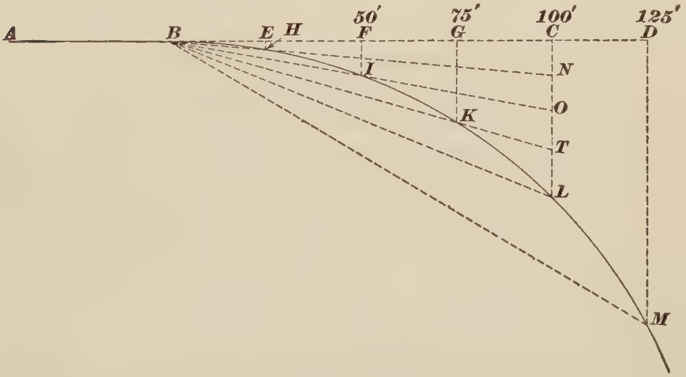


Fig. 27

The tangent to a curve is fixed in direction, being perpendicular to the radius at the point of tangency. Consequently, in a curve of given radius, the value of the tangent deflection depends wholly upon the length of the chord. Formula **14** shows that the tangent deflection is proportional to the square of the chord. Hence, knowing the tangent deflection for a chord of 100 feet, the following rule will give the tangent deflection for a chord of any other length:

Rule.—*Multiply the tangent deflection for a chord of 100 feet by the square of the given chord expressed as the decimal part of a chord of 100 feet.*

Example.—Let $B K M$, Fig. 27, represent a 2° curve. What are the tangent deflections for the chords $B H$, $B I$, $B K$, $B L$, and $B M$, having lengths of 25, 50, 75, 100, and 125 feet, respectively?

Solution.—For the chord $B L$ of 100 feet, the tangent deflection $C L$ is 1.745 feet. Hence, for the chords $B H = 25$ feet, $B I = 50$ feet, $B K = 75$ feet, and $B M = 125$ feet, the tangent deflections are, respectively, $E H = .25^2 \times 1.745 = .109$ foot, $F I = .50^2 \times 1.745 = .436$ foot, $G K = .75^2 \times 1.745 = .982$ foot, and $D M = 1.25^2 \times 1.745 = 2.727$ feet.

It is here assumed that the chords and corresponding tangents are of equal lengths. This is not strictly true, but is near enough when the degree of curve is small.

The above principle does not apply to chord deflections, however. The point G, Fig. 26, is in the prolongation of the chord $B C$, and the value of the chord deflection $G E$ is affected by the direction, and, consequently, by the length of $B C$. Since $O M$ and $O N$ are, respectively, perpendicular to the chords $B C$ and $C E$ at their middle points, and since the radius $O C$ is perpendicular to the tangent $C F$, we know that in the triangle $G C E$, the angle $G C F = \frac{1}{2} B O C$ and $F C E = \frac{1}{2} C O E$. Consequently, the triangle $G C E$ can be isosceles and similar to $C O E$ only when the angle $C O E = B O C$, that is, when $B C = C E$. Hence, formula **15** applies only *when the two chords preceding the station considered are of equal length.* When these chords are of different lengths, the chord deflection will be given closely by formula **15** if $\frac{1}{2} c\,(c + c')$ is substituted for c^2, where c' is the length of the second chord preceding the station. Or, if the *tangent* deflection f has been computed, the chord deflection d_o will be given closely by the formula

$$d_o = f\left(1 + \frac{c'}{c}\right) \qquad \textbf{(16)}$$

41. Laying Out Curves Without a Transit.—During construction, the engineer is often called upon to restore center stakes on a curve when the transit is not at hand. This can be accomplished reasonably well with a tape, as described in the following example.

Let it be assumed that, in Fig. 28, *A B* is a tangent and *B* is the P. C. of a 4° curve, and let it be required to locate each full station on the curve. The points *A* and *B* determine the direction of the tangent, the point *B* being the P. C., which is assumed to be at Station 8 + 25. For a 4° curve the regular chord deflection for 100 feet is $4 \times 1.745 = 6.98$ feet, and the tangent deflection is 3.49 feet.

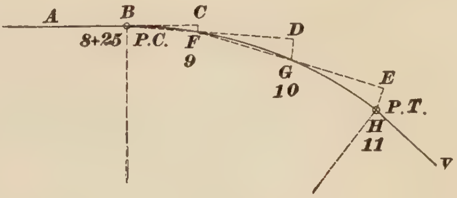


FIG. 28

The distance from the P. C. to the next station *C* is 75 feet; hence, the tangent deflection $CF = .75^2 \times 3.49 = 1.96$ feet (Art. **40**). The point *F* is found by first measuring 75 feet from *B*, thus locating the point *C* in the line *A B* prolonged, then from *C* measuring *C F*, = 1.96 feet, at right angles to *B C*; the point *F* thus determined will be Station 9. Next the chord *B F* is prolonged 100 feet to *D*; as *B F* is only 75 feet, formula **16** gives for *D G* the value $3.49 \times (1 + \frac{75}{100}) = 6.11$ feet. This distance is measured at right angles to *B D*; the point *G* thus determined will be Station 10. The point *H*, which is Station 11, and the P. T. of the curve, is determined in the same manner, except that, as the chords *F G* and *G H* are each 100 feet long, the regular chord deflection of 6.98 feet is used for *E H*. A stake is driven at each station thus located. Although a chord deflection is not at right angles to the chord theoretically, yet the deflection is so small, as compared with the length of the chord, that for curves of ordinary degree it is usually measured at right angles.

EXAMPLES FOR PRACTICE

NOTE.—In order that the student can compare his results, the answers to the following exercises are given to three decimal places, although two decimal places are sufficient in practice.

1. In a 5° curve, what are the tangent and chord deflections for a chord of 67 feet following one of 100 feet? Ans. $\left\{\begin{array}{l} f = 1.958 \text{ ft.} \\ d = 4.880 \text{ ft.} \end{array}\right.$

2. In a 7° 30′ curve, what are the tangent and chord deflections for a chord of 23.5 feet following one of 100 feet?

Ans. $\begin{cases} f = .361 \text{ ft.} \\ d = 1.897 \text{ ft.} \end{cases}$

3. In a 6° 15′ curve, what are the tangent and chord deflections for a chord of 100 feet following one of 84 feet?

Ans. $\begin{cases} f = 5.451 \text{ ft.} \\ d = 10.030 \text{ ft.} \end{cases}$

4. In an 8° 45′ curve, what are the tangent and chord deflections for a chord of 100 feet following one of 72 feet?

Ans. $\begin{cases} f = 7.628 \text{ ft.} \\ d = 13.120 \text{ ft.} \end{cases}$

MIDDLE ORDINATE

42. Relation Between Radius, Chord, and Middle Ordinate.—It is often convenient to know the ordinate to a curve at the middle point of a chord, commonly called the **middle ordinate** of the chord. Knowing the positions of two points on a curve, the point on the curve midway between them can be located easily by means of the middle ordinate of the chord connecting the two known points. Or, a point on the curve at a distance from either point equal to the distance between the points can be located, as will be explained farther on.

The radius of curvature, length of chord, and middle ordinate have a fixed relation, and when any two are given the other can easily be determined. The relation between these values can be established and expressed as follows:

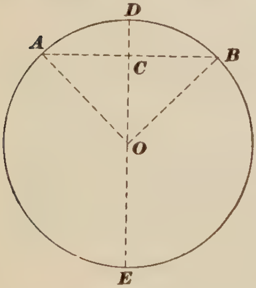


FIG. 29

Let AB, Fig. 29, be any chord of a circle whose radius is $OA = OB$, and let the chord DE be a diameter of the circle perpendicular to the chord AB at its middle point C. From geometry we know that when two chords of a circle intersect, the product of the two segments of one chord is equal to the product of the two segments of the other. Hence, we can at once write the equation

$$EC \times CD = AC \times CB$$

If we now denote the radius of the circle by R, the chord $A\,B$ by c and the middle ordinate $C\,D$ of this chord by m, then, since $C\,D = m$, $E\,C = 2\,R - m$, $A\,C = C\,B = \frac{c}{2}$, by substituting these values in the foregoing equation, we get

$$(2\,R - m)\,m = \frac{c^2}{4},$$

which easily becomes

$$2\,R\,m = \frac{c^2}{4} + m^2 \qquad \textbf{(17)}$$

From this equation we can easily write the value of any one of the quantities R, c, or m in terms of the other two, thus:

$$R = \frac{c^2}{8\,m} + \frac{m}{2} \qquad \textbf{(18)}$$

$$c = 2\sqrt{2\,R\,m - m^2} \qquad \textbf{(19)}$$

$$m = R - \sqrt{R^2 - \frac{c^2}{4}} \qquad \textbf{(20)}$$

These are fundamental formulas that apply exactly to all cases. Formula **20,** as derived algebraically from formula **17,** has both the + and − signs before the radical, but since m is always less than R, it is evident that the − sign must be used.

EXAMPLE 1.—What is the radius of a curve in which the middle ordinate to a chord of 60 feet is .71 foot?

SOLUTION.—Substituting known values in formula **18,** we have

$$R = \frac{60^2}{8 \times .71} + \frac{.71}{2} = 634.16 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 2.—In an 8° curve, what is the length of a chord whose middle ordinate is .69 foot?

SOLUTION.—The radius of an 8° curve as given in the Table of Radii and Deflections is 716.78. Substituting known values in formula **19,** we have

$$c = 2\sqrt{2 \times 716.78 \times .69 - .69^2} = 62.89 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. What is the radius of a curve in which the middle ordinate of a chord of 50 feet is .5 foot? Ans. 625.25 ft.

2. What is the length of the middle ordinate of a chord of 100 feet in a 6° curve? Ans. 1.31 ft.

3. The radius of a curve is 1,146.28 feet. What is the length of a chord whose middle ordinate is .55 foot? Ans. 71.01 ft.

4. What is the length of the middle ordinate to a chord of 100 feet in a curve whose radius is 800 feet? Ans. 1.56 ft.

TO DETERMINE DEGREE OF CURVE FROM MIDDLE ORDINATE

43. It is sometimes necessary to determine the radius or the degree of a curve in an existing track when no transit is available for measuring it. By measuring the middle ordinate of any convenient chord, the degree of the curve can be calculated from the relative values of the ordinate and chord. Since the track is likely not to be in perfect alinement, it is well to measure the middle ordinate of different chords in different parts of the curve. As also the middle ordinate of a chord measured to the inner rail will somewhat exceed the middle ordinate of the same chord measured to the outer rail, the ordinate of each chord should be measured to both rails and the average of the two taken as the value of the ordinate. Having measured the middle ordinate of one or more chords, the radius of curvature can be determined by applying formula **18.** For calculating the degree of curve either of the two following methods may be applied.

44. First Method.—Equating the value of R expressed by formula **11** with that expressed by formula **18,** we have

$$\frac{5{,}730}{D_c} = \frac{c^2}{8\,m} + \frac{m}{2}$$

Since any method of determining the degree of curve from the middle ordinate as measured on the rail can be only approximate by reason of the imperfect alinement of the track, and since also the last term of this equation has a very small value as compared with the other terms, it can be dropped from the equation without material error, so that by solving for D_c we shall have

$$D_c = \frac{45,840\ m}{c^2}, \qquad \textbf{(21)}$$

in which c and m are both measured in the same unit. If we denote the length of the chord, expressed as the decimal part of a station length of 100 feet, by c_{oo}, this may also be expressed by the closely approximate formula

$$D_c = \frac{55\ m}{12\ c^2_{oo}} \qquad \textbf{(22)}$$

Example.—Suppose that $A\ B$, Fig. 30, is a chord of 50 feet, and that its middle ordinate $a\ b$, as measured, is .44 of a foot. What is the degree of the curve?

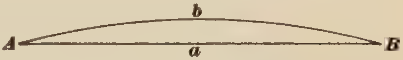


Fig. 30

Solution.—By substituting the given values in formula **21,** we have

$$D_c = \frac{45,840 \times .44}{50 \times 50} = 8.07 \text{ (nearly)}$$

Or, by substituting the given values in formula **22,** we have

$$D_c = \frac{55 \times .44}{12 \times .50 \times .50} = 8.07 \text{ (nearly)}$$

For this result it would be assumed that the original curve was an 8° curve.

45. Convenient Rules for Determining the Degree of Curve.—By reference to formula **21,** it will be seen that for any given length of chord c the degree of curve D_c varies directly as the middle ordinate m. Although this is not exactly the case, as was shown in the derivation of this formula, it is so nearly the case that for practical purposes the condition may be assumed. This assumption is of considerable practical value, as it affords a simple method of determining the degree of curve when the middle ordinate is measured to a chord of given length. For convenience, let formula **21** be written in the form

$$c^2\ D_c = 45,840\ m$$

For a chord of 100 feet in a 1° curve, c has a value of 100, while D_c is unity. By substituting these values in this equation and solving for the value of m, we get

$$m = \frac{100 \times 100}{45,840} = .218, \text{ very closely,}$$

which is the value of the middle ordinate to a chord of 100 feet in a 1° curve. As just stated, formula **21** shows that for any given length of chord the degree of curve varies directly as the middle ordinate. Hence, we have the following rule for determining the degree of a curve:

Rule I.—*Measure the middle ordinate to a chord of 100 feet, express it in feet and decimals of a foot, and divide by .218; the quotient will be the degree of the curve.*

This is a convenient rule except that the value .218 is not a very convenient divisor. We can choose any convenient value for the ordinate, however, and by substituting it in formula **21**, determine the length of chord in a 1° curve for which the value chosen is the middle ordinate. Remembering that D_c is equal to unity, by substituting a value of .2 for m in the above equation and solving for the value of c, we get

$$c = \sqrt{45,840 \times .2} = 95.75, \text{ very closely,}$$

which, in a 1° curve, is the length of chord, expressed in feet and decimals of a foot, whose middle ordinate has a value of .2 of a foot. Hence, in a curve of any degree, the degree of curve is equal to the middle ordinate of a chord of this length divided by .2, and since dividing .2 is the same as multiplying by 5, the following rule may be used for determining the degree of a curve:

Rule II.—*Measure the middle ordinate to a chord of 95.75 feet, express it in feet and decimals of a foot, and multiply by 5; the result will be the degree of the curve.*

In like manner, if we substitute a value of .1 for m in the above equation and solve for the value of c, we shall get

$$c = \sqrt{45,840 \times .1} = 67.71, \text{ very closely,}$$

which, in a 1° curve, is the length of chord, expressed in feet and decimals of a foot, whose middle ordinate has a value of .1 of a foot. Hence, in a curve of any degree, the degree of curve is equal to a chord of this length divided by .1, and since dividing by .1 is the same as multiplying by 10, we have the following convenient rule for determining the degree of a curve:

Rule III.—*Measure the middle ordinate to a chord of 67.71 feet, express it in feet and decimals of a foot, and multiply by 10; the result will be the degree of the curve.*

Again, if we substitute a value of 1 inch, $= \frac{1}{12}$ foot, for m in the preceding equation and solve for the value of c, we shall get

$$c = \sqrt{45{,}840 \times \tfrac{1}{12}} = 61.81, \text{ very closely,}$$

which, in a 1° curve, is the length of chord, expressed in feet and decimals of a foot, whose middle ordinate has a value of 1 inch. Hence, the following rule will give the degree of a curve directly:

Rule IV.—*Measure the middle ordinate to a chord of 61.81 feet, express it in inches and decimals of an inch; the result will be the degree of the curve.*

46. Second Method.—Let $C D$, Fig. 31, be the middle ordinate of any given chord $A B$ of the curve $A D B$, and let $A D$ be a chord drawn from the extremity A of the given chord to the point D, where its middle ordinate intersects the curve. If we draw the radial line $O E$ to the middle point E of the chord $A D$, it will evidently divide the angle $A O D$ into two equal parts, so that $A O E = E O D = \frac{1}{2} A O D = \frac{1}{4} A O B$. (See Art. **26,** 5.) From the principle stated in Art. **26,** 7, it is also evident that this radial line $O E$ is perpendicular to the chord $A D$, and since the triangles $A D C$ and $O D E$ each have one right angle, while

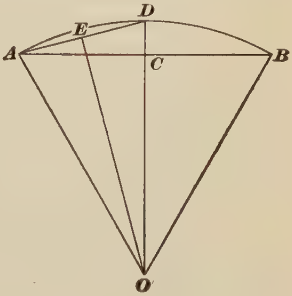


FIG. 31

the angle $A\,D\,O$ is common to both, the triangles are similar and the angle $D\,A\,C$ is equal to the angle $D\,O\,E$, which is equal to the angle $A\,O\,E$, and to one-fourth the angle $A\,O\,B$ subtended by the given chord. We can therefore write

$$\frac{D\,C}{A\,C} = \tan D\,A\,C = \tan \tfrac{1}{4}\,A\,O\,B$$

Since $A\,C$ is one-half the given chord, $D\,C$ is the middle ordinate and $A\,O\,B$ is the angle subtended by the given chord; if we denote the given chord by c, its middle ordinate by m, and the central angle subtended by the chord by a, this expression may be written

$$\frac{m}{\frac{1}{2}\,c} = \tan \tfrac{1}{4}\,a \qquad \textbf{(23)}$$

This is a general formula applying to any chord of a circle and may be expressed by the following general principle: *The quotient obtained by dividing the middle ordinate to any chord by one-half the chord is equal to the natural tangent of one-fourth the central angle subtended by the chord.*

Since the degree of a curve is the central angle subtended by a chord of 100 feet, and since dividing the middle ordinate by one-half the chord is the same as dividing twice the middle ordinate by the whole chord, we may write the following rule for determining the degree of curve by means of the middle ordinate to a chord of 100 feet:

Rule I.—*Twice the middle ordinate to a chord of 100 feet, divided by 100, is equal to the natural tangent of one-fourth the degree of curve.*

For chords of lengths other than 100 feet, the quotient obtained by dividing twice the middle ordinate by the chord, formula **23**, will be the tangent of one-fourth the central angle subtended by the chord. In order to obtain the degree of curvature in such cases, the following rule may be applied, which, though not exact, will give it very closely.

Rule II.—*Multiply the central angle by 100 and divide the product by the length of the chord employed.*

Example.—Suppose that $A B$, Fig. 30, is a chord of 50 feet, and that the middle ordinate $a b$ is .44 of a foot. What is the degree of curve?

Solution.—By substituting the given values in formula **23**, we have

$$\frac{2 \times .44}{50} = .0176 = \tan \tfrac{1}{4} a = \tan 60\tfrac{1}{2}' \text{ (closely)},$$

from which we get for the central angle the value

$$a = 4 \times 60\tfrac{1}{2}' = 242',$$

and by applying the preceding rule, we have for the degree of curve

$$242' \times \tfrac{100}{50} = 484' = 8° \, 4'. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

Note.—The answers to the following exercises can be obtained by either of the methods just explained. In order to obtain the exact answers by the second method, however, the angle must be determined to the nearest second, although results sufficiently accurate for the purpose can be obtained by taking the angle to the nearest minute.

1. The length of chord is 50 feet, the middle ordinate is .35 foot; what is the degree of curve? Ans. 6° 25′

(The original curve probably 6° 30′.)

2. The length of chord is 40 feet, the middle ordinate is .21 foot; what is the degree of curve? Ans. 6.02°

(The original curve probably 6°.)

3. The length of chord is 25 feet, the middle ordinate is .22 foot; what is the degree of curve? Ans. 16.13°

(The original curve probably 16°.)

4. The length of chord is 35 feet, the middle ordinate is .27 foot; what is the degree of curve? Ans. 10.10°

(The original curve probably 10°.)

OTHER USES OF MIDDLE ORDINATE

47. Middle Ordinate to Chord of Two Stations.—Let $A D B$, Fig. 32, be a portion of any circular curve, in which $A D, = D B$, is the regular distance between stations, usually 100 feet, and $A B$ is a chord connecting two alternate stations. If we let c_2 denote the chord $A B$, m_2 denote its middle

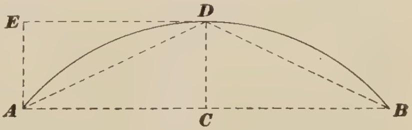


FIG 32

ordinate CD, and R denote the radius of the curve, then by formula **17** we have

$$2Rm_2 = \frac{c_2^2}{4} + m_2^2$$

But since $\frac{c_2}{2}$, $= AC$, is the base, and m_2, $= CD$, is the altitude, of the right triangle ACD, if by c we denote the regular station chord AD, which is the chord of half the arc ADB and the hypotenuse of the triangle, we can write the equation

$$\frac{c_2^2}{4} + m_2^2 = c^2,$$

and by substituting c^2 for the last member of formula **17** as written above, this equation becomes

$$2Rm_2 = c^2,$$

from which

$$m_2 = \frac{c^2}{2R} \qquad \textbf{(24)}$$

This equation gives the same value for m_2 as formula **14** gives for f, which should evidently be the case. For, if we draw a line DE tangent to the curve at the middle point D of the arc, it will be parallel to the chord of the whole arc AB, so that the perpendicular distance AE between the extremity of the chord and the extremity of the tangent will be equal to the middle ordinate CD.

48. To Lay Out a Curve by Middle Ordinate.—One of the most expeditious methods of laying out a curve without the aid of a transit is by means of the middle ordinate to a chord that subtends an arc of twice the length subtended by the chord for a regular station.

Let AB, Fig. 33, be a tangent, and B the P. C. of the curve $BCDE$, which it is desired to lay out on the ground when no transit is available. The distance BC', equal to 100 feet or one station length, is measured in line with the tangent AB, the head chainman keeping in range by means of a stake at B and a flag held at A, and a temporary stake is driven at the extremity C' of the measured distance. The tangent deflection $C'C$ is then calculated by formula **14**; this distance is measured at right angles to BC', the prolongation of the tangent, and a stake is driven at the point C on the curve. The middle ordinate CM of the long chord BD is then measured as nearly at right angles to the chord as can be judged and a temporary stake is set at M. The middle ordinate CM can be calculated by formula **24**; it is equal to the tangent deflection $C'C$, as has been shown. The rear flag is then held at B, the rear end of the chain is held at C, and the head chainman stretches out the chain and swings the forward end around until it is in range with the stake at M and the flag at B, thus determining the point D on the curve, at which a stake is then driven. Continuing, a temporary stake is set at the extremity N of the middle ordinate to the long chord CE as measured from D, and, with the rear end of the chain held at D, the forward end is brought in range with this temporary stake and the flag held at C, thus determining the point E. The remaining points on the curve are located in the same manner.

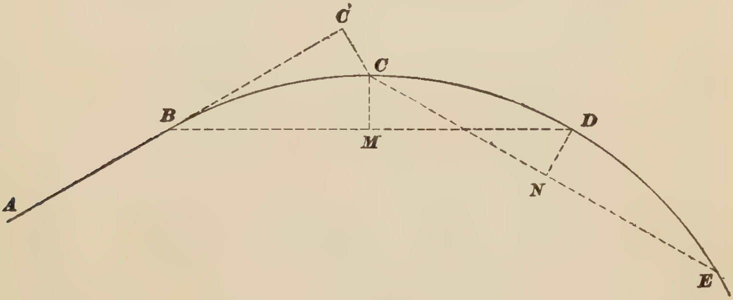


FIG. 33

49. Ranging in the Head Flag on Curves.—The facility with which a curve can be laid out in the field will depend largely on the ability of the head flagman, who is also the head chainman, to keep in line and find quickly the approximate positions of the various points on the curve at which stakes are to be driven, so that when lined in at each point by means of signals from the transitman, it will only be necessary to move his flag a few inches in order to be at the true point on the curve. In order to facilitate the field work of laying out curves, the following method is suggested to aid the head chainman in keeping the line.

Let us suppose that the curve is 6° and is to be laid out, as shown in Fig. 34, with stakes set at intervals of 50 feet.

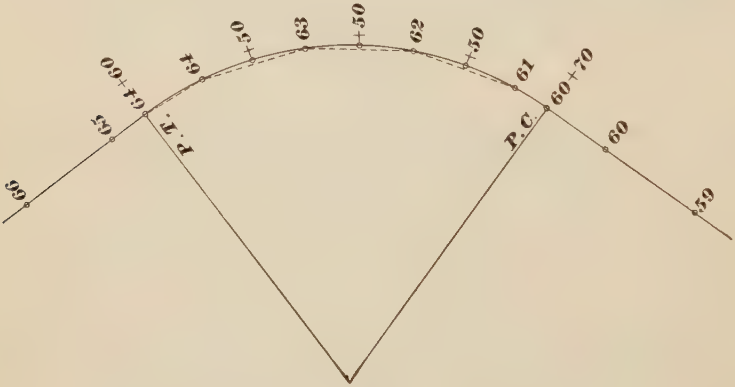


FIG. 34

With the instrument set up over the P. C., which is at Station 60 + 70, the first stake to be set is at Station 61, which is 30 feet from the P. C. The rear chainman holds the 30-foot mark on the chain at the P. C., and the head chainman holds his flag at the forward end of the chain and moves it to the right or left, according as the transitman may signal, until at the point for Station 61, at which point the stake for this station is driven. The rear chainman now moves up to this station and holds the 50-foot mark on the chain at the stake just set, while the head chainman with his flag held at the forward end of the chain gets the point for Station 61 + 50. The rear chainman remaining at Station 61 allows the chain to be drawn forward to its full length of 100 feet and then holds the rear end at the stake, while the head chainman measures the distance and is ready to get the point for Station 62. He ranges in his flag by holding it in such a position that by sighting from the flag to Station 61 the line of sight passes in front of the stake at Station 61 + 50 by an amount about equal to the middle ordinate of the given curve for a chord of 100 feet; the correct line is then given by the transitman. The rear chainman then moves up to Station 62 and holds the 50-foot mark on the chain at this station while the head chainman gets the point for Station 62 + 50, as before, and then holds the rear end of the chain at the same station while the head chainman gets the point for Station 63, and so on around the curve to Station 64, which is the last full station on the curve. He then holds the 60-foot mark on the chain at Station 64 while the head chainman gets the point for the P. T., which is at Station 64 + 60. In this case, the P. T. is only 10 feet beyond 64 + 50, and it will therefore be unnecessary to set a stake at the latter point, since the additional 10 feet in the length of the chord is unimportant.

Before starting to run out the curve, the head chainman should ascertain the value of the middle ordinate for a chord of 100 feet in a curve of the given degree. He can obtain this from the transitman, or, when the stakes are to be set at intervals of 50 feet, can himself calculate it by multiplying the degree of curve by the decimal .218, or, near enough for this purpose, by the decimal .22; the result will be the middle ordinate in feet. (See Art. **45.**) By estimating with the eye a distance from the preceding station equal to the ordinate, and ranging in the flag in the manner just described, the head chainman, after a little practice, can place his flag within a few inches of the required point, unless the curve happens to be on unusually rough ground.

If stakes are set on the curve only at the regular stations, 100 feet apart, substantially the same method can be employed for ranging in the head flag after the first two consecutive stakes 100 feet apart have been set. In this case the middle ordinate of the long chord, or the distance from the preceding stake to the line of sight from the flag to the second preceding stake, can be calculated by formula **24,** by merely substituting the value of c and R. In a 1° curve, the middle ordinate for a chord of two stations, as given by this formula, is equal to $\frac{100 \times 100}{2 \times 5{,}730} = .873$ foot. Hence, the ordinate for ranging in the head flag on a curve, when the stakes are set at the regular stations only, can be found near enough for the purpose by multiplying the degree of curve by the decimal .87; the result will be the ordinate in feet.

PASSING OBSTACLES ON CURVES

50. Different methods may be employed for passing obstacles on curves, depending on the conditions encountered and the ingenuity of the engineer in adapting the method to them. Three of the simplest and most common of the methods employed will be explained.

Suppose that it is required to run out the curve *A E H*, Fig. 35, with several obstacles in the direct line of the curve, as shown, Station 3 being the P. C., and the regular stations on the curve being in the positions indicated by the numbers 4, 7, 8, etc. The positions of Stations 5 and 6 are indicated by the letters *C* and *D*, the figures 5 and 6 being omitted for want of space. The stations are to be located in their proper positions on the curve, between the obstructions, wherever it is possible to do so. In addition to this, it is customary to mark with a tack or otherwise the point where the line of the curve intersects each obstruction.

Beginning at the point of curve *A*, which is at Station 3, we can run in the curve as far as the first obstruction, which is the building *P*, setting the stakes on the curve at Stations 4 and 5, and a tack in the side of the building *P* at the point where the line of curve intersects it, according to the deflection angle as determined by its distance from Station 5. We cannot proceed further in the regular manner, however, because Station 6 cannot be seen from the P. C. The most expeditious method of passing the buildings and locating the subsequent stations on the curve will depend on the conditions encountered. The three methods here explained are all susceptible of more or less modification.

51. First Method: by Equilateral Triangle.—The easiest way of passing a single obstacle in the line of the curve, such as the building *P*, Fig. 35, is by means of an equilateral triangle. The operations are performed as explained in the following paragragh.

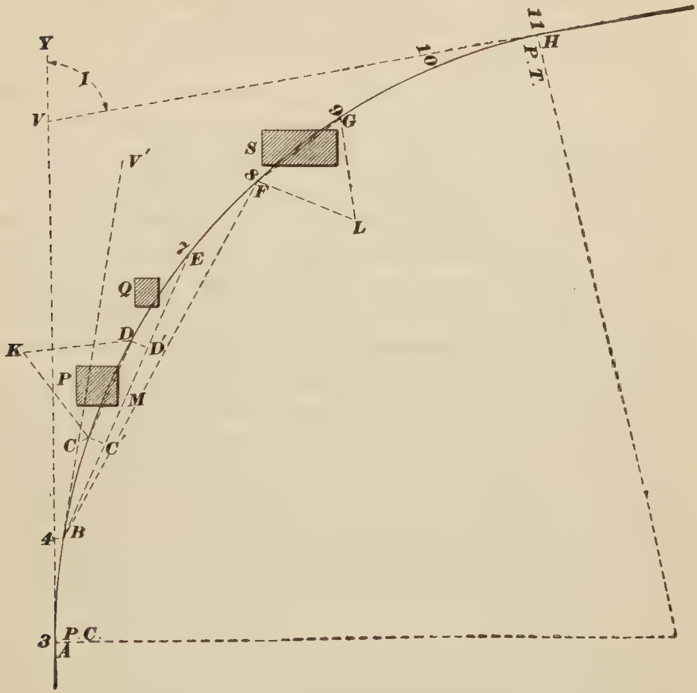


Fig. 35

The instrument is moved forward and set up at the point *C*, which is at Station 5, is sighted back to the P. C., which is at Station 3, the telescope is reversed, and the deflection angle for Station 6 is turned off the same as if no obstruction existed; the telescope will now be sighted on the line *C D*, although the point *D*, or Station 6, will not be visible, because the building *P* obstructs the view. The angle *D C K*, equal to 60°, is then turned in either direction, as may be most advantageous for passing the building; in the figure it is shown turned to the left. The distance CK is then measured on the line thus determined, equal to the distance CD, in this case 100 feet, and a transit point K is set in the line at the extremity of this distance. The transit is then moved forward and set up at K, sighted back on C, and the angle CKD, equal to 60°, turned to the left, giving the line KD. A distance equal to CK, in this case 100 feet, is measured on this line, and a transit point D is set at the extremity of the measurement. This point will be Station 6 on the line of the curve and in its proper position at a distance of 100 feet from Station 5. The transit is now moved forward and set up at D, the vernier set at the reading for Station 5, and an angle of 60° turned to the right from this reading. The telescope is then backsighted on K, and the vernier is turned back to the regular deflection for Station 6. The instrument is now sighted on a tangent to the curve at Station 6, and any point on the curve visible from this station can easily be established by turning off a deflection angle corresponding to its distance from the instrument. The point back on the curve where the line of curve intersects the building P is marked by a tack or otherwise, as is also the point forward on the curve where the line of curve intersects the building Q. No other stations can be seen from Station 6, however, though the point for Station 7 can be set from Station 6 by the method just explained, and from this point Station 8 can be located directly. Station 9 cannot be located from Station 7 by reason of the intervening building S, but by setting up the transit at Station 8, the building can be passed and this station located, and the remainder of the curve can then be completed regularly.

52. Second Method: by Long Chord.—With the conditions as shown in Fig. 35, however, Stations 5, 6, 7, and 8 can be located from the point B, which is Station 4. Station 7 can be located by means of the long chord BE and its deflection angle $V'BE$, turned from the tangent BV' to the curve at B. Stations 5 and 6 can then be located by means of the ordinates $C'C$ and $D'D$ measured to the curve at right angles to the chord BE from the points C' and D' located on the chord. Station 8 can be located by means of the deflection angle $V'BF$ and the chord EF, both of which are measured in the regular manner. The operations in detail and the calculations necessary to locate Stations 5 and 6 are as follows:

With the transit set at B, the method of continuous vernier is used as explained in Art. **38**, and the deflection corresponding to Station 7 is turned. If it is found that the line of sight will miss both obstructing buildings P and Q, the distance BE is measured, and the stake for Station 7 is set in the line given by the instrument. The length of the chord BE can be calculated from the deflection angle $V'BE$ that is turned from the tangent $V'B$, in order to give the direction of the chord. From formula **12**, we have for the length of the chord BE the value

$$c = 2R \sin D$$

In this case the distance from B, measured on the curve, is three full stations, and therefore the distance BE can be taken from a table of long chords, such as may be found in most engineers' field books.

If desired, Stations 5 and 6 can be located from the chord BE by calculating the ordinates $C'C$ and $D'D$ to the curve at Stations 5 and 6, respectively, and the distances MC' and MD' from the center M of the chord to each respective ordinate, as measured along the chord. In this case these ordinates are of equal length, since they are at equal distances from the center M of the long chord BE, but the same general method will apply to any case.

From Fig. 36, which represents in a somewhat exaggerated manner the portion of the curve between Station 4 and 7, it is evident that $C'C = MP - NP$. Since the angle BOP is equal to the angle $V'BE$, which is the deflection angle D for the long chord BE, and the distance $OM = OB \times \cos BOP = R \cos D$ we can write

$$MP = R\,(1 - \cos D)$$

Also, since in this case the arc CP is one-half of the arc CD, which latter arc is equal to the arc BC, the

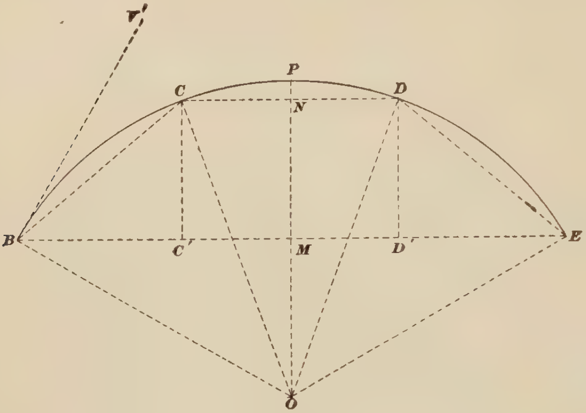


FIG. 36

angle COP is equal to one-third the angle BOP, or $\frac{1}{3}D$, we can also write

$$NP = R\,(1 - \cos\tfrac{1}{3}D)$$

Hence, we have for the ordinate $C'C$ the value

$$C'C = R\,(1 - \cos D) - R\,(1 - \cos\tfrac{1}{3}D) = R\,(\cos\tfrac{1}{3}D - \cos D)$$

The value of the ordinate $D'D$ is found in the same manner, and in this case is the same.

In order to locate the point C' on the chord BE, we have

$$MC' = NC = R\sin\tfrac{1}{3}D$$

The distance MD', locating the point D', is found in the same manner, and in this case is the same.

Having determined these values, in order to locate Stations 5 and 6, we have merely to bisect the chord BE and lay off the distances MC' and MD' on it from the middle point, then measure the ordinates $C'C$ and $D'D$ from the points so determined, thus locating the points C and D, which are the stations required. Or, what amounts to the same thing, we measure along the chord BE the distance BC', equal to one-half the length of the chord minus the distance MC', thus locating the point C', and from this point measure the distance $C'D'$, equal to twice the distance MC', thus locating the point D'; the ordinates $C'C$ and $D'D$ are then measured from these points.

53. Third Method: by Concentric Parallel Curve. Let it be required to run a curve $ABCD$, Fig. 37, connecting

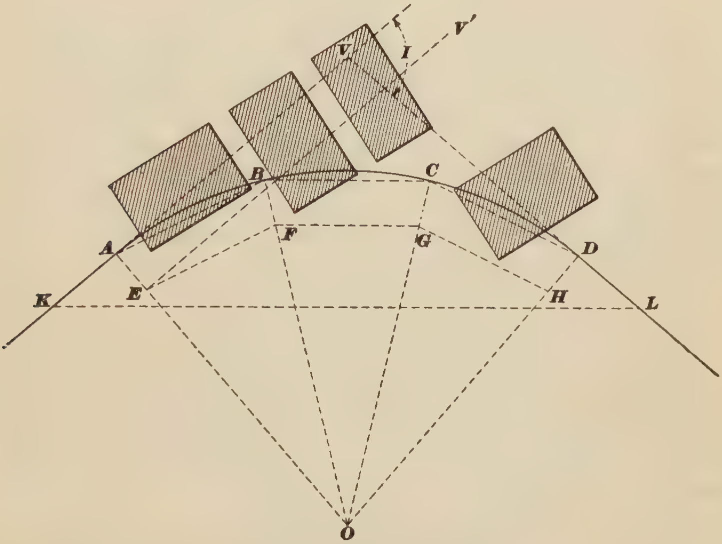


FIG. 37

the tangents KA and DL, and suppose that obstacles lie directly in the path of the curve, and that the point of intersection V is also inaccessible, as shown. A concentric parallel curve $EFGH$ may be run in such position as to avoid the obstacles, and from the stations on this parallel curve the corresponding stations on the required curve can be located. The intersection angle I is equal to the sum of the angles VKL and VLK, and by measuring these angles and the distance KL, the distances VK and VL can be calculated by the general principles of sines. The tangent distances VA and VD can then be calculated by formula **13.** The distance KA being the difference between the distances VK and VA, and the distance LD the difference between the distances VL and VD, the points A and D, which are, respectively, the P. C. and P. T. of the curve, can at once be located.

Having located the P. C. and P. T., the equal distances AE and DH are measured perpendicular, respectively, to the tangents KA and DL and of such length that the parallel curve $EFGH$ connecting their extremities E and H will avoid the obstacles, and transit points are set at E and H in the positions thus determined. It is evident that while the angle AOD remains constant, the chords EF, FG, and GH in the curve $EFGH$ are shorter than the corresponding chords AB, BC, and CD of the required curve, since they are nearer the common center O of the two curves. The triangles AOB and EOF are similar and therefore their sides are proportional. Hence,

$$OA : OE = AB : EF$$

If we denote the radius OA of the required curve by R_1, the radius OE of the parallel curve by R_2, the chord AB of the required curve by c_1, and the corresponding chord EF of the parallel curve by c_2, then from this proportion we can write the formula

$$c_2 = c_1 \frac{R_2}{R_1} \qquad \textbf{(25)}$$

The deflection angle $V'EF$ is equal to VAB, because the corresponding sides are parallel, and for similar reasons all the deflection angles of the parallel curve are the same as the corresponding ones for the required curve. The transit is set up at E, backsighted to A, and the angle AEV', equal to 90°, is turned, giving the direction of the tangent EV', from which the deflection angles for the curve $EFGH$ are turned off in the regular manner and a transit point is set at each station by measuring each chord EF, FG, and GH of the proper length, as calculated by formula **25.** The transit is then set at a point on the parallel curve, as at F, and by backsighting to another point on the curve, as *E*, and turning off the deflection angle, the line of sight is directed on a tangent to the curve at the instrument point. Then, by turning a right angle from this tangent and measuring from *F* the distance *F B* equal to *A E*, a stake may be set locating the point *B* on the required curve. In a similar manner each of the stations on the required curve can be located.

Example.—Suppose the required curve *A B C D*, Fig. 37, is a 7° curve and that the offset distance *A E*, = *D H*, necessary to avoid the obstacles, is 90 feet, what is the length of each of the chords *E F*, *F G*, and *G H* of the parallel curve in order that the corresponding chords of the required curve will each be 100 feet in length?

Solution.—The radius of a 7° curve is 819.02 feet, which is the radius $O A = R_1$; the radius $O E$ is therefore $819.02 - 90 = 729.02 = R_2$. The length of each of the chords *A B*, *B C*, and *C D* is 100 feet, $= c_1$. By substituting these values in formula **25**, the length of each of the chords *E F*, *F G*, and *G H* is found to be equal to

$$c_2 = 100 \times \frac{729.02}{819.02} = 89.01 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Determine the middle ordinate to a chord connecting two alternate stations in a 4° curve, the consecutive stations being 100 feet apart. Ans. 3.49 ft.

2. The degree of curve is 5°; what is the middle ordinate to a chord of 300 feet? Ans. 9.81 ft.

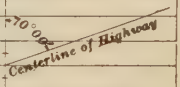
3. Assuming that *A E H*, Fig. 35, is a 6° curve, and that the stations are 100 feet apart, determine the length of the long chord *B E*, the ordinate *C′ C*, and the distance *M C′*.

$$\text{Ans.} \begin{cases} B E = 298.90 \text{ ft.} \\ C' C = 10.45 \text{ ft.} \\ M C' = 50.00 \text{ ft.} \end{cases}$$

4. Referring to Fig. 37, let the required curve *A B C D* be a 4° curve, let the chords *A B*, *B C*, and *C D* each be 50 feet in length, and let the offset distance *A E*, = *D H*, necessary to avoid the obstacles, be 75 feet. Determine the length of the chords *E F*, *F G*, and *G H* of the parallel curve. Ans. 47.38 ft.

FIELD NOTES FOR CURVES

54. As stated in another section, various styles of field notebooks are published, in which the pages are ruled to suit the different kinds and methods of field work. The following, which are the field notes of a portion of a line containing a curve, represents a good form for recording the field notes of a curve that is run in by the method of zero tangent explained in Art. **37.**

Station.	Deflection.	Tot. Angle.	Mag. Bearing.	Ded. Bearing.	Remarks.	June 20. 1894
9						
8						
7						
6+95	4°54′ P.T.	15°00′	N. 35°20′ E.	N. 35°15′ E.		
6+50	4°00′					
6	3°00′					
5+50	2°00′				5+80	
5	1°00′				5+60	
4+50	2°36′	5°12′				
4	1°36′				Int. Angle = 15°00′	4° Curve R
3+50	0°36′				T = 188.61 ft.	Def. Angle for 50 ft. 1°00′
3+20	P.C. 4° R				P.C. = 3+20	Def. Angle for 1 ft. = 1.2′
3					Length of Curve = 375 ft.	
2					P.T. = 6+95	
1						
0			N. 20°15′ E.	N. 20°15′ E.		

In the first column the station numbers are recorded. In the second column are recorded the deflections with the abbreviations P. C. and P. T., together with the degree of curve and the abbreviation *R* or *L*, according as the line curves to the right or left. At each transit point on the curve, the total or central angle from the P. C. to that point is calculated and recorded in the third column. This total angle is double the deflection angle between the P. C. and the transit point. In the above notes, there is but one intermediate transit point between the P. C. and the P. T. The deflection from the P. C. at Station 3 + 20 to the intermediate transit point at Station 4 + 50 is 2° 36′. The total angle is double this deflection, or 5° 12′, which is recorded on the same line in the third column. The record of total angles at once indicates the stations at which transit points are placed. The total angle at the P. T. will be the same as the angle of intersection, if the work is correct. When the curve is finished, the transit is set up at the P. T., and the bearing of the forward tangent taken, which affords an additional check upon the previous calculations. The magnetic bearing is recorded in the fourth column, and the deduced, or calculated, bearing is recorded in the fifth column.

When the method by continuous vernier is employed for running in the curve, the notes will be substantially the same as given, with the exception that the deflection angles will increase continuously from the P. C. throughout the entire curve by adding the proper amount for each station to the deflection angle for the preceding station. The deflection angles will then have the values 3° 36′, 4° 36′, 5° 36′, 6° 36′, and 7° 30′ for Stations 5, 5 + 50, 6, 6 + 50, and 6 + 95 of the foregoing notes, respectively, instead of starting anew for Station 5.

TABLE OF RADII AND DEFLECTIONS

Degree	Radii	Chord Deflection	Tangent Deflection
° ′			
0 5	68754.94	.145	.073
10	34377.48	.291	.145
15	22918.33	.436	.218
20	17188.76	.582	.291
25	13751.02	.727	.364
30	11459.19	.873	.436
35	9822.18	1.018	.509
40	8594.41	1.164	.582
45	7639.49	1.309	.654
50	6875.55	1.454	.727
55	6250.51	1.600	.800
1 0	5729.65	1.745	.873
5	5288.92	1.891	.945
10	4911.15	2.036	1.018
15	4583.75	2.182	1.091
20	4297.28	2.327	1.164
25	4044.51	2.472	1.236
30	3819.83	2.618	1.309
35	3618.80	2.763	1.382
40	3437.87	2.909	1.454
45	3274.17	3.054	1.527
50	3125.36	3.200	1.600
55	2989.48	3.345	1.673
2 0	2864.93	3.490	1.745
5	2750.35	3.636	1.818
10	2644.58	3.781	1.891
15	2546.64	3.927	1.963
20	2455.70	4.072	2.036
25	2371.04	4.218	2.109
30	2292.01	4.363	2.181
35	2218.09	4.508	2.254
40	2148.79	4.654	2.327
45	2083.68	4.799	2.400
50	2022.41	4.945	2.472
55	1964.64	5.090	2.545
3 0	1910.08	5.235	2.618
5	1858.47	5.381	2.690
10	1809.57	5.526	2.763
15	1763.18	5.672	2.836
20	1719.12	5.817	2.908
25	1677.20	5.962	2.981
30	1637.28	6.108	3.054
35	1599.21	6.253	3.127
40	1562.88	6.398	3.199
45	1528.16	6.544	3.272
50	1494.95	6.689	3.345
55	1463.16	6.835	3.417
4 0	1432.69	6.980	3.490
5	1403.46	7.125	3.563
10	1375.40	7.271	3.635
15	1348.45	7.416	3.708
20	1322.53	7.561	3.781
25	1297.58	7.707	3.853
30	1273.57	7.852	3.926
35	1250.42	7.997	3.999
40	1228.11	8.143	4.071
45	1206.57	8.288	4.144
50	1185.78	8.433	4.217
55	1165.70	8.579	4.289
5 0	1146.28	8.724	4.362
5	1127.50	8.869	4.435
10	1109.33	9.014	4.507

Degree	Radii	Chord Deflection	Tangent Deflection
° ′			
5 15	1091.73	9.160	4.580
20	1074.68	9.305	4.653
25	1058.16	9.450	4.725
30	1042.14	9.596	4.798
35	1026.60	9.741	4.870
40	1011.51	9.886	4.943
45	996.87	10.031	5.016
50	982.64	10.177	5.088
55	968.81	10.322	5.161
6 0	955.37	10.467	5.234
5	942.29	10.612	5.306
10	929.57	10.758	5.379
15	917.19	10.903	5.451
20	905.13	11.048	5.524
25	893.39	11.193	5.597
30	881.95	11.339	5.669
35	870.79	11.484	5.742
40	859.92	11.629	5.814
45	849.32	11.774	5.887
50	838.97	11.919	5.960
55	828.88	12.065	6.032
7 0	819.02	12.210	6.105
5	809.40	12.355	6.177
10	800.00	12.500	6.250
15	790.81	12.645	6.323
20	781.84	12.790	6.395
25	773.07	12.936	6.468
30	764.49	13.081	6.540
35	756.10	13.226	6.613
40	747.89	13.371	6.685
45	739.86	13.516	6.758
50	732.01	13.661	6.831
55	724.31	13.806	6.903
8 0	716.78	13.951	6.976
5	709.40	14.096	7.048
10	702.18	14.241	7.121
15	695.09	14.387	7.193
20	688.16	14.532	7.266
25	681.35	14.677	7.338
30	674.69	14.822	7.411
35	668.15	14.967	7.483
40	661.74	15.112	7.556
45	655.45	15.257	7.628
50	649.27	15.402	7.701
55	643.22	15.547	7.773
9 0	637.27	15.692	7.846
5	631.44	15.837	7.918
10	625.71	15.982	7.991
15	620.09	16.127	8.063
20	614.56	16.272	8.136
25	609.14	16.417	8.208
30	603.80	16.562	8.281
35	598.57	16.707	8.353
40	593.42	16.852	8.426
45	588.36	16.996	8.498
50	583.38	17.141	8.571
55	578.49	17.286	8.643
10 0	573.69	17.431	8.716
10	564.31	17.721	8.860
20	555.23	18.011	9.005
30	546.44	18.300	9.150
40	537.92	18.590	9.295

Degree	Radii	Chord Deflection	Tangent Deflection
° ′			
10 50	529.67	18.880	9.440
11 0	521.67	19.169	9.585
10	513.91	19.459	9.729
20	506.38	19.748	9.874
30	499.06	20.038	10.019
40	491.96	20.327	10.164
50	485.05	20.616	10.308
12 0	478.34	20.906	10.453
10	471.81	21.195	10.597
20	465.46	21.484	10.742
30	459.28	21.773	10.887
40	453.26	22.063	11.031
50	447.40	22.352	11.176
13 0	441.68	22.641	11.320
10	436.12	22.930	11.465
20	430.69	23.219	11.609
30	425.40	23.507	11.754
40	420.23	23.796	11.898
50	415.19	24.085	12.043
14 0	410.28	24.374	12.187
10	405.47	24.663	12.331
20	400.78	24.951	12.476
30	396.20	25.240	12.620
40	391.72	25.528	12.764
50	387.34	25.817	12.908
15 0	383.06	26.105	13.053
10	378.88	26.394	13.197
20	374.79	26.682	13.341
30	370.78	26.970	13.485
40	366.86	27.258	13.629
50	363.02	27.547	13.773
16 0	359.26	27.835	13.917
10	355.59	28.123	14.061
20	351.98	28.411	14.205
30	348.45	28.699	14.349
40	344.99	28.986	14.493
50	341.60	29.274	14.637
17 0	338.27	29.562	14.781
10	335.01	29.850	14.925
20	331.82	30.137	15.069
30	328.68	30.425	15.212
40	325.60	30.712	15.356
50	322.59	31.000	15.500
18 0	319.62	31.287	15.643
10	316.71	31.574	15.787
20	313.86	31.861	15.931
30	311.06	32.149	16.074
40	308.30	32.436	16.218
50	305.60	32.723	16.361
19 0	302.94	33.010	16.505
10	300.33	33.296	16.648
20	297.77	33.583	16.792
30	295.25	33.870	16.935
40	292.77	34.157	17.078
50	290.33	34.443	17.222
20 0	287.94	34.730	17.365

STADIA AND PLANE-TABLE SURVEYING

STADIA SURVEYING

PRELIMINARY DEFINITIONS AND PRINCIPLES

1. Definitions.—The term **stadia surveying** is commonly applied to a method of surveying in which distances are determined by observing what length of a graduated rod, held on a distant point, is included between two horizontal cross-wires in a telescope (usually that of a transit or a plane table) through which the rod is viewed. The rod is called a **stadia rod,** and the cross-wires are called **stadia wires.** The length of the stadia rod included between the stadia wires bears a certain fixed relation to the distance of the rod from the instrument.

Stadia measurement is of great value in all kinds of field work, especially in the minor operations of topographic and hydrographic surveys. It is the quickest and best way of measuring distances of any considerable length where great accuracy is not required.

2. Optical Principles Involved.—In Fig. 1, $L\,N$ represents a double convex lens, the faces of which are spherical surfaces having the same curvature. The line O_1O_2 passing through the centers of these surfaces is called the **principal axis** of the lens. It is shown in physics that rays of light r, r, r, parallel to the axis of the lens, converge, after being refracted, at a point F on the axis. This point is called

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the **principal focus** of the lens. The distance $F C_2$ from the principal focus to the lens is called the **principal focal distance** of the lens, and is generally represented by f.

It is also shown in physics that the rays from any point O_2 on the principal axis converge, after refraction, at another point O_1 on the axis. These two points are called **conjugate foci**, and their distances $O_1 C_2$ and $O_2 C_1$ from the lens are called **conjugate focal distances.** A law of optics is that the sum of the reciprocals of any two conjugate focal distances of a lens is equal to the reciprocal of the principal focal distance of the lens. Hence, representing $O_1 C_2$ by f_1 and $O_2 C_1$ by f_2,

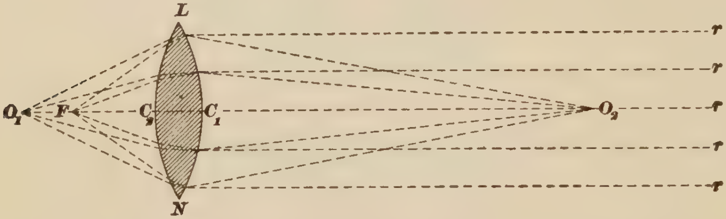

FIG. 1

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$$

3. The object glass of a transit or plane table consists of a combination of lenses forming a compound lens. The same general principles and formulas that are applicable to a single lens apply to a compound lens. As the lens is always very thin compared with the conjugate focal distances, its thickness may be neglected, and the lens represented by a single line, as in the following article.

STADIA FORMULAS

4. Horizontal Sights.—Let $L N$, Fig. 2, represent the objective lens, or object glass, of a telescope; $A B$, a portion of a vertical rod, the center of this portion being on the axis $m M$ of the lens; and $a b$, the image of $A B$ on the plane of the cross-wires. The axis of the lens is supposed to be horizontal and therefore perpendicular to $A\,B$ and $a\,b$. From the formula of Art. 2, f representing the focal distance,

$$\frac{1}{C\,M}+\frac{1}{C\,m}=\frac{1}{f},\text{ whence }\frac{1}{C\,m}=\frac{1}{f}-\frac{1}{C\,M}$$

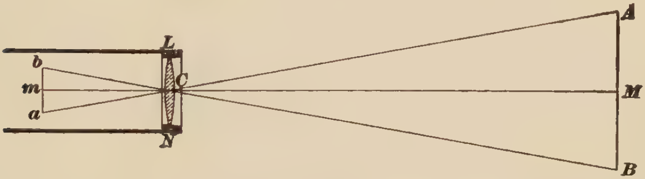

FIG. 2

Also, the triangles $A\,C\,B$ and $a\,C\,b$ are similar, and, therefore,

$$\frac{a\,b}{C\,m}=\frac{A\,B}{C\,M}$$

whence
$$\frac{1}{C\,m}=\frac{A\,B}{a\,b\times C\,M}$$

Equating this to the preceding value of $\frac{1}{C\,m}$, there results

$$\frac{A\,B}{a\,b\times C\,M}=\frac{1}{f}-\frac{1}{C\,M}$$

whence, solving for $C\,M$,

$$C\,M=\frac{f}{a\,b}\times A\,B+f \qquad (a)$$

As f is constant for any telescope, the distance $C\,M$ depends only on the two variable quantities $A\,B$ and $a\,b$. The distance $a\,b$ is made constant by placing two stadia wires $n\,o$ and $p\,q$, Fig. 3, on the reticule of the telescope parallel to and equidistant from the ordinary horizontal cross-wire $h\,k$, and taking always that part of the image included between a and b.

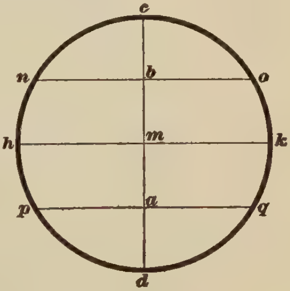

FIG. 3

To $C\,M$, which is the distance from the rod to the object glass, must be added the distance c from the object glass to the center of the instrument, in order to obtain the true distance d

from the rod to the center of the instrument. The distance c varies slightly as the object glass is moved in or out for focusing, but this variation is too small to be considered. The distance d is then given by the equation

$$d = CM + c$$

Substituting the value of CM from (a), and representing the distance ab between the stadia wires by r, and the rod reading AB by R, we have

$$d = \frac{f}{r} R + f + c \qquad \textbf{(1)}$$

The ratio $\frac{f}{r}$ is called the **stadia constant,** and $f + c$ is called the **instrument constant.** Representing the stadia constant by s and the instrument constant by i, formula **1** becomes

$$d = sR + i \qquad \textbf{(2)}$$

This formula assumes that the line of sight is perpendicular to the rod.

5. Inclined Sights.—As comparatively few lines of sight are horizontal, the transit used for stadia measurements should have a level tube attached to the telescope and an arc or circle for reading vertical angles.

In Fig. 4, let MN represent an inclined line of sight from a transit set over the point I to a point N on a rod held vertically on a point P, whose distance from I is required. Let MP_1 be the horizontal projection of IP or MN.

If the rod is held in the position $P'A'$ at right angles to the line of sight, and the portion $A'B'$ intercepted by the stadia wires is read, the length of MN can be found by formula **2**, Art. **4**. Representing $A'B'$ by R', that formula gives

$$MN = sR' + i \qquad (a)$$

The right triangle NMP_1, in which V represents the vertical angle NMP_1, as measured by the vertical arc of the transit, and d represents the horizontal distance MP_1, gives

$$d = MN \cos V$$

or, substituting the value of MN from (a),

$$d = (sR' + i) \cos V$$

6. It is inconvenient and inaccurate to hold the rod perpendicular to an inclined line of sight. In practice, the rod is usually held vertical, as shown at PA, Fig. 4, and the horizontal distance MP_1 or d is expressed in terms of the rod reading $R (= AB)$ determined by the stadia wires on the rod thus held. As the angle AMB is very small, no appreciable error will result if A' and B' are considered to be right angles. Assuming this to be the case, the right triangles ANA' and BNB' give

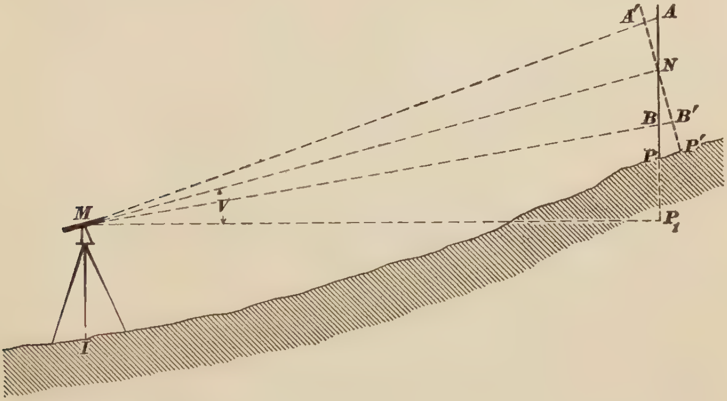

FIG. 4

$$A'N = AN \cos ANA'$$
$$NB' = NB \cos BNB'$$

Adding these equations member to member and noticing that the angle BNB' is equal to ANA',

$$A'N + NB' = (AN + NB) \cos ANA'$$

that is, $A'B' = AB \cos ANA'$, or

$$R' = R \cos ANA' \qquad (a)$$

As the sides of the angle ANA' are, respectively, perpendicular to those of NMP, or V, these two angles are equal, and equation (a) may be written

$$R' = R \cos V$$

Substituting this value of R' in the last equation of the preceding article,

$$d = (sR \cos V + i) \cos V$$

7. Vertical Distances.—In Fig. 4, NP_1 is the difference of elevation between the center M of the transit (the intersection of the horizontal and the vertical axis of the telescope) and the point N. Denoting this difference by v, the right triangle NMP_1 gives

$$v = MP_1 \tan V = d \tan V$$

or, substituting the value of d from the formula of Art. **6**, and performing the multiplications,

$$\begin{aligned} v &= sR\cos^2 V \tan V + i\cos V \tan V \\ &= sR\cos^2 V \frac{\sin V}{\cos V} + i\cos V \frac{\sin V}{\cos V} \\ &= sR\sin V\cos V + i\sin V \end{aligned}$$

or, writing $\frac{1}{2}\sin 2V$ instead of $\sin V\cos V$ (for $\sin 2V = 2\sin V\cos V$),

$$v = \tfrac{1}{2} sR\sin 2V + i\sin V$$

Fig. 4 shows a line of sight inclined above the horizontal. This formula and the formula of Art. **6** apply equally well when the line of sight is inclined below the horizontal. When the line of sight is inclined above the horizontal, the vertical angle is an angle of elevation, and is recorded in the notes as plus (+); when the line of sight is inclined below the horizontal, the vertical angle is an angle of depression, and is recorded in the notes as minus (−).

8. It is usually desired to determine the difference of elevation between the point over which the instrument is set and that on which the rod is held, whereas the formula of Art. **7** gives the difference of elevation between the center of the instrument and the point observed on the rod. If the middle cross-wire is made to intersect the rod at a point whose height above the ground is equal to that of the instrument, the result obtained by the formula of Art. **7** will be equal to the difference of elevation between the point over which the instrument is set and that on which the rod is held. In ordinary work, the observer estimates a distance on the rod about equal to the height of the instrument. If the difference of elevation is desired as accurately as possible, the observer generally measures the height of the instrument by holding a rod alongside of it or in some other convenient manner. He calls out the height to the rodman, who marks an equal height on the rod, usually by setting a target or tying a piece of cloth around the rod at the required height

Example.—The length intercepted on the rod is 7.00 feet, and the angle that the line of sight makes with a horizontal line is 18° 23′. If the stadia constant is 100 and the instrument constant 1.00 foot: (*a*) what is the horizontal distance of the rod from the center of the instrument? (*b*) what is the difference of elevation between the center of the transit and the point where the line of sight intersects the rod, as indicated by the center cross-wire?

Solution.—(*a*) Here $s = 100$, $R = 7$, $i = 1$, and $\cos V = \cos 18° 23' = .94897$. Substituting these values in the formula of Art. **6**,

$$d = (100 \times 7 \times .94897 + 1) \times \cos 18° 23' = 631.3 \text{ ft. Ans.}$$

(*b*) Here $\sin V = \sin 18° 23' = .31537$, and $\sin 2V = \sin 36° 46' = .59856$. Substituting these values and those given above in the formula of Art. **7**,

$$v = \tfrac{1}{2} \times 100 \times 7 \times .59856 + 1 \times .31537 = 209.8 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1. The length intercepted on the stadia rod when held at right angles to the horizontal line of sight is 4.45 feet, the instrument constant is 1.00 foot, and the stadia constant, 100; what is the distance of the stadia rod from the center of the instrument? Ans. 446.0 ft.

2. The length intercepted on the stadia rod is 8.67 feet, and the angle that the line of sight makes with the horizontal is 25° 21′. If the stadia constant is 100 and the instrument constant 1.00 foot: (*a*) what is the horizontal distance of the rod from the center of the instrument? (*b*) what is the difference of elevation between the center of the instrument and the point where the line of sight intersects the rod, as indicated by the center cross-wire? Ans. $\begin{cases} (a) \ 709.0 \text{ ft.} \\ (b) \ 335.9 \text{ ft.} \end{cases}$

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9. Values of the Constants.—The values of the instrument and stadia constants are usually determined by the instrument maker. The instrument constant varies from about .75 to 1.33 feet in different transits according to the size and power of their telescopes. Its value is usually marked on a card attached to the inside of the instrument box.

The stadia constant is customarily made equal to 100; so that, in a horizontal line of sight, the stadia wire will intercept a distance of 1 foot on a rod whose distance from the instrument is 100 feet plus the instrument constant. Thus, if the stadia wires intercept a distance of 8.37 feet on the rod, the distance from the rod to the transit would be 837 feet plus the instrument constant. For ordinary topographical work, especially for long distances, it is sufficiently close to take for the distance 100 times the length intercepted on the rod, the instrument constant being disregarded.

10. Determination of the Constants.—If it is desired to determine or to check the constants, a line, from 400 to 800 feet long, is run on ground as level as practicable, taking care that the atmospheric conditions are favorable. (Refraction is least between about 8 and 10 A. M. and increases with the temperature.) The rod is held on this line at distances of 50, 100, 200 feet, etc. from the center of the instrument, and the space intercepted by the stadia wires at each setting of the rod is carefully read. Then the distances and the corresponding readings are taken in pairs and substituted in formulas given in this article; from each pair a value is found for s and one for i. Finally, the means are taken of all the values of s and of all the values of i.

Let d_1 and d_2 be two of the measured distances, and R_1 and R_2 the corresponding readings of the rod. Then, from formula **2**, Art. **4**,

$$d_1 = sR_1 + i \qquad (a)$$

$$d_2 = sR_2 + i \qquad (b)$$

Subtracting (a) from (b),

$$d_2 - d_1 = s(R_2 - R_1)$$

whence

$$s = \frac{d_2 - d_1}{R_2 - R_1} \qquad (1)$$

also,

$$i = d_1 - sR_1 = d_1 - \frac{(d_2 - d_1)R_1}{R_2 - R_1}$$

or, reducing,

$$i = \frac{d_1 R_2 - d_2 R_1}{R_2 - R_1} \qquad (2)$$

In determinations of this kind, the rod should be read to thousandths by means of a vernier. Although it is easier to find i by the formula $i = d_1 - sR_1$, it is advisable to find it independently by formula **2** of this article.

EXAMPLE.—To determine the stadia and the instrument constant from the following data:

DISTANCE MEASURED, FEET	ROD READING, FEET
50	.488
100	.988
200	1.988
300	2.991
400	3.986

SOLUTION.—Taking 50 feet for the value of d_1 and 100 feet, 200 feet, etc. successively for the values of d_2, and applying formulas **1** and **2**, we find:

First, for $d_1 = 50$ ft. and $d_2 = 100$ ft.,

$$s = \frac{100 - 50}{.988 - .488} = 100.000$$

$$i = \frac{50 \times .988 - .488 \times 100}{.988 - .488} = 1.200 \text{ ft.}$$

Second, for $d_1 = 50$ ft. and $d_2 = 200$ ft.,

$$s = \frac{200 - 50}{1.988 - .488} = 100.000$$

$$i = \frac{50 \times 1.988 - .488 \times 200}{1.988 - .488} = 1.200 \text{ ft.}$$

Third, for $d_1 = 50$ ft. and $d_2 = 300$ ft.,

$$s = \frac{300 - 50}{2.991 - .488} = 99.880$$

$$i = \frac{50 \times 2.991 - .488 \times 300}{2.991 - .488} = 1.258 \text{ ft.}$$

Fourth, for $d_1 = 50$ ft. and $d_2 = 400$,

$$s = \frac{400 - 50}{3.986 - .488} = 100.057$$

$$i = \frac{50 \times 3.986 - .488 \times 400}{3.986 - .488} = 1.172 \text{ ft.}$$

Tabulating these results,

	s	i
	100.000	1.200
	100.000	1.200
	99.880	1.258
	100.057	1.172
	4)399.937	4)4.830
means	99.984	1.208

$$\text{Ans.}\begin{cases} s = 99.984 \\ i = 1.208 \end{cases}$$

If very accurate determinations are desired, observations as above should be taken at different hours of the day and an average of the results should be used.

EXAMPLE FOR PRACTICE

Determine the stadia and the instrument constant from the following data:

DISTANCE MEASURED, FEET	ROD READING, FEET
100	1.000
150	1.504
200	2.011
250	2.516
300	3.019

Ans. $\begin{cases} s = 99.030 \\ i = .970 \end{cases}$

11. Precision.—The precision of stadia measurements depends on the accuracy of the determination of the constants, the care with which the rod readings are taken, and very largely on the state of the atmosphere. Other things being equal, greatest accuracy is obtained between 8 and 10 A. M., when refraction is least. If the atmosphere is hazy or unsteady from the effects of heat, as is often the case in summer, especially along a railroad track, a less degree of precision can be obtained than under the opposite conditions. Long sights should be avoided. With a good transit and under favorable atmospheric conditions, the error should not exceed $\frac{1}{200}$, and with very careful work may be as low as $\frac{1}{1000}$.

STADIA RODS

12. Kinds of Rods Used in Stadia Work.—An ordinary self-reading leveling rod is very often used in making stadia measurements. However, more satisfactory and more rapid work can be done with a rod made especially for the purpose. Various patterns have been designed, the principal idea being so to mark the graduations of the rod that they will be most easily and quickly read by the instrumentman. Fig. 5 shows four patterns of graduations in three of which the tenths of a foot are indicated by the principal angular points. Pattern (*a*) is one of the styles used by the United States Coast and Geodetic Survey, and patterns (*b*) and (*c*) are used by the United States Lake Survey. Pattern (*d*) is easily graduated, each foot being subdivided into tenths by the black rectangles and the white spaces between them. One foot is here shown as the unit of graduation, although a yard or a meter is sometimes used. In pattern (*a*), each twentieth of a foot, and in pattern (*c*) each fiftieth of a foot, is also designated by an angular point. If readings to hundredths of a foot are desired, as is generally the case, they are estimated along the diagonal lines of the graduations. Each diagonal line represents in pattern (*a*), $\frac{5}{100}$ foot; in pattern (*b*), $\frac{10}{100}$ foot; and in pattern (*c*), the long diagonals represent $\frac{4}{100}$ and the short diagonals $\frac{2}{100}$ foot. Thus, if the full line crossing each stadia rod, as shown in Fig. 5, represents the intersection of the line of sight, the readings on patterns (*a*), (*b*), (*c*), and (*d*) are, respectively, .73, .46, 1.07, and .31. In pattern (*d*), as in an ordinary self-reading leveling rod, the number of hundredths of a foot from the nearest tenth mark to the intersection of the line of sight must be estimated. With a little

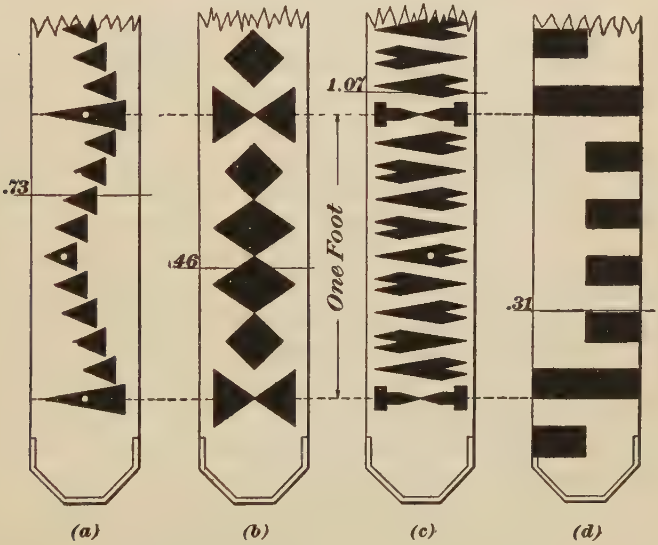


FIG. 5

practice, close reading of any of the various patterns of stadia rods is readily acquired. Stadia rods are generally about 12 feet long and are made in two equal sections, which are connected by a hinge for convenience in handling when the rod is not in use.

13. Regraduating a Stadia Rod.—In order that the stadia measurements may readily be determined from the rod readings when the stadia constant differs considerably from 100, the stadia rod may be regraduated so that one of its full divisions shall be intercepted between the stadia wires when the distance from the rod to the center of the instrument is 100 feet plus the instrument constant. The length of the new main graduations will be equal to 100 feet divided by the stadia constant. Thus, if the stadia constant is found to be 98, the length of the new graduations on the rod will be $100 \div 98 = 1.02$ feet. The entire length of the stadia rod should thus be laid off in equal divisions 1.02 feet long, and each division should be subdivided so that decimal portions can be read. Regraduating a rod is objectionable in that the rod is thus made unfit for use with any other transit or for leveling purposes. It is always preferable to employ a rod divided into true feet and fractions of a foot, and compute the distances by means of formula **2**, Art. **4**, and the formulas of Arts. **6** and **7**, or, still better, by means of stadia tables, as will be explained further on.

PRACTICAL STADIA WORK

MAKING THE OBSERVATIONS

14. Organization of Party.—The stadia party may consist merely of an **instrumentman**, or **observer**, and a **stadia rodman.** If the survey is large and rapid progress is desired, a **recorder** and one or more additional stadia rodmen are necessary. In this case the observer merely makes the observations and calls them out to the recorder. The observer generally has charge of the work of the party and should indicate to the rodmen the points on which the stadia rod should be held for observation. The recorder keeps all the notes and should make sketches of objects that it is desired to locate, showing their position with reference to the points located by the transit. He should repeat to the observer the result of each observation as given out by the observer, before recording it in the notebook.

15. Location of Points.—A point over which the instrument is set is called an **instrument point.** A point on which a stadia rod is held for observation is called a **stadia point.** A stadia point is generally located with reference to the meridian or to some other line. The azimuth of the line of sight to the point to be located is obtained from the reading of the vernier. The distance along this line of sight to the point is obtained from the stadia reading and (if the sight is inclined) the vertical angle. From the stadia reading and the vertical angle is also obtained the difference of elevation between the stadia point and the instrument point.

16. Taking the Readings.—Observing the length intercepted on a rod by the stadia wires is called **taking a stadia reading.** To take a stadia reading on a point where a rod is held, the telescope is first directed toward the rod, and then raised or lowered by means of the tangent screw until one of the stadia wires, generally the lower, coincides with a full division mark on the rod, and the number of full divisions and fractions of a division to the point intersected on the rod by the other stadia wire is read. The difference between the readings of the two wires is the stadia reading. Thus, if the lower wire is made to coincide with the 3-foot mark on the rod and the upper wire intersects the rod at a point whose reading is 5.35, the stadia reading is 5.35 − 3.00, or 2.35. The process is the same if a regular stadia rod is used.

17. As the stadia wires are placed equidistant from the regular cross-wire of the telescope, the latter will intersect the rod at a point midway between the points intersected by the stadia wires. The distance on the rod intercepted between the middle cross-wire and one of the stadia wires is, therefore, one-half of the regular stadia reading. In very long sights, where the distance intercepted between the stadia wires is greater than the length of the rod, or where a portion of the rod is not visible, the middle wire and one of the stadia wires are used in taking the stadia reading, this reading being practically one-half the true reading. Thus, if the rod is held on a distant point and the lower stadia wire is made to read 1 foot and the middle cross-wire reads 8.24, the difference 8.24 − 1, or 7.24, is one-half the true reading, and the true stadia reading is 7.24 × 2, or 14.48. Such half readings are frequently taken as checks on regular readings.

18. If the line of sight is inclined, the angle that it makes above or below the horizontal should be read from the vertical limb. It should be recorded in all cases if differences of elevation are to be determined, but it is not usually considered in connection with horizontal distances unless the vertical angle is more than 3°. When the angle is less than 3°, the line of sight may be assumed to be horizontal and the distance found by formula **2**, Art. **4**.

If the notes are kept by a recorder, the observer calls out the stadia reading as soon as obtained. If the observer keeps the notes, as soon as the stadia reading is taken, but before it is recorded, he should move the telescope up or down until the middle cross-wire intersects the rod at a point whose height above the ground is equal to the height of the instrument above the ground. The stadia reading is then recorded, after which the observer reads and records the vertical angle and the azimuth, if the latter has not already been recorded.

Care should be taken not to touch the lower clamp or tangent screw until all the sights from any instrument point have been taken and their azimuths recorded. It is generally best to check the sights from a station after the last one has been taken by sighting again to the first point observed from that station and noting if the vernier reading is the same as when the first sight on that station was taken. If one instrument point is located from another, the stadia reading and vertical angle should be checked by backsights. The readings given by a backsight should be about the same as those given by the corresponding foresight, and the means between them are assumed to be the correct readings.

STADIA NOTES

19. Form of Notes.—For stadia notes, a regular transit book is commonly used. The numbers and letters designating the stadia stations and points on which stadia readings are taken are recorded in the first column of the left-hand page, the azimuths in the second column, the stadia readings in the third column, and the vertical angles in the fourth column. These four columns are filled out in the field when the observations are made. The horizontal distances are recorded in the fifth column and the elevations in the sixth column. These two columns are filled out when the notes are reduced, which is usually done in the office.

The notes are generally begun at the top of the page and the observations from each station are grouped as shown in the form on next page. The right-hand page of the book is used for sketches and remarks. Each instrument station is usually designated, both in the notes and sketches, by a dot enclosed in a small square, followed by the number of the station; thus, ⊡ 1, ⊡ 2, mean, respectively, *Station 1*, *Station 2*. A plain triangle or a square is often used for the same purpose.

At the top of the first left-hand page are written the location of the tract surveyed and the date the survey is begun. On the opposite page are written the names and positions of the members of the party. At the end of the notes is recorded the date on which the survey is completed. It is also advisable to record the date on which the stadia reductions are made and the notes platted, and the names of the persons who do this part of the work.

20. Reducing the Notes.—Before the notes can be platted, the horizontal and vertical distances must be determined from the stadia readings and the vertical angles. This operation is usually called **reducing the notes.** The

Sta.	Azimuth	Stadia	Vert. Angle	Hor. Dist.	Elev.	⊡ 1 = Old corner stone. Elev. assumed as 100.0.
	Readings from ⊡ 1 Elev. = 100.0					Property line from ⊡ 1 to A
A	230° 0′	6.18	0° 0′			Corner post.
B	100° 24′	4.47	−18° 10′			Artesian well.
⊡ 2	125° 14′	12.10	−11° 45′			
	Readings from ⊡ 2 Elev. =					
⊡ 1	305° 14′	12.16	+11° 53′			

vertical distances are not recorded, but they are used when obtained to determine the elevations of the stadia points with reference to some known or assumed datum. The elevation of any stadia point is obtained by first obtaining the difference in elevation between that point and the instrument point from which the readings on the former point are taken, and then adding that difference to or subtracting it from the elevation of the instrument point, according as the vertical angle is one of elevation or one of depression.

In stadia work, it is customary to calculate and record horizontal distances to the nearest foot only, and elevations to the nearest tenth of a foot only, except on instrument stations, where the differences of elevation are determined and recorded to the nearest hundredth of a foot. Such results are as close as are justified by the degree of accuracy usually attained in stadia measurement.

21. If the line of sight is horizontal, the distance corresponding to a stadia reading is obtained by formula **2**, Art. **4.** This formula is generally used if the angle of inclination does not exceed 3°. Since the stadia constant is commonly 100, the distance required is 100 times the stadia reading, plus the instrument constant, which, if considered, is generally taken as 1 foot. Thus, if the stadia reading in a horizontal sight is 6.18, the distance is $100 \times 6.18 + 1$, or 619 feet. If the line of sight is inclined, the horizontal distance may be obtained by the formula of Art. **6** and the difference of elevation by the formula of Art. **7.** These calculations are generally and more conveniently made by the aid of a table, the use of which is explained in the following article.

It is sometimes desirable to reduce and plat the notes in the field. The platting is done on a sheet attached to a light drawing board. The azimuths are laid off with a protractor, as that method is sufficiently accurate for this work.

22. Stadia Reduction Table.—The formula of Art. **6** giving the horizontal distance between an instrument and a stadia point, may be written

$$d = sR\cos^2 V + i\cos V \qquad (a)$$

Let e be the difference between the stadia constant s and 100, so that $s = 100 \pm e$, the upper sign applying when s is greater than 100, the lower when s is less than 100. This value substituted in equation (a) gives

$$\begin{aligned} d &= (100 \pm e)\, R\cos^2 V + i\cos V \\ &= \left(100\cos^2 V \pm 100\cos^2 V \times \frac{e}{100}\right) R + i\cos V \qquad (b) \end{aligned}$$

In order to facilitate the application of this equation, the Stadia Reduction Table given at the end of this Section has been prepared. That table contains, in the column headed Horizontal Distances, values of $100\cos^2 V$ for values of V varying by intervals of 2′ from 0° to 31°. At the bottom of each column of horizontal distances are given values of $i\cos V$ for three values of i; namely, .75, 1.00, and 1.25 feet, and for an angle equal to that at the top of the column plus 30′, this being the mean between the angle at the top and the angle plus 60′; a mean value that can be used with sufficient accuracy in place of any value of V included between those limits.

Let the value of $100 \cos^2 V$ taken from the table, for any given value of V, be denoted by d_1, and the corresponding value of $i \cos V$ (the mean value mentioned above being here used instead of V) be denoted by I_d. Equation (b) may then be written

$$d = \left(d_1 \pm d_1 \times \frac{e}{100}\right) R + I_d \qquad (1)$$

If, as is usually the case, the stadia constant is 100, or very nearly 100, the term containing e is omitted, and the formula becomes

$$d = d_1 R + I_d \qquad (2)$$

23. The formula of Art. **7** for the difference of elevation between the instrument and the stadia point may be written

$$v = \frac{(100 \pm e) R \sin 2V}{2} + i \sin V$$

$$= \left(\frac{100 \sin 2V}{2} \pm \frac{100 \sin 2V}{2} \times \frac{e}{100}\right) R + i \sin V \qquad (a)$$

The stadia table contains, in the column headed Difference of Elevation, values of $\frac{100 \sin 2V}{2}$ for values of V, as stated in the preceding article. At the bottom of each of these columns are given values of $i \sin V$, for the three values of i and the mean angles mentioned in the article just referred to. If the values of $\frac{100 \sin 2V}{2}$ and $i \sin V$, as taken from the table, are denoted by v_1 and I_v, respectively, equation (a) takes the form

$$v = \left(v_1 \pm v_1 \times \frac{e}{100}\right) R + I_v \qquad (1)$$

When the stadia constant is 100, or very nearly 100, this formula reduces to the form

$$v = v_1 R + I_v \qquad (2)$$

24. It has been stated that the angles in the table vary by intervals of 2′; in other words, the table contains only even numbers of minutes. For any odd number of minutes, it is sufficiently accurate to substitute either of the two even numbers between which it lies. Thus, if the angle V is 23° 15′, either 23° 14′ or 23° 16′ may be used in its place. Should greater accuracy be desired, a mean of the values (horizontal distance or difference of elevation) corresponding to 23° 14′ and 23° 16′ may be taken.

EXAMPLE 1.—The stadia reading is 3.96 feet; the vertical angle, when the line of sight intersects the rod at a height equal to the height of the instrument, is 10° 26′; the stadia constant is 100; and the instrument constant is 1 foot. To determine by the table: (*a*) the horizontal distance from the center of the instrument to the stadia point; (*b*) the difference of elevation between the instrument point and the stadia point.

SOLUTION.—(*a*) The horizontal distance given in the table for 10° 26′ is found to be 96.72($= d_1$). At the bottom of the 10° column of horizontal distances and opposite a value of $i = 1.00$ (the instrument constant) is found .98($= I_d$). As the stadia constant is 100, formula **2**, Art. **22**, should be used. In this case, we have,

$$d_1 = 96.72,\ R = 3.96,\ I_d = .98$$

Substituting these values in the formula,

$$d = 96.72 \times 3.96 + .98 = 383.99 \text{ ft.} \quad \text{Ans.}$$

(*b*) Likewise, the difference of elevation corresponding to 10° 26′ is found to be 17.81($= v_1$). The value of I_v for a value of $i = 1.00$ is found at the bottom of the column of differences of elevation, under 10°, to be .18. To apply formula **2**, Art. **23**, we have,

$$v_1 = 17.81,\ R = 3.96,\ I_v = .18$$

Substituting these values in that formula,

$$v = 17.81 \times 3.96 + .18 = 70.71. \quad \text{Ans.}$$

EXAMPLE 2.—If in the preceding example the elevation of the instrument point is 102.46 feet, and the vertical angle is one of depression, what is the elevation of the stadia point?

SOLUTION.—The difference of elevation between these points was found to be 70.71 ft., and, since the vertical angle is one of depression, this distance is subtracted from the elevation of the instrument point. The elevation of the stadia point is, therefore, 102.46 − 70.71, or 31.75 ft. Ans.

EXAMPLE 3.—The stadia reading being 8.24; the vertical angle, + 20° 40′; the elevation of the instrument point, 240.72 feet; the stadia constant, 97.8; and the instrument constant, 1.25, to find: (*a*) the horizontal distance d from the instrument point to the stadia point; and (*b*) the elevation of the stadia point.

SOLUTION.—(*a*) The distance d is determined by formula **1**, Art. **22**. In this case, $e = 100 - 97.8 = 2.2$, and $\frac{e}{100} = .022$. The value of d_1 corresponding to an angle of 20° 40′ is 87.54. At the bottom of the column of horizontal distances, under 20°, and horizontally opposite the expression $i = 1.25$, is found $1.17 (= I_d)$. As $R = 8.24$, formula **1**, Art. **22**, gives

$$d = (87.54 - 87.54 \times .022)8.24 + 1.17 = 706.60 \text{ ft. Ans.}$$

(*b*) The values of v, and I_v, to be substituted in formula **1**, Art. **23**, are found from the table to be 33.02 and .44, respectively; therefore, applying that formula,

$$v = \left(33.02 - 33.02 \times \frac{2.2}{100}\right) 8.24 + .44 = 266.51 \text{ ft.}$$

Adding this to the elevation 240.72 of the instrument point (since the vertical angle is +), 507.23 ft. is obtained as the elevation of the stadia point.

EXAMPLES FOR PRACTICE

NOTE.—In calculating the horizontal distances and differences in elevation in the following examples, use the Stadia Reduction Table.

1. The intercepted distance on the vertical stadia rod is 4.28 feet, and the vertical angle is −20° 25′. Assuming the instrument constant to be .75 foot and the stadia constant to be 100, what are: (*a*) the horizontal distance from the center of the instrument to the stadia rod; (*b*) the difference in elevation between the center of the instrument and the point observed on the rod? Ans. { (*a*) 376.6 ft. (*b*) 140.2 ft.

2. The intercepted distance on the vertical stadia rod is 3.27 feet, and the vertical angle is −28° 16′. Assuming the instrument constant to be 1 foot and the stadia constant to be 100: (*a*) what is the difference in elevation between the center of the instrument and the point observed on the rod? (*b*) If the height of the center of the instrument above datum is 256.28 feet, and the reading of the center cross-wire on the stadia rod is 8.39 feet, what is the elevation of the stadia point above datum? Ans. { (*a*) 136.9 ft. (*b*) 111.0 ft.

3. The intercepted distance on the stadia rod is 5.47 feet, and the vertical angle is +18° 14′. Assuming the instrument constant to be 1.25 feet, the stadia constant 100, and the elevation of the instrument point 126.00 feet, find: (*a*) the elevation of the stadia point, if the line of sight intersects the rod at a height equal to the height of instrument; (*b*) the horizontal distance from the center of the instrument to the stadia rod. Ans. { (*a*) 289.0 ft. (*b*) 494.6 ft.

4. All the other conditions being the same as in the preceding example, what is the horizontal distance, if the stadia constant is 99? Ans. 489.7 ft.

PLANE-TABLE SURVEYING

THE PLANE TABLE

25. **Description.**—The **plane table** is an instrument used in the preparation of topographical maps. It consists of a drawing board mounted on a tripod, and a device for sighting in any direction and transferring the line of sight to a sheet or roll of drawing paper on the board. Fig. 6 shows the Johnson plane table, which is the one most generally used in private work.

26. The **drawing board** is made of well-seasoned pine with a piece dovetailed on each end at right angles to the grain of the board, so as to overcome the tendency to warp.

27. The instrument for sighting and transferring the line of sight to the paper on the board is called the **alidade.** It consists of a metal ruler r from which rises an upright carrying the telescope T. The telescope has a vertical movement in a plane that contains the edge of the ruler or is parallel to it. It is equipped with a level tube and a vertical circle for the measurement of vertical angles, and generally has two stadia wires in addition to the regular horizontal and vertical wires. The alidade merely sets on the drawing board when in use; when not in use, it is kept in the instrument box.

28. The **tripod** is of the same general design as that of a transit, except that the legs are heavier and shorter, so that the table may be set firmly and at such a height as is convenient for the purpose of drawing or for sighting through the telescope. The device by which the drawing board is connected to the tripod head, permitting the separate motions necessary for leveling the board and for turning it horizontally, is called the **movement.** Fig. 6 shows the board with

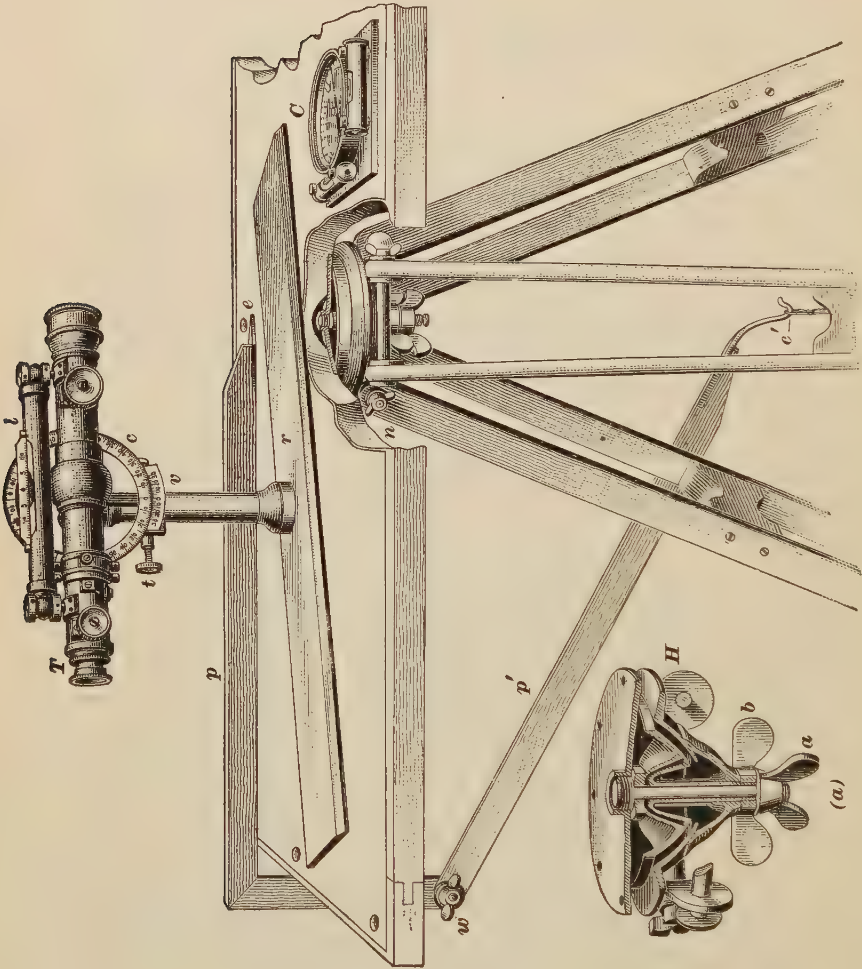


FIG. 6

a portion cut away to show the tripod head and movement. The latter is also shown separately with a portion cut away so that its construction may be seen more plainly. In some plane tables, the leveling is accomplished by means of regular leveling screws.

29. The **declinator** *C*, Fig. 6, is a compass needle mounted in a flat box, whose sides are straight edges parallel to the line determined by the zero marks of the compass graduations. In this type of plane table, the level tubes are attached to the declinator, as shown in the figure. The level tubes are sometimes attached to the alidade ruler.

30. The **plumbing arm** $epp'e'$, Fig. 6, is a device for suspending a plumb-bob so that it will be directly under a point e on the paper representing the point determined on the ground by the plumb-bob.

31. Plane-Table Paper.—The drawing paper for plane-table work is sometimes used in rolls, the portions not in use being rolled under the ends or sides of the board. More often the paper is prepared in the form of sheets about the size of the board. It is held on the board by heavy brass clamps or by strong tacks or screws.

The paper should be of good quality, as the expansion and contraction due to atmospheric changes is very large in the poorer grades of paper. It may be seasoned by being exposed for several days alternately to a very damp and to a very dry atmosphere. This considerably lessens the effect of atmospheric changes.

If a great degree of accuracy is desired, doubly mounted sheets of the best paper are used. They are made by pasting a sheet of paper on each side of a piece of tightly stretched muslin with the grain of one sheet at right angles to the grain of the other. Celluloid sheets are sometimes used where there is likely to be an accumulation of moisture on the sheet, such as rain, dew, or water dropping from the trees.

TESTS AND ADJUSTMENTS

32. The Level Tubes.—The level tubes attached to the declinator are tested by placing the declinator in the center of the table and bringing each bubble to the center of the tube by the leveling apparatus. The declinator is then reversed in azimuth; if it is in adjustment, the bubbles will be in the center of the tubes. If not, one-half the error is corrected by means of the adjusting screws, and the other half by again leveling the table. This adjustment is similar to that of the plate levels of a compass or transit. The same method is used if the level tubes are attached to the alidade.

33. The Telescope.—The adjustments of the telescope are the same as the second, third, fourth, and fifth adjustments of a transit.

34. The Drawing Board.—If the drawing board is warped or uneven, it should be planed off until a true plane surface is obtained. The board should revolve in a truly horizontal plane. To determine this, after the level tubes have been tested, the table should be leveled carefully and then turned around about 180°. If the bubbles remain in the center of the tubes, the test is satisfactory. If not, one-half the error should be corrected by inserting thin washers between the board and the arms or plate of the movement to which it is attached on that side of the center which it is necessary to raise, in order to bring the bubble toward the center of the tube. This adjustment is repeated until the bubbles remain in the center of the tubes during a complete revolution of the table. Except for very important work, it is not necessary that this adjustment should be perfect.

PLANE-TABLE FIELD WORK

35. Organization of Party.—A plane-table party consists of an instrumentman, generally called a **topographer,** and one or more rodmen. Additional helpers are needed if the survey is of a large area or the country is very rough. The topographer makes the observations and does the platting and sketching. The rodmen should be able to determine what points are necessary to be located, in order to properly represent on the map the ground covered by the survey.

36. Plane-Table and Triangulation Stations.—The point on the ground over which the plane table is set is called a **plane-table station,** and is generally designated by a small circle with a dot in the center (⊙). In plane-table work, it is customary to denote points or stations on the ground by capital letters, and their platted positions by small letters. Thus, if a point on the ground is called A, its platted position, or its position on the map, is called a.

Plane-table work is generally based on an imaginary line connecting two known points that are visible from each other. The distance between these points is measured or calculated, and the line connecting them is platted to the scale decided on for the map on such a portion of the sheet as is indicated by the relation of the known points to the rest of the area to be covered by the survey. In the survey of a large area, reference lines forming a net of triangles are generally run with a transit, and from them other points of the survey are located by means of the plane table. The vertexes of these triangles are called **triangulation stations,** and are often designated by a small triangle with a dot in the center (◬). In order that a triangulation station can be seen from the plane-table stations in various directions, a **signal** is often erected after the point has been occupied by the transit. This generally consists of a straight pole, braced so as to stand in a vertical position, and with a piece of cloth attached to the top to make it more easily discernible.

37. Setting Up the Plane Table.—In setting up the plane table, the legs of the tripod are so placed that the tripod head is about level. The board is then made level by means of the level tubes attached to the declinator or to the alidade. If the movement is an adaptation of the ball and socket joint, the board is moved up or down by pressure on the sides until the bubble is in the center of each tube.

By means of the plumbing arm (see Art. **30**), the table can be so set that a special point on the map shall be directly over a given station on the ground. In order to accomplish this, it may be necessary to move the tripod legs, or the tripod as a whole, several times. Usually, however, if the platted point is nearly over the station on the ground that it represents, the table is considered to be correctly set, as the map is generally drawn to so small a scale that an error, even of 1 foot, would not show on the map.

The level movement should be clamped as soon as the board is leveled, and the horizontal movement as soon as the desired position of the board is secured.

38. Orienting the Plane Table.—If the survey is based on lines connecting two or more points whose positions are platted on the board, the latter, after being leveled, must be clamped in such a position that each line platted corresponds in direction with, or is parallel to, the line on the ground that it represents. When this condition obtains, the plane table is said to be **oriented**, or **in orientation**, or **in position**.

Unless the platted point is in the center of the board, any turning of the table to orient it will change the position of the platted point with reference to the station on the ground. Hence, the plane table is oriented as closely as possible by the eye before attempting to set a platted point accurately over a point on the ground. Thus, in Fig. 7, the Stations A and B on the ground are platted on the plane-table sheet at a and b. The plane table is placed at Station A and is oriented approximately by sighting along the line $a\,b$ and turning the board until the line $a\,b$ is about in the same direction as the line $A\,B$ on the ground. The point a is brought over Station A by moving the tripod and using the plumbing arm, care being taken to keep the board approximately oriented. The table is then leveled and the edge of the alidade ruler is made to coincide with the line $a\,b$, the objective end of the telescope being directed toward B. The table is next turned in azimuth until the telescope is accurately directed to *B*. This turning of the board changes the position of *a* with reference to *A*; but the error is generally so small that it is not considered. When the edge of the ruler is along *a b*, and the telescope directed to *B*, as just explained, the plane table is oriented. The board is then clamped; and lines of sight to any points, such as *C* and *D*, that are to be located and are visible from the station occupied, are platted in the manner explained in the following article.

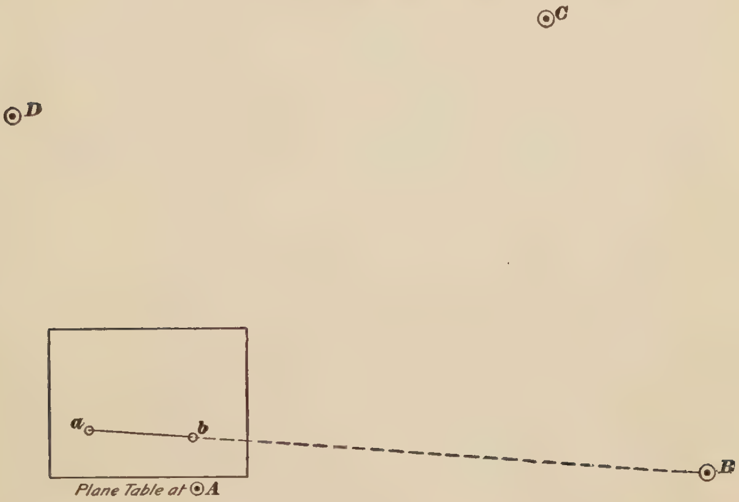


FIG. 7

39. Platting the Line of Sight.—After the plane table is oriented, the telescope is directed to the point to be located, the edge of the alidade ruler being kept in contact with the platted position of the station occupied. A line is then drawn along the edge of the ruler, beginning at the platted point and extending in the direction of the point to be located. Thus, in Fig. 7, the plane table is oriented at Station *A*, the telescope is directed to the point *C*, with the edge of the ruler in contact with the point *a*, which is over Station *A* on the ground. A line representing this line of sight is then drawn along the edge of the ruler, as shown at *a x* in the left-hand portion of Fig. 8. In platting a line of sight, a hard pencil having a fine point should be used, and the line should be drawn lightly and close to the edge of the ruler.

LOCATION OF POINTS

40. Location by Distance.—If the plane table is oriented at any station, any point visible from that station may be located on the plat by laying off to scale, on the platted line of sight to the point, the distance from the point to the station occupied. That distance may be determined by either stadia or direct measurement. Thus, in Fig. 8, the plane table having been oriented at Station *A* and the line of sight to the point *C* having been platted at *a x*, the point *c* may be located by obtaining the distance from *A* to *C* and laying off this distance to the scale of the map on the line *a x*.

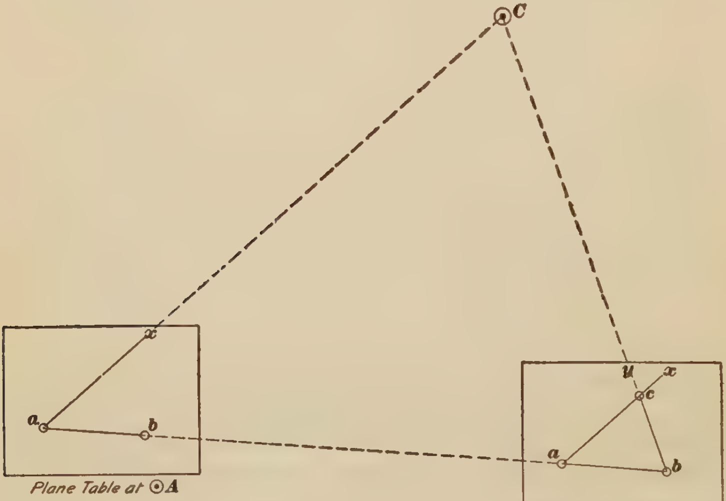


FIG. 8

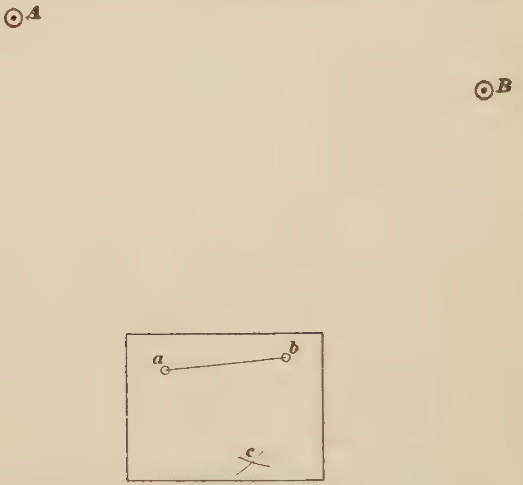
41. Location by Intersection.—A point may be located on the plat by sighting to it with the plane table oriented from two stations whose locations are platted on the board. The intersection of the platted lines of sight from those two stations to the point will determine the position of the point on the plat. Thus, in Fig. 8, the plane table is shown in two locations at Stations *A* and *B*, these stations being platted at *a* and *b*. To locate the point *C*, the plane table is set up at *A* and oriented by sighting to *B* along *a b*, as explained in Art. **38.** The board being clamped, the telescope is directed to the point *C* with the edge of the ruler in contact with the platted position *a* of the station occupied, and a line *a x* is drawn representing this line of sight. The plane table is then set up at *B* and oriented by sighting to *A* along *b a*, and a line of sight *b y* is platted to *C*. The intersection *c* of this line with the line *a x* drawn from *a* will be the platted location of *C*.

This method of location is based on the principle of geometry that two triangles are similar when two angles of one are equal to two angles of the other. Thus, in Fig. 8, the triangle *a b c* is similar to the triangle *A B C* on the ground, since the angles *a b c* and *b a c* are platted equal to the angles *A B C* and *B A C*. The sides of these triangles are therefore proportional, and as *a b* represents *A B* to the scale of the map, *a c* and *b c* will represent, respectively, *A C* and *B C* to the same scale.

To secure a good determination of the point of intersection, the lines should, if possible, meet at an angle not less than 30° nor greater than 150°. If a point located by intersection is an important one, sights should be taken to it, if practicable, from more than two known points, in order to check its position on the map.

42. Location by Resection.—This method is of great advantage in an extensive survey, as it permits the point occupied by the plane table to be arbitrarily selected. It consists in setting the table over a convenient point from which two or more of the points already platted can be seen, determining the position on the plat of the point thus occupied, and orienting the table at that point, so that other points may be observed from it and platted.

If two points that have already been platted are near at hand and accessible, the station occupied may be located on the plat by determining the distances from it to those two points. Using these distances, to the scale of the map, as radii, the intersection of arcs swung from the platted points will locate on the map the station occupied. Thus, in Fig. 9, the point occupied by the plane table is Station C. It is desired to locate on the map the position of Station C, with reference to the platted positions a and b of the points A and B. The distances of the points A and B from Station C are determined either by stadia or by direct measurement; then, with a and b as centers and with radii equal, to the scale of the map, to the distances AC and BC, respectively, arcs are swung. The point of intersection c of these arcs is the platted position of the point C.



Plane Table at ⊙C

FIG. 9

If it is desired to locate additional points from this station, the plane table is oriented by placing the edge of the ruler in contact with ca or cb and directing the telescope to A or B, as the case may be.

43. If the station occupied and to be located on the map is on a line that has been platted from another station, its position on the map is determined as follows: Let C, Fig. 10, be a point on the ground to which a line of sight was directed when the table was set and oriented at *A*, and let *a x* be the platted position of that line of sight. The plane table is oriented by placing the edge of the ruler in contact with the line *a x* and directing the telescope to Station *A*. Having clamped the board in this position, the edge of the ruler is placed in contact with another platted point *b*, and the ruler turned about this point until the telescope is directed to the corresponding point (*B* in this case) on the ground, and the line of sight is platted. The intersection *c* of this line with the line *a x* is the platted position of the point *C*.

When the point occupied has not been sighted to before, and its distances from other known points cannot be conveniently obtained, its position on the map is determined by the solution of one of two problems, known, respectively, as the *three-point problem* and the *two-point problem*—the former occurring when three known stations are visible from the station occupied, and the latter when only two stations are visible. The former condition is preferable to the latter, and, whenever possible, the station occupied should be so chosen that three known stations already located on the map can be seen from it.

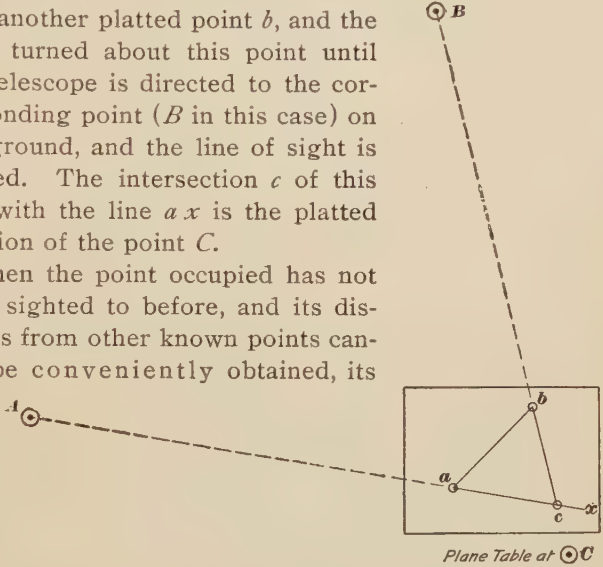


FIG. 10

44. The Three-Point Problem.—Let the plane table be set over a point *P*, Fig. 11, from which three points *A*, *B*, *C*, platted at *a*, *b*, and *c*, respectively, are visible. The

three-point problem consists in the determination of the point *p* on the map corresponding to the point *P* on the ground; that is, of a point on the map whose position with reference to *a*, *b*, and *c* shall be the same as that of *P* with reference to *A*, *B*, and *C*. For convenience, the board is turned so that the points *a*, *b*, and *c* occupy about the same relative positions on the board as are occupied by the points *A*, *B*, and *C* on the ground; in other words, the table is approximately oriented by the eye. A piece of tracing cloth or paper *s s s s*, large enough to cover the three platted points and the estimated location of the station occupied, is fastened to the board over the plane-table paper. A point *p′* is chosen on the tracing cloth approximately in the same position with reference to the points *a*, *b*, and *c* that Station *P* has with reference to the points *A*, *B*, and *C*. With the edge of the ruler in contact with the point *p′*, the telescope is directed successively to *A*, *B*, and *C*, and the lines of sight are platted, as shown at *p′ a′*, *p′ b′*, and *p′ c′*. The alidade is then removed and the tracing cloth is unfastened and shifted on the drawing paper to a position in which the lines *p′ a′*, *p′ b′*,

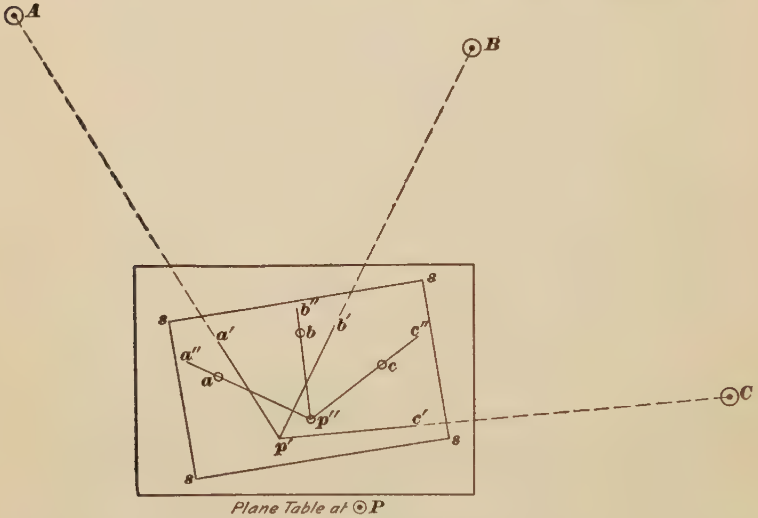


FIG. 11

and $p'c'$ pass through the platted points a, b, and c, as shown at $p''a''$, $p''b''$, $p''c''$, which are then the positions of $p'a'$, $p'b'$, and $p'c'$, respectively, p'' being the position of p'. The point p'' is over the exact position of p and can be pricked through with a fine needle point. The tracing paper is then removed and the alidade is replaced on the table. With the edge of the ruler in contact with the pricked point p and one of the platted points, such as a, the table is turned in azimuth until the telescope is directed to the point on the ground that the platted point (a in this case) represents. By this operation, the plane table is oriented and the board is then clamped. If desired, the position of p may be checked by sighting to the points B and C with the edge of the ruler in contact, respectively, with the points b and c, and platting the lines of sight. These lines will intersect at p if the work has been accurately done.

Care should be exercised to keep the tracing cloth free from wrinkles and not to stretch it in the process of fastening over the drawing paper.

45. The Two-Point Problem.—Let C, Fig. 12 (a), be a station from which two points A and B, platted at a and b, are visible. It is required to determine the platted position of C. To do this, a fourth point D is chosen in such a position that A, B, and C are visible from it and that the lines of sight from C and D to A and B will make with each other angles sufficiently large for good intersections. The plane table is set up at D, approximately oriented by the eye, and clamped. A point d is chosen on the board about in the same position with reference to a and b that the point occupied has with reference to A and B. The point D is located directly under d by means of the plumbing arm. With the edge of the ruler in contact with d, the telescope is directed successively to A, B, and C and the lines of sight are platted, as shown at da', db', and dc'. The plane table is then set up at Station C (the station to be located on the map) and by placing the edge of the ruler on the line $c'd$ and directing the telescope to Station D, the board is oriented with reference to the line *C D*. A point *c''* on the line *c' d* is chosen to represent the Station *C*. (This is generally done by estimating the distance from *C* to *D* and laying it off from *d* to the scale of the map, but the distance *d c''* does not affect the result, and, therefore, *c''* may be taken anywhere on *d c'*.) With the edge of the ruler in contact with *c''*, the telescope is directed successively to the points *A* and *B*, and the lines of sight are platted. These lines will intersect the lines *d a'* and *d b'* at *a''* and *b''*, respectively. The points *A* and *B* are thus located by intersection, with reference to *D C*, at *a''* and *b''*, and the line *a'' b''* is parallel to *A B*. To orient the plane table, the platted line *a b* must be made parallel to *A B*. This is now readily accomplished by placing the edge of the ruler on the line *a'' b''* and setting a rod or other mark in line at any convenient point at least 500 feet from the plane table. The board is then unclamped, and, with the edge of the ruler on the line *a b*, turned until the line of sight bisects the rod.

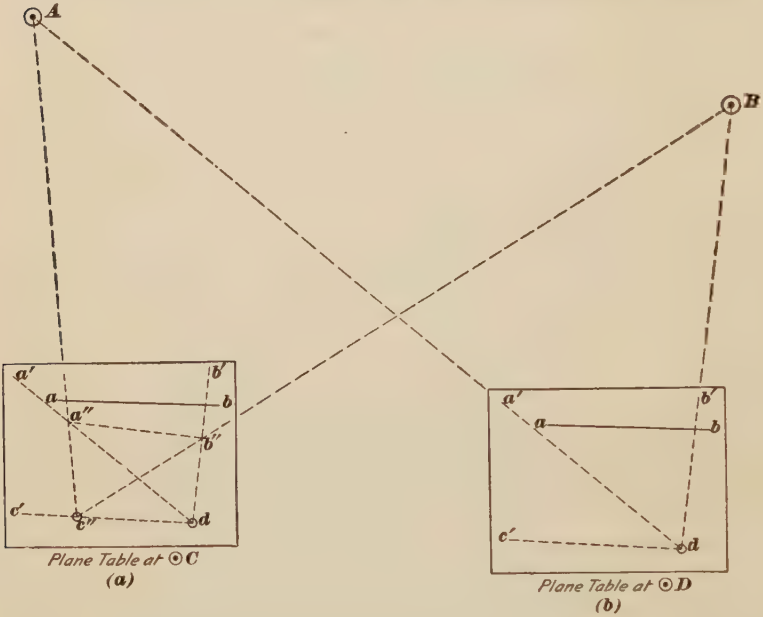


FIG. 12

The distance between the points a and a'' is so small compared with the length of the sight to the rod, that the line $a\,b$ may be considered to coincide with the former position of the line $a''\,b''$, which was parallel to $A\,B$. Therefore, the board may be considered as oriented. By sighting to A with the alidade in contact with a, and to B with the alidade in contact with b, the platted lines representing these sights will intersect at a point c, which is the platted position of the station occupied. The board is now in position to locate any additional points from Station C.

The auxiliary lines necessary for the solution of this problem are sometimes drawn on tracing cloth fastened to the board, and the position of the point c when obtained is pricked through to the drawing paper. The tracing cloth is then removed.

46. Compass Orientation.—When the plane table is in orientation, a magnetic meridian may be established by means of the declinator, which is placed on the board and moved until the needle points to the north point of the compass. A line drawn along the side of the declinator box that is parallel to the needle will indicate a magnetic meridian. The plane table may be oriented roughly, that is, within the degree of accuracy of the compass, at any succeeding station by placing the side of the declinator box in contact with the magnetic meridian as drawn on the board and turning the board until the needle points to the north point of the compass box. **Compass orientation** is often used in the preliminary operation of the solution of the two-point and the three-point problem, and also in running rough traverse lines to locate unimportant roads, streams, fence lines, etc.

STADIA REDUCTION TABLE

Minutes	0°		1°		2°		3°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	100.00	.00	99.97	1.74	99.88	3.49	99.73	5.23
2	100.00	.06	99.97	1.80	99.87	3.55	99.72	5.28
4	100.00	.12	99.97	1.86	99.87	3.60	99.71	5.34
6	100.00	.17	99.96	1.92	99.87	3.66	99.71	5.40
8	100.00	.23	99.96	1.98	99.86	3.72	99.70	5.46
10	100.00	.29	99.96	2.04	99.86	3.78	99.69	5.52
12	100.00	.35	99.96	2.09	99.85	3.84	99.69	5.57
14	100.00	.41	99.95	2.15	99.85	3.89	99.68	5.63
16	100.00	.47	99.95	2.21	99.84	3.95	99.68	5.69
18	100.00	.52	99.95	2.27	99.84	4.01	99.67	5.75
20	100.00	.58	99.95	2.33	99.83	4.07	99.66	5.80
22	100.00	.64	99.94	2.38	99.83	4.13	99.66	5.86
24	100.00	.70	99.94	2.44	99.82	4.18	99.65	5.92
26	99.99	.76	99.94	2.50	99.82	4.24	99.64	5.98
28	99.99	.81	99.93	2.56	99.81	4.30	99.63	6.04
30	99.99	.87	99.93	2.62	99.81	4.36	99.63	6.09
32	99.99	.93	99.93	2.67	99.80	4.42	99.62	6.15
34	99.99	.99	99.93	2.73	99.80	4.47	99.61	6.21
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.32
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38
42	99.99	1.22	99.91	2.97	99.78	4.71	99.58	6.44
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61
50	99.98	1.45	99.90	3.20	99.76	4.94	99.55	6.67
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.79
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96
i = .75	.75	.01	.75	.02	.75	.03	.75	.05
i = 1.00	1.00	.01	1.00	.03	1.00	.04	1.00	.06
i = 1.25	1.25	.02	1.25	.03	1.25	.05	1.25	.08

STADIA REDUCTION TABLE—*Continued*

Minutes	4°		5°		6°		7°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4	99.50	7.07	99.22	8.80	98.88	10.51	98.49	12.21
6	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.27
8	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30	99.38	7.82	99.08	9.54	98.72	11.25	98.30	12.94
32	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
i = .75	.75	.06	.75	.07	.75	.08	.74	.10
i = 1.00	1.00	.08	1.00	.10	.99	.11	.99	.13
i = 1.25	1.25	.10	1.24	.12	1.24	.14	1.24	.16

STADIA REDUCTION TABLE—*Continued*

Minutes	8°		9°		10°		11°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
i = .75	.74	.11	.74	.12	.74	.14	.73	.15
i = 1.00	.99	.15	.99	.17	.98	.18	.98	.20
i = 1.25	1.24	.18	1.23	.21	1.23	.23	1.22	.25

STADIA REDUCTION TABLE—*Continued*

Minutes	12°		13°		14°		15°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24	95.39	20.97	94.63	22.54	93.82	24.09	92.95	25.60
26	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
$i = .75$	.73	.16	.73	.18	.73	.19	.72	.20
$i = 1.00$	.98	.22	.97	.23	.97	.25	.96	.27
$i = 1.25$	1.22	.27	1.22	.29	1.21	.31	1.20	.33

STADIA REDUCTION TABLE—*Continued*

Minutes	16°		17°		18°		19°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24	92.03	27.09	91.06	28.54	90.04	29.95	88.97	31.33
26	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34	91.87	27.33	90.89	28.77	89.86	30.18	88.78	31.56
36	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58	91.48	27.91	90.49	29.34	89.44	30.74	88.34	32.09
60	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
i = .75	.72	.21	.72	.23	.71	.24	.71	.25
i = 1.00	.96	.28	.95	.30	.95	.32	.94	.33
i = 1.25	1.20	.36	1.19	.38	1.19	.40	1.18	.42

STADIA REDUCTION TABLE—*Continued*

Minutes	20°		21°		22°		23°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
$i =$ .75	.70	.26	.70	.27	.69	.29	.69	.30
$i =$ 1.00	.94	.35	.93	.37	.92	.38	.92	.40
$i =$ 1.25	1.17	.44	1.16	.46	1.15	.48	1.15	.50

STADIA REDUCTION TABLE—*Continued*

Minutes	24°		25°		26°		27°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92
30	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
i = .75	.68	.31	.68	.32	.67	.33	.67	.35
i = 1.00	.91	.41	.90	.43	.89	.45	.89	.46
i = 1.25	1.14	.52	1.13	.54	1.12	.56	1.11	.58

STADIA REDUCTION TABLE—*Continued*

Minutes	28°		29°		30°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	77.96	41.45	76.50	42.40	75.00	43.30
2	77.91	41.48	76.45	42.43	74.95	43.33
4	77.86	41.52	76.40	42.46	74.90	43.36
6	77.81	41.55	76 35	42.49	74.85	43.39
8	77.77	41.58	76.30	42.53	74.80	43.42
10	77.72	41.61	76.25	42.56	74.75	43.45
12	77.67	41.65	76.20	42.59	74.70	43.47
14	77.62	41.68	76.15	42.62	74.65	43.50
16	77.57	41.71	76.10	42.65	74.60	43.53
18	77.52	41.74	76.05	42.68	74.55	43.56
20	77.48	41.77	76.00	42.71	74.49	43.59
22	77.42	41.81	75.95	42.74	74.44	43.62
24	77.38	41.84	75.90	42.77	74.39	43.65
26	77.33	41.87	75.85	42.80	74.34	43.67
28	77.28	41.90	75.80	42.83	74.29	43.70
30	77.23	41.93	75.75	42.86	74.24	43.73
32	77.18	41.97	75.70	42.89	74.19	43.76
34	77.13	42.00	75.65	42.92	74.14	43.79
36	77.09	42.03	75.60	42.95	74.09	43.82
38	77.04	42.06	75.55	42.98	74.04	43.84
40	76.99	42.09	75.50	43.01	73.99	43.87
42	76.94	42.12	75.45	43.04	73.93	43.90
44	76.89	42.15	75.40	43.07	73.88	43.93
46	76.84	42.19	75.35	43.10	73.83	43.95
48	76.79	42.22	75.30	43.13	73.78	43.98
50	76.74	42.25	75.25	43.16	73.73	44.01
52	76.69	42.28	75.20	43.18	73.68	44.04
54	76.64	42.31	75.15	43.21	73.63	44.07
56	76.59	42.34	75.10	43.24	73.58	44.09
58	76.55	42.37	75.05	43.27	73.52	44.12
60	76.50	42.40	75.00	43.30	73.47	44.15
i = .75	.66	.36	.65	.37	.65	.38
i = 1.00	.88	.48	.87	.49	.86	.51
i = 1.25	1.10	.60	1.09	.62	1.08	.63

TOPOGRAPHIC SURVEYING

TOPOGRAPHY AND SLOPE MEASUREMENT

DEFINITIONS AND METHODS

1. Topography is the detailed representation of the physical features of a region, including not only the geographical location of its boundaries and all important divisions of, and objects on, its surface, but also the form of its surface with respect to its elevations and depressions. A point is located on a plane surface when the length and direction of a line from that point to a point of reference are known, or when its coordinates with respect to two axes are known. The topographical location of a point includes also its elevation above a given level surface. In determining the topography of a region, points are located topographically in sufficient number and in such positions that the change in the surface between any two adjacent points will be reasonably uniform, so that the form of the intervening surface can be inferred from the points located. From this it is evident that the points located should be at the most abrupt changes, both in the outline, such as angles, corners of conspicuous objects, etc., and in the configuration of the surface, such as the crests of ridges, bottoms of ravines, etc.

2. Topographic surveying is the operation of determining the topographical features of any portion of the earth's surface, including the location of points within the limits of the district surveyed and the relative elevations or depressions of the surface at the different points. It thus

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determines the positions and forms of prominent objects and the inequalities of the surface. Three general methods, differing with regard to the instruments used, are employed in making topographic surveys; namely, the transit and level method, the stadia method, and the plane-table method.

3. Transit and Level Method.—In this method, objects are located by means of a compass or transit for the azimuths and a chain or tape for the linear measurements, while the relative elevations are determined by means of a leveling instrument, sometimes supplemented by a clinometer or slope level.

This method is well adapted to surveys for the location of railroads and to similar surveys that relate to lines rather than to areas, and in which the topography is required to cover only comparatively narrow strips of country contiguous to the lines. In such surveys, the entire process is based on the line of the survey, which is usually alined with a transit and measured with a chain or tape, and the elevations are taken over it with a leveling instrument.

4. Stadia Method.—Points are located by means of a transit for the azimuths. The transit is equipped with a level on the telescope, a vertical arc or circle, and stadia wires. The distances and the differences of elevation are determined by stadia measurement. This method is adapted to all kinds of surveys in which a great degree of accuracy is not required. It is without question the best method of making a general topographical survey of considerable extent, and is especially convenient for preliminary railroad location surveys. The stadia method was officially adopted by the United States Lake Survey in 1864.

5. Plane-Table Method.—Points located by the plane table are at once platted on the map, which is thus prepared in the field without the intermediate process of reading and recording angles and distances. This method is well adapted to mapping, especially for filling in the details after the principal lines of a survey have been determined by other means. It has been used extensively for this purpose by the United States Coast and Geodetic Survey and the United States Geological Survey. It is also adapted for smaller surveys, such as that of a park, in which it is desired to locate very numerous objects within a small area, and in surveys for rough maps, the time for making which is limited and in which only some of the principal points are located accurately, the other features being sketched in by eye.

If the area to be covered is long and narrow, as in a railroad survey, the line of survey is taken as a line of reference for the location of all points. The elevations are determined from a line of levels, which is generally run along with the survey line. At suitable intervals along the line, generally at each station (unless the ground is very regular), cross-sections are taken at each side of the line and at right angles to it; in these the rates of slope of the surface are determined.

6. The Rate of Slope.—The rate of slope is determined either by measuring the horizontal and vertical distances between two points in the slope or by measuring the angle of the slope. In the former method, the vertical distance that some point of the slope is above or below the beginning of the slope is measured with a level and rod, and the horizontal distance with a tape. A hand level, described in Art. **7**, is generally used in this work. The angle of the slope is determined by means of a clinometer, described in Art. **9**. Slopes of the natural surface are generally designated by the vertical rise or fall in any given number of feet measured horizontally. Thus, a slope of 1 in 20 indicates a rise or fall of 1 foot in 20 feet measured horizontally. In earthwork, such as a railroad or other embankment, a slope is generally designated by the ratio of the horizontal measurement to the vertical measurement, the latter being reduced to 1. Thus, a slope of 20 feet vertical in 30 feet horizontal is designated as a $1\frac{1}{2}$ to 1 slope, and is usually expressed thus: $1\frac{1}{2} : 1$. Slopes are sometimes designated by the slope angle. Thus, a 45° slope indicates an angle of 45° with the horizontal.

7. **The Hand Level.**—The usual form of this level is shown in Fig. 1; it is called the **Locke level,** from the name of the inventor. It consists of a brass tube *A B* about 6 inches long, usually finished in bronze or nickel plated, and having on the top near the object end a small spirit level *C*. Beneath the level is an opening in the tube through which, by means of a reflecting prism placed below it, the bubble can be seen when the eye is placed at the small opening *D* in the eye end. The reflecting prism occupies one-half the cross-section of the tube and is set at an angle of 45° to the line of sight; any object toward which the level is directed can be seen through the part of the tube unobstructed by the mirror. A cross-wire is placed directly below the center of the level in such position that, as reflected, it will bisect the bubble when the line of sight is horizontal. If the level is held to the eye and directed toward a distant object, and its object end is raised or lowered until the bubble is bisected by the cross-wire, the point on the distant object with which the cross-wire coincides, as seen through the unobstructed portion of the tube, is at the same elevation as the eye.

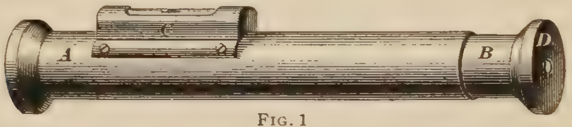


FIG. 1

8. **Cross-Sectioning With a Hand Level.**—This may be done by a topographer and one rodman. Usually, however, the topographer has three assistants—one rodman and two tapemen. The latter merely make the horizontal measurements as directed by the topographer or rodman. A metallic tape, 50 or 100 feet long, and a regular self-reading level rod are generally used; though for extensive cross-sectioning, a special rod, similarly graduated, but lighter and longer, is sometimes preferred. Before beginning the work, the topographer, by standing alongside the rod, measures the height of his eye above the ground, which height is a constant quantity to be subtracted from all rod readings except

when the observations are extended by means of a backsight and foresight. The topographer, standing at the station on the line of survey where a cross-section is to be taken, determines by the eye the right-angle line on which the elevations are to be taken, and generally selects a point some distance off in line with which the rodman is kept when the rod readings are taken. The readings are taken at the points where the slope seems to change or at points where the top or bottom of the rod is level with the eye of the topographer. The points where elevations are taken are located by horizontal measurements from the survey line or from points already located. On a steep ascending slope the topographer, after determining the right-angle line, walks along it up the slope until his eye is about level with the top of the rod, when the latter is held at the station on the survey line. The rod reading on the station, less the height of the topographer's eye above the ground, determines the height above the station of the point where he is standing. On an ascending slope that is not steep, and on a descending slope, the topographer stands at the station while the rodman holds the rod on the first point on which a reading is to be taken. The topographer then stands at the point formerly occupied by the rod, and the rodman proceeds up or down the slope to give another sight. By continuing the sights, the cross-section is made to cover as much ground on each side of the survey line as may be desired. The topographer, by taking a rod reading on a point whose elevation has already been determined, determines the height of his eye above the known point. It is often desirable to continue this sight to a point still further from the survey line. The eye of the topographer remains at the same height as he turns around and takes a rod reading on the latter point, the elevation of which above or below the preceding point is equal to the difference between the two rod readings. These are similar to a backsight and foresight in direct leveling.

It is sometimes desired to take a sight on the rod when its top is below the level of the topographer's eye. This is accomplished by "shinning the rod"; that is, by the rodman raising the rod off the ground so that the top is sufficiently high. The bottom is generally held against his legs or body to steady it. After the reading has been taken by the topographer, the rodman measures with the rod the distance from the ground to the point to which the bottom of the rod was raised. This distance is called out to the topographer, who adds it to the rod reading. If the ground is flat, the length of a sight is limited by the distance away that the figures on the rod can be plainly read. The topographer must decide, from the nature of the ground and from the purpose for which the topography is desired, how far on each side of the survey line the cross-section should extend. Ascending slopes are recorded as + and descending slopes as −.

9. The Clinometer.—By means of a **clinometer,** or **slope level,** the angle that a slope makes with the horizontal is determined. A simple form of this instrument is shown in Fig. 2; it consists of a straight bar *a b* about 6 inches long, to which is attached the movable arm *m n*, which carries a spirit level *l* and turns on a hinge at *m*. The direction of the arm with reference to the bar *a b* is shown by the quadrant scale *q r*, which is graduated to degrees. If the bar *a b* is placed on any sloping surface and the arm *m n* raised until

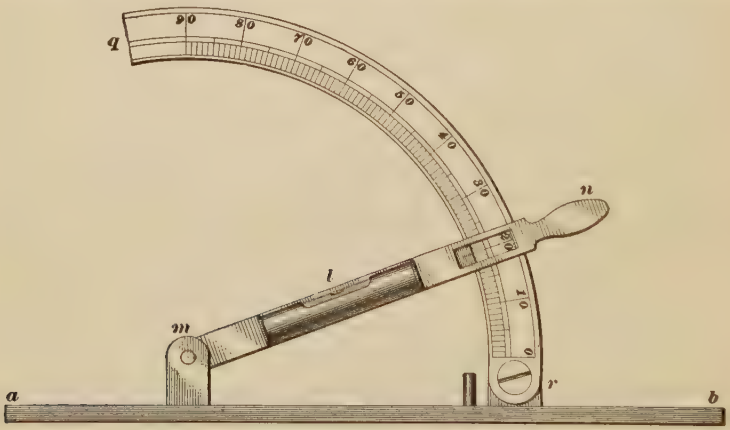


FIG. 2

the bubble is at the center of the level tube, the arm will be horizontal and its reading on the graduated quadrant will be the angle that the slope makes with the horizontal. Since the bar *a b* is short and the surface of the ground uneven, in order that the slope of the surface of the ground, as measured, shall be its average slope, a board about 10 feet long, called a **slope board,** or **slope rod,** is used with the clinometer. This board has one straight edge with a portion of the opposite edge parallel. The straight edge is placed on the sloping surface of the ground and the bar *a b* of the clinometer is placed on the opposite parallel edge of the board. It is sometimes convenient to attach permanently the clinometer to the slope board.

10. Abney Level and Clinometer.—As now commonly made, the clinometer is combined with the hand level, and the combined instrument is known as the **Abney level and clinometer.** Such an instrument is shown in Fig. 3. It is similar to an ordinary hand level in every way, except that the small spirit level on top of the main tube is movable in a vertical plane, so that when the tube is given any inclination, the level can be turned to a horizontal position, as indicated by the bubble, when the angle of the inclination of the tube with the horizontal is shown by means of a graduated arc attached to the instrument. The main tube is square in cross-section, and its lower side is straight, so that it can be applied to any surface. When the spirit level is set in a

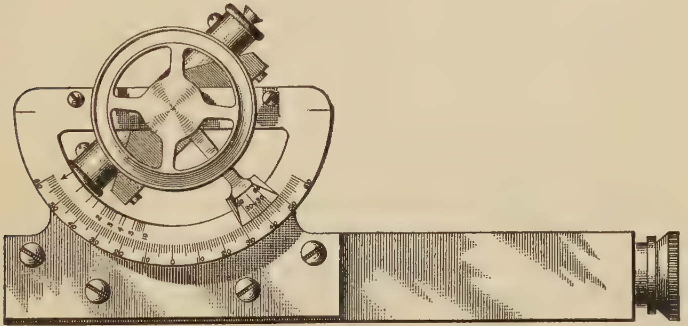


FIG. 3

position parallel to the main tube so as to read zero on the graduated arc, the bubble can be seen through the eyepiece, as in any hand level, and the instrument can be used as a hand level.

The graduations on the inner arc to the left of the center indicate the slope ratio, the left-hand edge of the vernier arm being used as an index. The extreme left-hand graduation indicates a slope of 1 to 1, the next $1\frac{1}{4}$ to 1, etc. up to 10 to 1.

11. Cross-Sectioning With the Clinometer.—If the simple form of clinometer is used, the angle of each slope of the cross-section is measured with it, and the horizontal distance with a tape. Thus, if the angle of a descending slope is found to be 5° and the horizontal distance is 90 feet, it is recorded $-\frac{5°}{90}$. Each of the different slopes of a cross-section is thus measured and recorded. If the Abney level and clinometer is used to determine the angle of the slope, it is directed to a point as far above the ground as the instrument is when held to the eye of the observer. This is readily done by sending an assistant up or down the slope and sighting about to the top of his head. While sighting thus on a line parallel to the surface of the ground, the level tube and the vernier arm, which is attached to it, are moved until the bubble is in the center of the tube. The angle is then read on the graduated arc, the vernier reading to 5 minutes. The advantage of this instrument is that it may be used as a hand level when differences of elevation are desired, as in slopes that are rough and steep, and as a clinometer when the slopes are long and flat. In the latter case, it is sometimes not even necessary to do any horizontal measuring, the angle of the slope determining the elevation for any desired distance.

12. Computing Slope Distances From Slope Angles.—Having determined the slope angle, the horizontal distance of the slope equals the difference of elevation between the top and the bottom of the slope multiplied by the natural cotangent of the slope angle. The difference of elevation between the top and the bottom of the slope equals the horizontal distance multiplied by the natural tangent of the slope angle. Thus, if the slope angle is 5°, the horizontal distance for a difference of elevation of 10 feet is 10 cot 5°, or $10 \times 11.4 = 114$ feet, and the difference of elevation in a horizontal distance of 100 feet is 100 tan 5°, or $100 \times .087 = 8.7$ feet. If a clinometer is used in determining slopes, it is sometimes convenient to prepare a table for field use giving the natural tangent and cotangent, correct to three figures, of angles up to 45°, each quarter degree up to 2°, each half degree up to 15°, and each degree up to 45°. Such a table can be inserted in the topographer's field book.

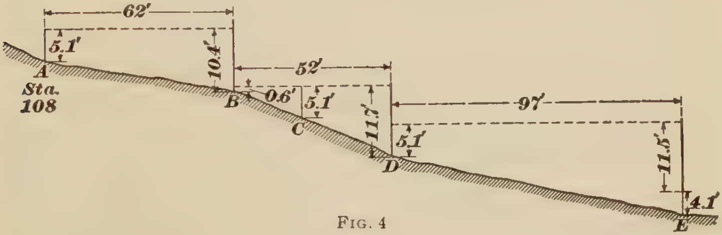


FIG. 4

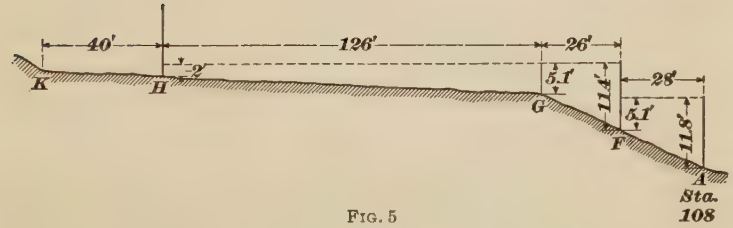


FIG. 5

13. Cross-Sectioning Illustrated.—Fig. 4 represents the **right slope** at Station 108 of a railroad survey; that is, the slope on the right-hand side of the survey line. Fig. 5 represents the **left slope**. The topographer, having determined that his eye is 5.1 feet above the ground, stands at the station and, by looking along the line of survey, generally both forwards and backwards, determines a line about at right angles to the survey line. The right slope, Fig. 4, is taken first. The rodman walks down the slope until he reaches *B*, where the slope changes. The rod is held at this point, and the topographer, by means of the hand level, finds that 10.4 feet on the rod is level with his eye. From this is deducted 5.1 feet, the height of his eye, and the remainder, 5.3 feet, is recorded as the difference in elevation between the points *A* and *B*. If there are tapemen in the party, they measure the horizontal distance from *A* to *B* while the reading is being taken. If not, the rodman takes one end of the tape while the topographer keeps the other and the measurement is made after the sight is taken on the rod. The distance from *A* to *B* is found to be 62 feet, and the slope, which is a descending one, is recorded $-\frac{5.3}{62}$.

The topographer then proceeds down the slope to *C*, where his eye is about level with the bottom of the rod at *B*. The rod reading on *B* is 0.6 foot. The rodman proceeds to *D*, where the slope again changes. The topographer turns around at *C* and obtains the rod reading on *D*, which is 11.7. The difference of these rod readings, $11.7 - 0.6 = 11.1$, is the difference in elevation between *B* and *D*. Since the elevation of point *C* is not desired, its location is not recorded. The distance from *B* to *D* is found to be 52 feet, and the second slope is recorded $-\frac{11.1}{52}$.

The topographer moves forwards to the point *D*, and the rodman holds the rod at *E*, the foot of the slope. The top of the rod is below the level line of sight from the topographer's eye, so the rodman "shins the rod," holding it against his body sufficiently high to be intersected by the level line of sight. The rod reading is found to be 11.5 feet. The rodman then measures with the rod the distance from the ground to the point on his body to which the bottom of the rod was raised. This distance, 4.1 feet, is called out to the topographer, who adds it to the rod reading and then deducts the height of the eye. The distance from *D* to *E* is found to be 97 feet. This slope is recorded $-\frac{10.5}{97}$.

Having previously determined to continue the cross-section for about 200 feet on each side of the line, no further sights are necessary, since the topographer sees from the notes that a distance of $62 + 52 + 97 = 211$ feet has already been covered by the sights taken. The party then returns to the station to determine the left slope. Since the slope is quite steep, the topographer decides, after having determined a line at right angles to the survey line, to proceed up the slope to *F* until his eye is about level with the top of the rod when held on the station. The rod reading is 11.8 feet and the distance from *A* to *F* is 28 feet. Deducting the height of the eye from the rod reading, the difference in elevation between *A* and *F* is found to be 6.7 feet. The rodman then holds the rod on *F* and the topographer goes to *G*, the top of the slope. The rod reading on *F* is 11.4 feet and the distance from *F* to *G* is 26 feet. Deducting the height of the eye, the elevation of *G* above *F* is found to be 6.3 feet. These two sights are $+\frac{6.7}{28}$ and $+\frac{6.3}{26}$. Since they are part of the same slope, they are sometimes added together and recorded thus, $+\frac{13.0}{54}$. The topographer, wishing to continue his line of sight at the same level, turns around and takes a reading on the rod at *H*, which is as far away as he can read the figures on the rod. The rod reading is 2.0 feet, and the distance from *G* to *H* is 126 feet. Since the height of the eye was 5.1 feet above the ground, this height minus the rod reading on *H* gives 3.1 feet as the difference of elevation between *G* and *H*. This is recorded $+\frac{3.1}{126}$.

The same slope continues to *K*, so the distance from *H* to *K* is measured and found to be 40 feet. This is added to the distance *GH* and is recorded with the slope as determined, $+\frac{3.1}{126}$ for 166 feet, which means that the ground slopes at the rate of 3.1 feet in 126 feet for a distance of 166 feet. The horizontal measurements on the left slope (28 + 26 + 126 + 40 = 220 feet) exceed the distance from the survey line that it is desired to cover. Therefore, no further sights are necessary, and the cross-section at Station 108 is completed.

At a station where the line of survey changes direction, the cross-section is generally taken on a line that bisects the angle formed by the two courses of the survey line.

14. Cross-Section Notes.—The field notes of cross-section work are usually kept in a regular transit book. A convenient form is shown on the following page, the left-hand page of the note book being shown. This page is usually ruled off in six columns. The numbers of the stations are recorded in the first column. In the second column is put the elevation of the station. The third and fourth columns are used for the cross-section notes to the left of the line; and the fifth and sixth columns, for the cross-section notes to the right of the line. For clearness, a heavy pencil line may be drawn over the third and the fifth vertical line of the book so as to mark the slope columns. It is sometimes preferred to record the elevations in the fourth column, leaving the second, third, fifth, and sixth columns for the cross-section notes. The right-hand page is used for sketches and general notes, such as the location and description of bench marks, the width and depth of streams and the direction of their currents, general character of the soil, timber, vegetation, buildings, and other important features. It is usually ruled and cross-ruled and has a red line down the center. The latter is used to represent the line of the survey in any sketches made on this page. The notes should begin at the bottom of the page so that when facing in the direction in which the survey line is being run, the right slope will be on the right-hand

Sta.	Elev.	Left	Slope	Right	Slope
109	112.1	$+\frac{4.7}{58}$	$+\frac{7.3}{91}+\frac{5.8}{70}$	$-\frac{2.4}{51}-\frac{8.9}{40}$	$-\frac{11.2}{85}$ for 140′
108	106.4	$+\frac{3.1}{126}$ for 166′	$+\frac{6.3}{26}+\frac{6.7}{28}$	$-\frac{5.3}{62}-\frac{11.1}{52}$	$-\frac{10.5}{97}$

side of the topographer and objects on either side of the line can be sketched in their natural positions with reference to the line.

15. Sketches and Eye Measurements.—In topographical surveying, the work can often be much facilitated by estimating certain unimportant distances by the eye. The distances thus estimated may be recorded by written notes or represented by means of sketches drawn freehand. Such notes, or sketches, as the case may be, are placed on the right-hand page of the notebook. After some practice, the topographer is able to estimate distances by the eye with some degree of approximation, though this method is not to be relied on when accurate results are required. In taking slopes, the length of the last one may usually be estimated by the eye. More distant objects, especially those that lie beyond the probable limits within which the location will be made, may be sketched in by the aid of the eye alone.

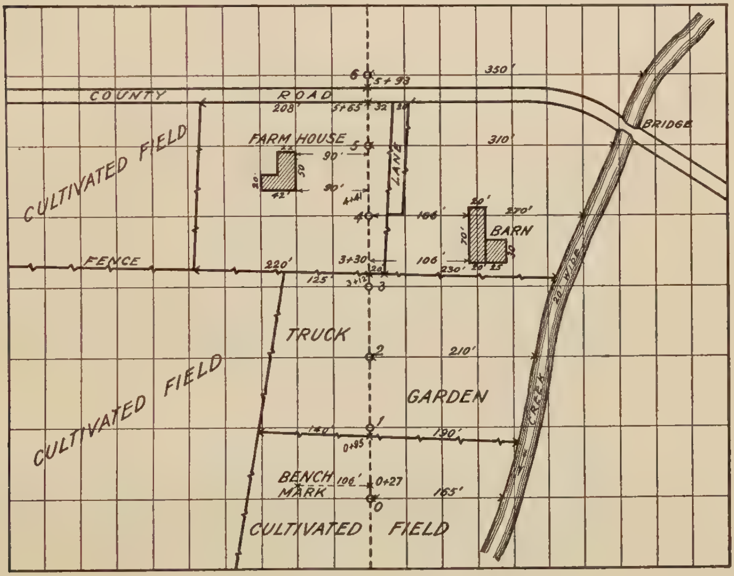


Fig. 6

Freehand sketches, when made intelligently, are of much value in topographical work, especially when supplemented by a few judicious measurements. The measurements are written in connection with the sketches and represent certain distances or dimensions accurately, and by the aid of these, other distances or dimensions can be sketched in with a reasonable degree of approximation. Such sketches frequently enable the topographical draftsman to plat correctly features concerning which he would be in doubt if they were recorded only by the written notes. Many topographical features can be represented by means of rough freehand sketches, with dimensions and distances marked, much more quickly and clearly than they can be recorded by means of written descriptions.

Fig. 6 represents a portion of the right-hand page of a topographer's notebook, on which is drawn a rough freehand sketch showing the topographical features of a farm through which the line of survey passes. This sketch, in connection with the regular notes written on the left-hand page of the notebook, gives very complete information regarding the locality. The center line of the page is assumed to represent the line of survey.

TOPOGRAPHY OF LIMITED AREAS

16. General Description of Method.—The method here described may be used for making the topographical survey of a limited area that is to be laid out, as a new town site, an addition to a city, a park, a cemetery, or is to be devoted to any other purpose requiring a knowledge of the topography of the surface.

If the boundaries are not clearly defined, the entire tract is first surveyed and its boundary lines determined and marked. Then, in order to determine the topography, the tract is usually divided, as far as possible, into squares or parallelograms of uniform size, whose sides may have any dimensions from about 25 to 100 feet or more, as the conditions may require. The form chosen for these will depend somewhat on the form of the tract, and their size will depend on the physical features of the ground and on the degree of accuracy required. The corners of the squares or parallelograms are either defined by stakes or are located by ranging out and measuring from stakes already set.

Fig. 7 is assumed to represent a tract of land that is to be surveyed for the purpose of determining the topography of its surface. It is assumed that a traverse survey locating the boundaries has already been made. The tract is to be divided by means of lines in two perpendicular directions, and 100 feet apart. In dividing tracts in this manner, it is customary to designate by letters the lines that extend in one direction, and by figures the lines at right angles to that direction. The point at the intersection of any two lines is then designated by the letter and figure of the respective intersecting lines. Thus, in Fig. 7, the lines perpendicular to the side decided on for a base are designated as lines *A*, *B*, *C*, etc., while the lines parallel to the base are designated as lines *1*, *2*, *3*, etc. The intersection of the line *D* and the line *1* is designated as *D 1*, the intersection of the line *E* and

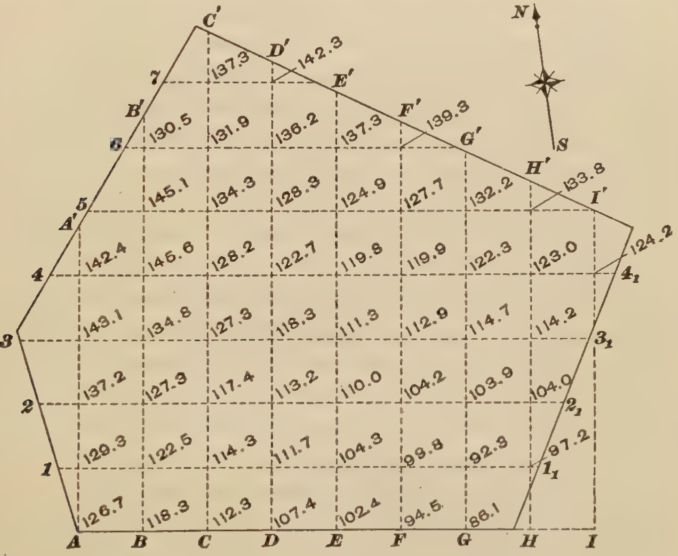


FIG. 7

the line 5 is designated as *E* 5, etc. The intersections of the boundary lines with the dividing lines are designated by the letters or numbers by which the latter lines are designated, affected with an accent or subscript, as *A*′, *B*′, 2_1, 3_1, etc. (See Fig. 7.)

17. Staking Out the Tract.—Different methods may be followed in laying out a tract. Any method is satisfactory that accurately defines the positions of the points of intersection, so that they can be readily located when the levels are being taken. For this purpose it is not usually necessary to mark all the points of intersection, but a sufficient number of points should be marked by stakes so that the remaining points can be located easily and quickly by merely ranging them in from the points that are marked. The rougher and more irregular the surface of a tract, the more stakes must be set, and a tract of irregular form usually requires a comparatively greater number of stakes than a tract of rectangular form. If the tract is rectangular in form and its boundary lines are complete and unbroken, so that the stakes can be set on each boundary line along the entire length of the side, and if the tract is comparatively level, so that the stakes on one boundary line can be seen from the corresponding stakes on the opposite boundary line, it will not usually be necessary to set stakes at the points on the interior of the tract. For, in taking the levels over such a tract, the leveling rod can be ranged in between the stakes on the boundary lines.

The surface of the tract represented in Fig. 7 is somewhat irregular, as shown by the elevations written at the intersections of the lines. For these conditions, the following method may be adopted in staking out the tract: The south side is taken as a base line, and stakes are set along this line at intervals of 100 feet, these stakes being marked *A*, *B*, *C*, etc. At each of these points lines are run across the tract at right angles to the base line, stakes being set at intervals of 100 feet. The base line is extended beyond the tract so that the lines *H* and *I* may be drawn. Each interior stake is marked by the number of intervals from, and the letter of, the stake from which the line is run. Thus, the stakes in the line run from the point *A* are marked *A 1*, *A 2*, *A 3*, etc. No cross-lines are run, the squares being completed in the sketches and on the map by drawing lines through *A 1*, *B 1*, *C 1*, etc. and *B 1*, *B 2*, *B 3*, etc. After the stakes at *A 1* and *B 1* have been set, the stake at *1* is put in line with them. Likewise, the stake at *2* is put in line with *A 2* and *B 2*, and so on, locating the stakes that are not at the corners of the squares. Care should be taken to have the stakes, as shown by the sketch in the notebook, numbered in the same manner as on the ground. If the surface of the tract were comparatively level, or of uniform slope, so that the stake on any line could be seen from the corresponding stake on the opposite line, these would be the only stakes necessary to be set, since all the remaining points of intersection could be located by ranging them in from these stakes with sufficient accuracy for the purpose of taking the levels.

18. Taking the Levels.—After the required number of stakes have been set, the levels are taken over the tract, determining the elevations at all points of intersection and at any intermediate points where the slope changes abruptly. Such an intermediate point is generally located in a direct line between two intersections by its distance from the intersection having the lower letter or number. This distance is measured with a tape, approximated by pacing, or merely estimated by the eye, according to the conditions and to the degree of accuracy required, and is recorded as a plus. The tops of knolls and the low points of depressions are generally located when they are not crossed by the line between two intersections.

The levels should be taken in such order as is advantageous, which will depend on the nature of the ground, the object being to take rod readings at each of the intersections and other points with as few settings of the level as possible. To be sure that rod readings are taken at all the intersections, those taken from each setting are checked off on the sketch or sketches in which they are all shown.

19. Form of Notes.—The notes are substantially the same as ordinary level notes, with the addition on the right-hand page of sketches, showing the form and dimensions of the tract surveyed, the manner in which it is divided, and the method of numbering the stations. Each station is designated by its letter and number, since the levels are not usually taken successively along one line.

TOPOGRAPHY BY TRANSIT AND STADIA

20. Survey of a Large Area.—In making the survey of a large area by stadia measurement, it is customary, before beginning the stadia work proper, to establish a number of controlling points whose positions with reference to each other are determined and from which the various stadia lines can be begun. The azimuths and lengths of the lines joining these established points are determined and a line of levels is run to ascertain the elevations of these points with reference to a common datum. The tract is generally divided into a series of triangles, the angles and sides of which are carefully measured, or computed, thus forming an accurate framework on which to build the map by filling in the details determined by stadia measurement.

This framework sometimes consists of two or more random lines carefully run through the tract, with tie-lines connecting them at convenient points.

21. Survey of a Small Area.—In the survey of a small tract, it is not usually necessary to establish a framework for the map. The magnetic bearing of any line of the survey can be observed, the azimuth of the line calculated from its bearing, and this azimuth taken as a basis from which to determine the azimuths of the other lines of the survey.

Fig. 8 represents a small tract to be surveyed. It is assumed that the magnetic bearing of that part of Mead

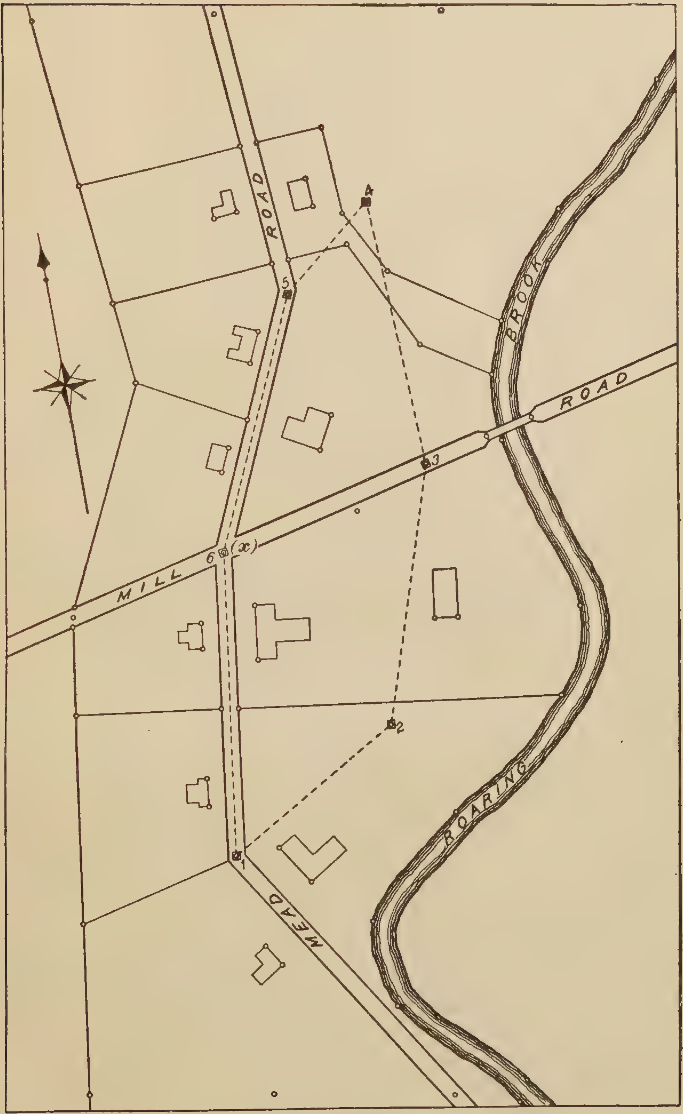


FIG. 8

Road, south of Mill Road, as observed from the angle in Mead Road, is found to be N 7° 30′ E. It is also assumed that from the records of another survey, the elevation of the center of Mill and Mead Roads at their intersection is known to be 173.2 feet. It is required to locate the brook forming the eastern boundary of the tract, the roads, fences, and buildings, and also to determine the elevations of a sufficient number of points to show the topography of the tract.

The party consists of an observer, a recorder, and two rodmen. The survey is conducted as follows: The transit is first set up over a point in the center of Mead Road at the angle south of Mill Road, which point is Station 1 of the survey, and is so marked in the notes and figures. It is decided to close the survey at the intersection of the center lines of Mead and Mill Roads, and, since the number of intervening stations is not yet known, this point is temporarily marked x, as shown in the notes and sketches following this description.

Considering the azimuth to be measured from the north point clockwise, the azimuth of the center line of Mead Road is 7° 30′, since the bearing of this line is N 7° 30′ E. The vernier is set at 7° 30′, the telescope is directed to the rod held on the point x, and the lower clamp is set. By this operation, the transit is oriented on the center line of Mead Road. The stadia reading is taken and called out to the recorder. The vertical angle is read and called out. The plate clamp is then loosened and the telescope is directed to the stadia rod held successively on the points to be located. The stadia rod, vertical angle, and azimuth are read and recorded for each sight. These sights, called **side shots,** are preferably taken in succession by turning to the right, but this plan cannot usually be adhered to, as it is desirable that the rodmen shall cover the ground with as little unnecessary walking as possible. The points to which side shots are taken are commonly indicated in the notes and sketches by the small letters of the alphabet, beginning with a at each instrument point. Thus, side shots a and b are taken to two adjacent corners of the tannery (see notes and sketches on pages

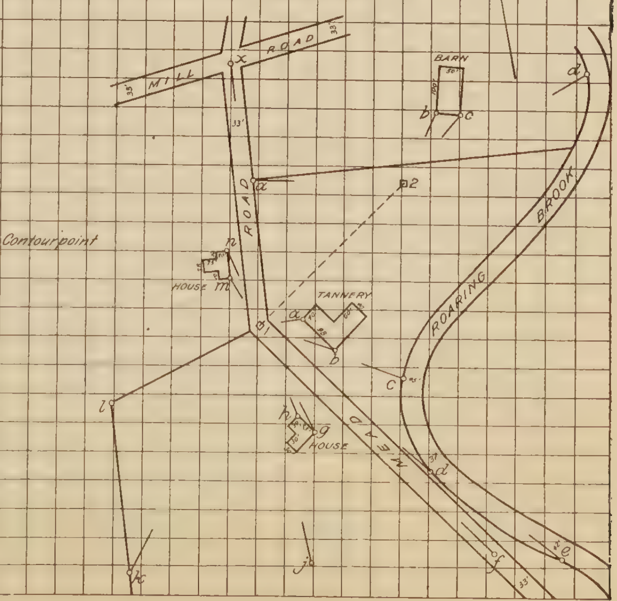
22 to 25), *c*, *d*, and *e*, to points on the edge of Roaring Brook; *f*, to the center of Mead Road; *g* and *h*, to corners of the house south of the station occupied; and *j*, to a point where the slope of the ground changes. Such a point, located merely to show the elevation of the ground, is commonly called a **contour point** (marked *c. p.*), as it is used only to determine the contours of the tract, as will be explained later. Contour points are located when, in the judgment of the observer, the slope of the ground is not sufficiently well shown by the elevation of the other points that have been located. Side shots *k* and *l* are taken to locate the fence line, and *m* and *n* to locate the house north of Station 1.

All the desired points adjacent to Station 1 having been located, one of the rodmen selects a position suitable for Station 2. Before sighting to this point, the observer sets the vernier again at 7° 30′ and checks the position of the instrument by sighting again on the rod held at point *x*. If it is found that the instrument has moved slightly out of position, the telescope is again directed to the rod by means of the lower tangent screw. The upper plate is then unclamped and the telescope is directed to the rod held on the point selected for Station 2. The stadia reading is taken and the vertical angle and the azimuth are read. The latter is found to be 60°. The transit is then set up at Station 2 and oriented in azimuth by taking a backsight on Station 1. The vertical angle and stadia reading from Station 2 to Station 1 are taken as a check. Thus, the stadia reading from Station 1 to Station 2 is 4.20 and the vertical angle is 1° 33′, whereas, on the backsight, the stadia reading is 4.22 and the vertical angle is 1° 37′. The means between the two stadia readings and the two vertical angles are generally taken as the correct readings. Side shots to points adjacent to Station 2 are taken, and Station 3 is then located. In a similar manner, Stations 4 and 5 and points adjacent to them are located. When the instrument is at Station 3, it is noticed that the point *x* is plainly visible, so a stadia reading is taken on it. The distance thus obtained is used as a check on the

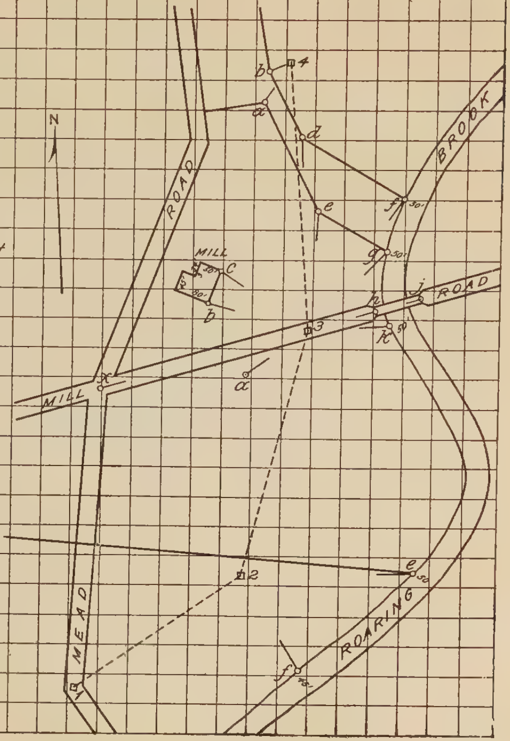
Stadia Survey of Meadville Suburb
May 27th 1901

Observer-J.R.Gavin
Recorder-Gonaver
Rodmen — Griffith
Steele

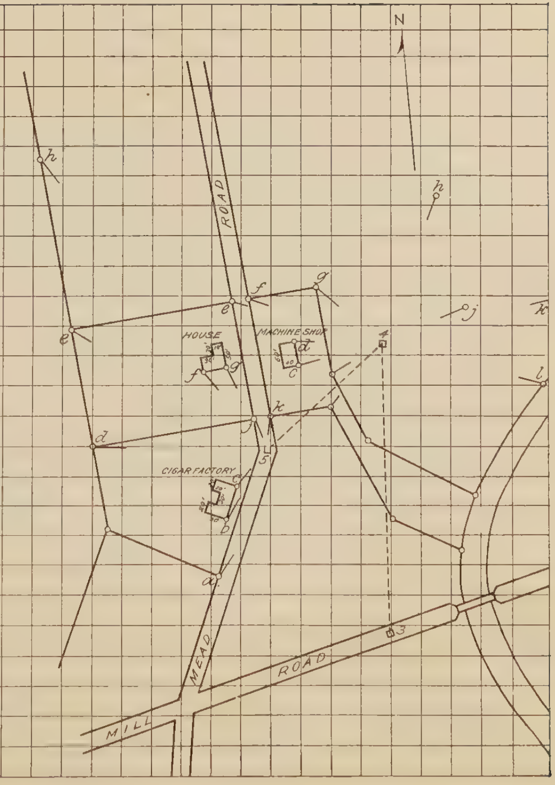
Sta.	Azimuth	Stadia	Vertical Angle	Horiz. Distance	Elevation	
	Readings	from ⊡1,	Elev. 177.42.			⊡1 Angle in Center Mead Road, South of Mill Road.
x	7° 30′	6.30	− 0° 23′	631	173.2	= Center Mill & Mead Roads as per Wilson's Survey
a	90° 43′	.91	− 2° 04′	92	174.1	
b	120° 18′	1.66	− 2° 12′	167	171.0	
c	126° 31′	3.15	− 2° 06′	316	165.8	
d	143° 43′	4.60	− 1° 25′	461	166.0	
e	141° 32′	7.85	− 0° 38′	786	168.7	
f	148° 04′	6.74	− 0° 34′	675	170.7	
g	167° 19′	2.47	− 0° 50′	248	173.8	
h	172° 22′	1.97	− 1° 20′	198	172.8	
j	181° 17′	4.99	+ 0° 12′	500	179.2	Contour point
k	221° 45′	5.79	+ 1° 02′	580	187.8	
l	256° 01′	3.47	+ 1° 50′	348	188.5	
m	342° 03′	1.17	+ 1° 16′	118	180.0	
n	350° 16′	1.71	+ 0° 52′	172	180.0	
⊡2	60° 00′	4.20	− 1° 33′	M = 422	165.77	
	Readings	from ⊡2,	Elev 165.77			
⊡1		4.22	+ 1° 37′			
a	286° 01′	3.21	+ 2° 01′	322	177.1	
b	32° 02′	2.36	− 0° 48′	237	162.5	
c	41° 49′	2.60	− 1° 03′	261	161.0	
d	68° 32′	4.61	− 1° 22′	462	154.8	



Sta	Azimuth	Stadia	Vertical Angle	Hor. Distance	Elevation	
	Readings from ⊡2 (cont.) Elev. = 165.77.					
e	90° 45′	3.59	− 1° 18′	360	157.6	
f	154° 53′	2.28	− 0° 54′	229	162.2	
⊡3	17° 30′	5.47	+ 0° 17′	M = 545	168.61	
	Readings from ⊡3, Elev. = 168.61.					
⊡2		5.42	− 0° 19′			
a	245° 30′	1.73	− 1° 53′	174	162.9	Contour Point
x	256° 45′	4.58	+ 0° 40′	459		Tie Line
b	287° 43′	2.19	− 1° 16′	220	163.7	
c	307° 24′	2.21	− 1° 41′	222	162.1	
d	359° 04′	4.06	− 1° 26′	407	158.4	
e	7° 33′	2.49	− 2° 50′	249	156.3	
f	38° 21′	3.39	− 3° 07′	339	150.1	
g	47° 20′	2.31	− 4° 18′	231	151.2	
j	76° 45′	2.39	− 0° 20′	240	167.2	
h	76° 46′	1.39	− 0° 34′	140	167.2	
k	82° 52′	1.66	− 5° 41′	165	152.9	
⊡4	357° 35′	5.61	− 0° 46′	M = 559	161.29	
	Readings from ⊡4, Elev. = 161.29					
⊡3		5.55	+ 0° 44′			
a	215° 41′	0.96		97	161.3	
b	255° 10′	0.53	+ 1° 13′	54	162.5	

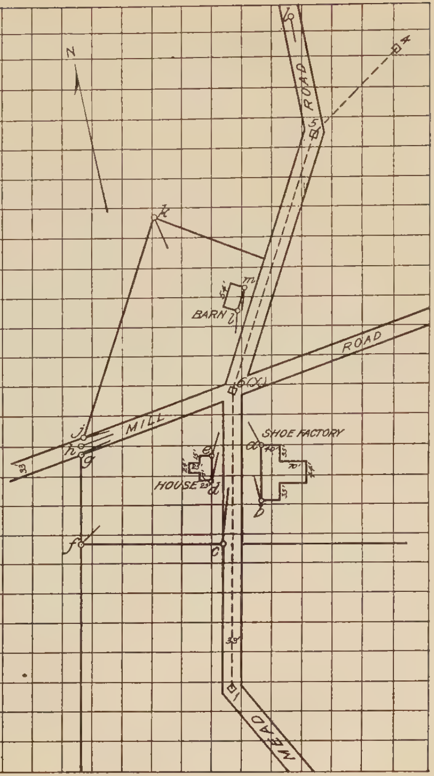


Sta.	Azimuth	Stadia	Vertical Angle	Horiz. Distance	Elevation
	Readings	from ⊡4,	(cont.) Elev. =	161.29	
c	275° 17′	1.09	+ 1° 19′	110	163.8
d	301° 11′	1.34	+ 0° 54′	135	163.4
e	303° 46′	2.84	+ 0° 47′	285	165.2
f	308° 36′	2.57	+ 0° 43′	258	164.5
g	339° 25′	1.79	+ 0° 08′	180	161.7
h	32° 19′	4.29	− 1° 24′	430	150.8
j	70° 03′	2.67	− 1° 38′	268	153.7
k	77° 32′	6.61	− 1° 21′	662	145.7
l	102° 12′	4.03	− 1° 54′	404	147.9
⊡5	230° 54′	2.53	+ 0° 44′	M = 252	164.51
	Readings	from ⊡5,	Elev. = 164.51		
⊡4		2.49	− 0° 45′		
a	208° 02′	2.73	+ 0° 28′	274	166.7
b	218° 53′	1.63	+ 0° 38′	164	166.3
c	228° 02′	0.96	+ 0° 53′	97	166.0
d	277° 15′	3.68	+ 1° 59′	369	177.3
e	307° 47′	4.87	+ 1° 04′	488	173.6
f	326° 20′	2.18	+ 1° 02′	219	168.5
g	338° 43′	1.99	+ 0° 48′	200	167.3
h	327° 47′	7.71	− 0° 15′	772	161.1
j	343° 48′	0.71	+ 0° 19′	72	164.9
k	13° 31′	0.72	− 0° 34′	73	163.8



Sta.	Azimuth	Stadia	Vertical Angle	Horiz. Distance	Elevation
	Readings from ⊡5, (cont.) Elev. = 164.51				
l	355° 45′	6.02	+ 0° 27′	603	169.2
x=⊡6					173.35
	Readings from ⊡6, Elev. = 173.20				
⊡5		5.55	− 0° 59′		
a	159° 12′	1.27	− 0° 14′	128	172.7
b	172° 20′	2.34	+ 0° 22′	235	174.7
c	190° 14′	3.22	+ 0° 53′	323	178.2
d	201° 46′	2.02	+ 1° 12′	203	177.5
e	207° 24′	1.51	+ 1° 13′	152	176.4
f	232° 18′	4.52	+ 1° 25′	453	184.4
g	254° 03′	3.44	+ 1° 18′	345	181.0
h	256° 45′	3.39	+ 0° 50′	340	178.1
j	260° 02′	3.29	+ 1° 14′	330	180.3
k	342° 47′	3.96	+ 0° 04′	397	173.7
l	10° 14′	1.65	− 1° 07′	166	170.0
m	13° 11′	2.18	− 1° 07′	219	168.9
⊡1	187° 30′	6.27	+ 0° 25′		
	Survey finished May 28th 1901				
	Stadia Readings reduced by Gonaver—June 1st 1901				
	Notes platted by Griffith — June 1st 1901				

Should be 173.20
as per Wilson's Survey
= Original B.M.



platting of the survey. The last point sighted from Station 5 is the point x at the intersection of Mead and Mill Roads which was the first point sighted from Station 1 and now becomes Station 6. When the transit is set up and oriented at this station, the azimuth readings on all the instrument points can be checked by sighting to Station 1. If the field work has been performed accurately, the azimuth reading for this sight should be $7° 30' + 180° = 187° 30'$. As the survey progresses, the recorder makes sketches showing the lines run, the objects located, etc., being careful to designate each point in the sketches by the same letter that is used for the same point in the first column of the notes. In the location of a building, two adjacent corners are generally located by stadia measurements, and the sides are measured by the rodmen or by the recorder and one rodman. The complete notes and sketches of this survey are shown in the preceding pages. The platting of these notes is explained in *Mapping*, Part 2.

TOPOGRAPHY BY PLANE TABLE

22. Survey of a Large Area.—The tract is generally divided into a system of triangles called the **control.** The vertexes of these triangles are called **controlling stations.** The triangles are carefully located by the transit and the chain or tape, while a line of levels is run to determine the elevations of the controlling stations. After they have been established and platted, generally on a number of plane-table sheets, additional points within the triangles are determined by some of the plane-table methods already explained.

From these points, which are so selected as to obtain the best outlook, the topographical features adjacent to each are located by stadia measurement and at once platted. The difference of elevation between the station occupied and each point observed is determined by means of the vertical angle and the Stadia Reduction Table. The elevation of the observed point is then obtained by adding or subtracting the difference of elevation to the elevation of the station occupied, as explained in *Stadia and Plane-Table Surveying*.

The elevation of each point located is marked on the map as soon as determined, and the topographer, with the ground before him, is able to determine if a sufficient number of points have been located to represent the slope of the ground with the degree of accuracy desired for the map. He is also able to see from the map if his locations from the station occupied cover the ground sufficiently close to the area previously mapped. Random lines are sometimes run through heavily wooded sections of country where the surface is not visible from any of the principal points.

23. Survey of a Small Area.—A survey of a small area is made with the plane table in a similar manner to that used for a small stadia survey, no preliminary triangulation being necessary unless a great degree of accuracy is desired. Traversing with the plane table is sometimes done to locate roads, streams, and other features, or to locate and determine the elevation of points in heavily wooded country where the surface is not visible from the principal stations.

CONTOURS

LOCATION OF CONTOURS

24. Definition.—A **contour** is a line that connects all points having the same elevation. It may be also defined as the intersection of the surface of the ground with a horizontal plane.

25. Contour Intervals.—The vertical distance between two adjacent contours is called a **contour interval.** This distance is generally the same for each map, but varies with the degree of accuracy with which it is desired to show the surface. If it is desired to represent the surface accurately, a small interval, such as 2 feet, 5 feet, or 10 feet, is used. The interval customarily used in mapping by the United States Coast and Geodetic Survey is 20 feet. In mountains or very rough country, where sketching is largely relied on, an interval of 100 feet is usually sufficiently close to show all necessary features.

Contours are generally taken at elevations that are multiples of the interval. Thus, 10-foot contours are taken at elevations of 10, 20, 30, 40 feet, etc., and 5-foot contours at elevations of 5, 10, 15, 20 feet, etc.

26. Determining the Contours.—The contours between two adjacent points whose elevations have been determined are generally platted as a part of the office work, as will be explained in *Mapping*, Part 2. Since the slope of the surface is assumed to be about uniform from each point to the one adjacent in each direction, it is necessary to locate a sufficient number of points to show the surface with a degree of accuracy sufficient for the purpose and in harmony with the scale of the map.

27. Sketching the Contours.—This method is used as an auxiliary to the location of points when the contours are platted in the field and is especially advantageous when the map is a rough one and is hastily made. Sketching is an important part of the topographer's work and a difficult one in which to acquire proficiency. It affords a wide scope for the exercise of judgment and skill, and its accuracy depends on the ability to see the principal features of the country and to represent them on paper so that they give the same impression as is given by the natural features.

28. Direct Location of Contours.—It is sometimes advantageous to determine on the ground the location of each contour by locating a sufficient number of points of equal elevation to properly represent the contour when platted. The number of points located depends on the roughness of the country and the accuracy with which it is desired to represent the topographical features. The plane table is most commonly used in the direct location of contours, and it is generally supplemented by a **Y** level. The transit with stadia wires is sometimes used. The process with plane table and level is as follows: Fig. 9 represents a portion of a tract of ground on which the contours at intervals of 20 feet are to be located directly. The points *A*, *B*, and *C* have been previously located and their elevations have been determined, as shown in the figure. The plane table is set up and oriented at a point *P*, from which is visible as much as possible of the ground to be covered, and the station occupied is platted on the map. It is decided to locate contour *100*; so the level is set up above this elevation at a point of good outlook. A backsight reading of 4.2 is taken on station *A* and the height of instrument is thus found to be 97.5 + 4.2 = 101.7. All points in contour *100* are 101.7 − 100 = 1.7 feet lower than the level line of sight. The target is set at 1.7 feet on the rod and the contour is then traced on the ground as follows: The first point *1* of the contour is determined by moving the rod up or down the slope until the target is intersected by the line of sight. This point is then sighted on from the plane table and a stadia reading taken. The line of sight is platted lightly and the distance is laid off on it to the scale of the map. The rodman then determines about where it is necessary to locate another point to properly show the contour. Here he again

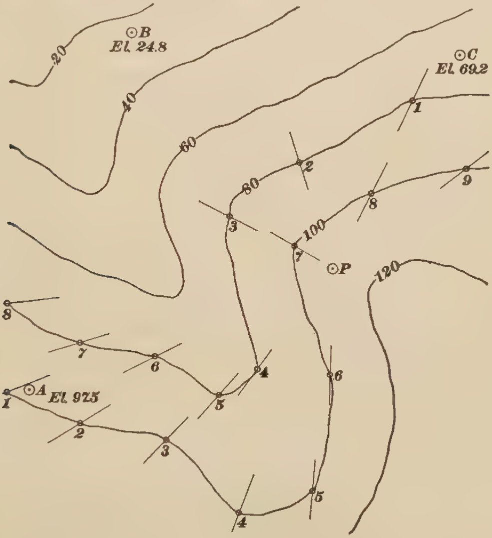


Fig. 9

moves up or down hill until he reaches a point where the target is cut by the line of sight. This point *2* is located by the plane table, in a manner similar to that used for locating the first point. Points *3*, *4*, *5*, *6*, and *7* are located in the same manner. After point *7* is located, it is noted by the levelman that his position will have to be changed to trace the contour further, so he moves to another position beyond the last point located, takes a backsight on point *7*, and proceeds to locate the other points as before. Each contour point when platted is marked lightly with its elevation and not with numbers *1*, *2*, *3*, etc. When a number of points of equal elevation have been platted, the topographer delineates the contour by drawing a continuous line through the contour points, curving it where it appears to change its direction. No more points on contour *100* being visible from the plane table, the leveler sets a temporary turning point by a foresight of 11.8, the elevation of which is 102.3 – 11.8 = 90.5, and again by a backsight of 1.4 and a foresight of 10.9, a temporary turning point is set whose elevation is 90.5 + 1.4 – 10.9 = 81.0. Selecting a point with a good outlook, the level is set up and a backsight of 1.8 gives a height of instrument of 81 + 1.8 = 82.8. The target on the rod is then set at 2.8 and points *1*, *2*, *3*, *4*, *5*, *6*, and *7* are located on contour *80*. In a similar manner portions of contours *60*, *40*, and *20* are platted, after which another position is chosen for the plane table and the contours are extended still farther over the tract. The farthest point located on each contour in the direction the contours are being extended is generally marked with a temporary stake, and this point is sighted on first from the next succeeding position of the plane table. Thus, point *9* on contour *100*, and point *8* on contour *80* will each be marked with a stake. It is often of advantage to have a stadia rodman who holds a rod on the contour point when determined by the leveler and his rodman. While the topographer is taking the reading, the level rodman is selecting the next contour point. If the contour intervals are 5 feet, two rodmen are sometimes employed, one tracing one contour and the other the next one above; the leveler generally determines points in two contours from the same setting of the instrument, the topographer alternately locating points in each contour.

29. Contours in Cross-Sectioning.—In cross-sectioning, 5-foot contours are sometimes located directly with the hand level or rod at a point 5 feet above the ground. The process is as follows: Fig. 10 represents a cross-section at Station 102 in a survey. The elevation of the station is 177.2 feet, and it is desired to locate 5-foot contours for 200 feet on each side of the station.

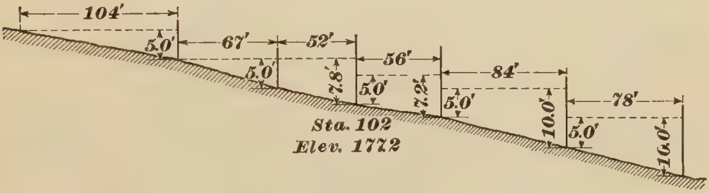


FIG. 10

The topographer, standing at the station, determines by the eye the right-angle line on which the cross-section is to be taken. The elevation of the eye, when the hand-level staff is held at the station, is 177.2 + 5.0 = 182.2. The first contour below the station is 175 and a point at this elevation is determined by the rodman moving the rod down the slope until the level line of sight intersects the rod at 182.2 − 175 = 7.2. This point is then located by measurement from the station and found to be 56 feet. It is recorded thus: $\frac{175}{56}$. The topographer then moves to the point just located, determines a point in contour *170* in substantially the same manner as explained for contour *175*, and so on.

CONTOUR LINES

30. Description.—In representing topography by means of a map, the area surveyed is conceived to be projected on a horizontal plane, represented by the plane of the paper, on which the inequalities of surface, important natural features and other conspicuous objects are shown in their true relative positions. The simplest method and the one most generally employed for representing the topography of a given surface is by means of lines joining points of equal elevation, called **contour lines,** or simply **contours.** A map containing the outlines of a given area or tract, together with the contour lines showing the relative elevations of different portions of its surface, is called a **contour map** of that surface.

31. Example of Contour Lines.—The manner in which the relative elevations of different portions of a given surface can be represented by contour lines is illustrated in Fig. 11. Let *M P N O* represent the top view of a hill projected on a horizontal surface, and let *A B C* represent a vertical section of the hill on the line *M N*, and *D E F* represent a vertical section on the line *O P*. In the horizontal outline *M P N O*, the lines *a a*, *b b*, *c c*, *d d*, and *e e* may represent imaginary lines drawn around the hill through points of equal elevation; these lines are assumed to be at intervals 10 feet apart vertically. Suppose that this hill is surrounded by water, the surface of which reaches just to the line *a a*.

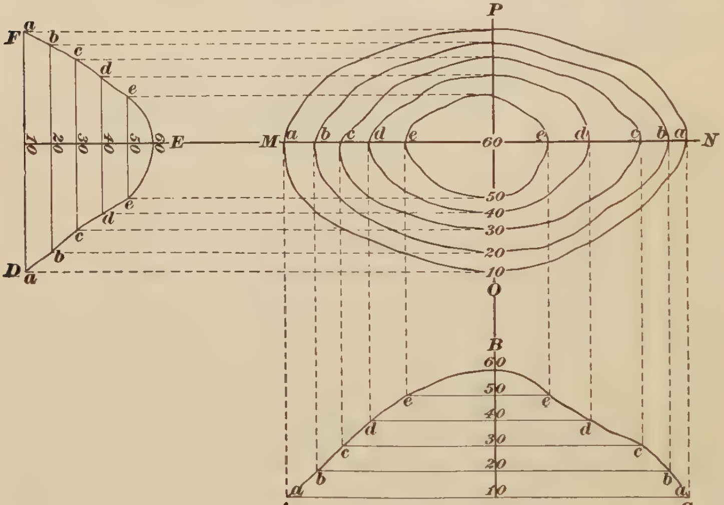


FIG. 11

Since this line passes around the hill through points of equal elevation, and since the surface of the water is level, and consequently has the same elevation at all points, if the surface of the water reaches the line *a a* at any point, it will just touch it at all points around the hill, so that this line will represent the flow line or shore line of the water. In the vertical section *A B C*, the elevation of the surface of the water will be represented by the line *a a*; it will also be represented by the corresponding line *a a* in the vertical section *D E F*. Now, suppose the hill to be gradually submerged in water by the water rising in successive heights of 10 feet, corresponding to the vertical intervals between the lines. At each successive rise of the water, its flow line, or shore line around the hill, corresponds to one of these lines. The horizontal lines shown in the vertical sections *A B C* and *D E F* correspond, respectively, to the positions of the surface of the water when at the successive elevations. Thus, when the water has risen to the line *b b* in the horizontal plan *M P N O*, this line will represent the shore line of the water around the hill. The horizontal line *b b* in the vertical section *A B C* and the corresponding horizontal line *b b* in the vertical section *D E F* are lines in these respective sections that represent the elevation of the surface of the water at such a stage.

Each of the lines *a a*, *b b*, *c c*, *d d*, and *e e* that represent the shore line of the water around the hill at the successive stages is evidently a line through points of equal elevation and therefore corresponds to a contour line. These contour lines can be constructed from the vertical sections, or profiles, in the following manner: The points *a*, *b*, *c*, *d*, and *e*, where the respective horizontal lines intersect the surface of the hill in the vertical section *A B C*, are projected on the line *M N*, and the corresponding points in the vertical section *D E F* are also projected on the line *O P*. The corresponding points thus projected on *M N* and *O P* are connected by continuous lines, as shown in the figure; these lines are lines of equal elevation, or contour lines. In order to determine the position of the contour lines and the approximate form of the hill with a reasonable degree of accuracy and completeness, the elevations of the surface should be taken on a sufficient number of sections across the hill so that the contour lines can be drawn with the required degree of accuracy. The two vertical sections represented in the figure are sufficient for the purpose of illustration, but in actual practice a greater number will usually be required.